LOCALISATION DE $\mathfrak{g}$-MODULES

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Abstract. Notes on the equivalence between $\mathfrak{g}$-modules and D-modules on the flag variety.

1. Important results

Notation: $\mathcal{M}$ is a D-module on the flag variety X (we include the twist, and assume the twisting parameter satisfies a dominance condition).

Lemma 1.1. *Let μ be a dominant weight. The canonical quotient map from $\mathcal{M} \otimes V_\mu$ to $\mathcal{M}(\mu)$ splits in the category of sheaves on X.*

Proof. An important central character computation that should be done. □

Lemma 1.2. *If $\mathcal{M} \neq 0$, then $\Gamma(X, \mathcal{M}) \neq 0$*

Proof. Since $\mathcal{M}$ is quasicoherent, it contains a non-zero coherent submodule $\mathcal{N}$. Since $\mathcal{N}$ is coherent, there exists an ample μ such that $\Gamma(X, \mathcal{N}(\mu)) \neq 0$. Therefore $\Gamma(X, \mathcal{M}(\mu)) \neq 0$. By Lemma 1.1, we get $\Gamma(X, \mathcal{M} \otimes V_\mu) = \Gamma(X, \mathcal{M}) \otimes V_\mu \neq 0$, as required. □

References

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