Syllabus Semester 1, 2019 (Last update: February 20, 2019)

SUBJECT

Measure Theory (MAST90012)

Meeting times:

All *lectures* are in the Evans Williams theatre, at the following times:
Tuesdays at 14h15, and Wednesdays and Fridays at 9h00.
I have *office hours* in Peter Hall 204, at these times:
Tuesdays 16h-17h, and Wednesdays 14h-15h; or Fridays 14h-15h (upon request)

CONTACT

Course coordinator:

Volker Schlue, *Office:* Peter Hall 204, *Email:* `volker.schlue@unimelb.edu.au`

ASSESSMENT

Assessment will consist of two written assignments worth `20%` each, and a written examination worth `60%` of the final mark.

Description:
In addition to being made available on LMS, the problems and exercises that make up the assignments will be mentioned and handed out in class. This gives you the opportunity to start working on the assignments early. Starred problems ($*$) are optional, and do not contribute to the final mark.

Assignment 1 – *20%*

- *Due:* Wednesday, April 17 (Week 7)

Assignment 2 – *20%*

- *Due:* Wednesday, May 29 (Week 12)

Examination – *60%*

- *Date:* in the examination period.

PROGRAM

Description:

Measure Theory is at the foundation of modern Analysis. This course covers a range of topics, from the basic construction of the Lebesgue measure, to abstract measure theory with applications to ergodic theory and spectral theory.

Outline:

Basic Measure Theory, Integration & Differentiation: Exterior measure; Lebesgue measure; Measurable functions; Brunn-Minkowski inequality. — Lebesgue integral; the space L^1; Fubini's theorem. — Lebesgue differentiation theorem; Hardy-Littlewood maximal function; Bounded variation functions; Absolutely continuous functions; Rectifiable curves and isoperimetric inequality.

Hilbert spaces: the space L^2; orthogonal projections; Riesz representation theorem; adjoints.

Abstract Measure Theory: Carathéodory's theorem; metric exterior measures. — Integration on a measure space; product measures; Borel measures; Lebesgue-Stieltjes integral. — Signed measures; absolute continuity of measures; Radon-Nikodym theorem.

Applications: Ergodic theory; mean ergodic theorem; pointwise ergodic theorem; measure-preserving transformations — Spectral theorem; positive operators; spectrum.

Additional topics: Hausdorff Measure; Fractals; Hausdorff dimension; space-filling curves.

Prerequisites:

Group Theory and Linear Algebra (MAST20022); Metric and Hilbert spaces (MAST30026).

Course Material

Recommended reading:

Elias M. Stein & Rami Shakarchi, **Real Analysis**: Measure Theory, Integration, and Hilbert Spaces, *Princeton Lectures in Analysis*, III, Princeton University Press (2005).

Further reading:

Terence Tao, **An Introduction to Measure Theory**, *Graduate Studies in Mathematics*, Volume 126, AMS (2010).

Donald L. Cohn, **Measure Theory**, Birkhäuser (1980).