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Preservation of depth

Let T/F torus splits over a tamely

ramified \wedge extension F'/F F : p -adic field
finite

$$T' = \text{Res}_{F'/F} (T \otimes_F F')$$

$$T' \twoheadrightarrow T$$

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$$\oplus \text{Res}_{F'/F} (G_m)$$

$T(F)$ carries a Moy-Prasad filtration $T(F)_r$

For $T'(F) = (F')^*$ the M-P filtr.

$$T'(F)_0 = \mathcal{O}_{F'}^* \quad T'(F)_r = \{ x \in (F')^* \mid \text{val}_F(x-1) \geq r \}$$

val_F : val on $(F')^*$ s.t. $\text{val}(F^*) = \mathbb{Z}$ $r > 0$

Now $T(F)_r = \text{Im}(T(F)_r)$

(2)

$$\chi: T(\bar{F}) \rightarrow \mathbb{C}^\times$$

$$\text{depth}(\chi) = \inf \{ r \geq 0 \mid T(\bar{F})_s \subset \ker(\chi) \text{ for all } s > r \}$$

The Weil gp carries a filtration by upper indexing.

$$\rightarrow \varphi \in H^1(W_F, \check{T})$$

$$\text{depth } \varphi = \inf \{ r \geq 0 \mid W_F^s \subset \ker \varphi \quad \forall s > r \}$$

Class field theory

$$F^\times \cong W_F^{\text{ab}}$$

$$U_F^r \cong (W_F^r)^{\text{ab}}$$

$$\cong 1 + \pi_F^r \mathcal{O}_F$$

(3)

Thm The LLC for tori split over tamely ramified ext preserves depth.

Easy to reduce to $T' = \text{Res}_{F'/F}(G_m)$

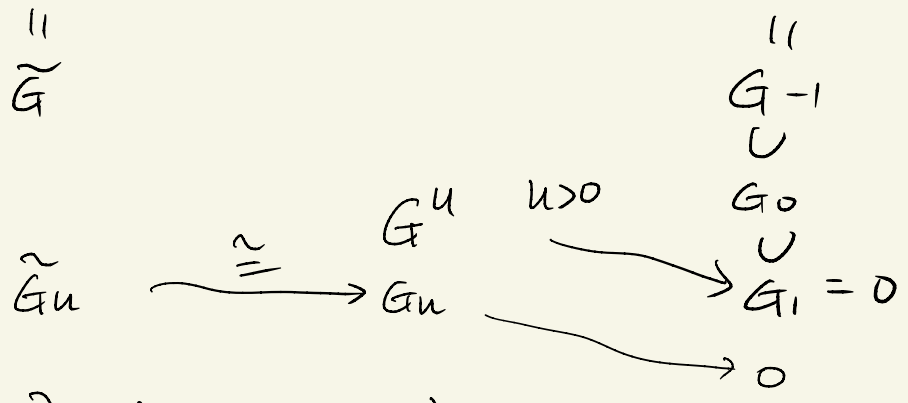
K/F Galois ext of local fields. $G = \text{Gal}(K/F)$

$$\chi \in G_s \text{ iff } \chi|_{\mathcal{O}_K/m_K^{st}} = 1$$

The lower indexing is functorial in subs.

K'/F' Galois

$$1 \rightarrow \text{Gal}(K'/F') \rightarrow \text{Gal}(K/F) \rightarrow \text{Gal}(F'/F) \rightarrow 1$$



$$\tilde{\varphi}(u) = e\varphi(u)$$

$$e = |G_0|$$

$$\Rightarrow \tilde{G}^{eu} = G^u \quad u > 0$$

④

$$u > 0 \quad G^{\varphi(u)} = G_u$$

$$\varphi(u) = \frac{1}{g_0} (g_1 + \dots + g_m + (u-m) g_{m+1}) \quad m \leq u \leq m+1$$

$$g_i = |G_i|$$

Upper indexing is functorial in quotients.

(lower indexing is in terms of L)
upper ————— // — F)

$$\underline{LLC} \quad H^1(W_F, \mathbb{Y}) = H^1(W_F, \text{Ind}_{W_{F'}}^{W_F} \mathbb{C}^x)$$

$$= H^1(W_{F'}, \mathbb{C}^x) = \text{Hom}(W_{F'}, \mathbb{C}^x)$$

$$= \text{Hom}((F')^*, \mathbb{C}^x) = \text{Hom}(T(F), \mathbb{C}^x)$$

$$T(F)_s \subset T(F)$$

$$\parallel \quad \parallel$$

$$U_{F'}^{es}$$

$$F^x$$

$$(W_{F'}^{es})^{ab}$$

$$\Rightarrow \ker(x) = \ker(\varphi)$$

$$\parallel$$

$$(W_F^s)^{ab}$$