Nov 12, 2019

Kari V

Preservation of depth

Let T/F torus splits over a tamely ramified finite extension F'/F F: p-adic field

$$T' = \mathrm{Res}_{F'/F}(T \otimes_F F')$$

$$\begin{array}{c} T' \twoheadrightarrow T \\ \| \\ \bigoplus \mathrm{Res}_{F'/F}(\mathbb{G}_m) \end{array}$$

$T(F)$ carries a Moy-Prasad filtration $T(F)_r$

For $T'(F) = (F')^*$ the M-P filtr.

$$T'(F)_0 = \mathcal{O}_{F'}^\times \quad T'(F)_r = \{ x \in (F')^* / \mathrm{val}_F(x-1) \geq r \}$$

$r>0$

val_F : val on $(F')^*$ s.t $\mathrm{val}(F^\times) = \mathbb{Z}$

Now $T(F)_r = \mathrm{Im}(T'(F)_r)$

$\chi : T(F) \to \mathbb{C}^\times$

$$\mathrm{depth}(\chi) = \inf\{r \geq 0 \mid T(F)_s < \ker(\chi) \text{ for all } s > r\}$$

The Weil gp carries a filtration by upper indexing.

$\to \varphi \in H^1(W_F, \check{T})$

$$\mathrm{depth}\,\varphi = \inf\{r \geq 0 \mid W_F^s < \ker\varphi \quad \forall s > r\}$$

<u>Class field theory</u>

$$\begin{array}{ccc} F^\times & \simeq & W_F^{ab} \\ \cup & & \cup \\ U_F^r & \cong & (W_F^r)^{ab} \\ \parallel & & \\ 1 + \pi_F^r \mathcal{O}_F & & \end{array}$$

<u>Thm</u> The LLC for tori split over tamely ramified ext preserves depth.

Easy to redue to $T' = \mathrm{Res}_{F'/F}(\mathbb{G}_m)$

L/F Galois ext of local fields. $G = \mathrm{Gal}(L/F)$

$x \in G_s$ iff $x|_{\mathcal{O}_L/\mathfrak{m}_L^{s+1}} = 1$

The lower indexing is functorial in subs.

L/F' Galois

$$1 \to \underset{\tilde{G}}{\underset{\parallel}{\mathrm{Gal}(L/F')}} \to \mathrm{Gal}(L/F) \to \underset{G_{-1}}{\underset{\parallel}{\mathrm{Gal}(F'/F)}} \to 1$$

$$G_{-1} \supset G_0 \supset G_1 = 0$$

$$G^u \xrightarrow{u>0} G_1 = 0$$

$$\tilde{G}_u \xrightarrow{\simeq} G_u \to 0$$

$$\tilde{\varphi}(u) = e\varphi(u)$$

$$e = |G_0|$$

$$\Rightarrow \tilde{G}^{eu} = G^u \quad u > 0$$

$u > 0 \qquad G^{\varphi(u)} = G_u$

$$\varphi(u) = \frac{1}{g_0}\left(g_1 + \cdots g_m + (u-m)\, g_{m+1}\right) \qquad m \le u \le m+1$$

$$g_i = |G_i|$$

Upper indexing is funtorial in quotients.

(lower indexing is in terms of L

upper ——————— " — F)

<u>LLC</u> $\quad H^1(W_F, \check{T}) = H^1(W_F, \mathrm{Ind}_{W_{F'}}^{W_F} \mathbb{C}^\times)$

$$= H^1(W_{F'}, \mathbb{C}^\times) = \mathrm{Hom}(W_{F'}, \mathbb{C}^\times)$$

$$= \mathrm{Hom}((F')^*, \mathbb{C}^\times) = \mathrm{Hom}(T(F), \mathbb{C}^\times)$$

$$\begin{array}{ccc}
T(F)_s & \subset & T(F) \\
\| & & \| \\
U_{F'}^{es} & & F^\times \\
\| & & \\
(W_{F'}^{es})^{ab} & & \\
\| & & \\
(W_F^{s})^{ab} & &
\end{array}$$

$\exists\ \ker(\chi) = \ker(\varphi)$