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Musical Form and Architectural Structure: Some Considerations on *Qwalala*

Maria Mannone and Federico Favali

Mathematics provides tools to investigate music and different types of artworks. At the same time, mathematics can also lead to the creation of new works of art. Thus, it is possible to state that the mathematical analysis of the structure and of the internal dynamics of an artwork, such as a sculpture or an installation, can inspire the creation of a new artwork in another medium, for example, music.

In the interdisciplinary field of sonification studies (in which several notable examples have recently appeared),[1] the objective is to translate data into sound, without necessarily creating music. There are degrees of freedom in each sonification—the result of data being translated into sound—that depend on the choice of mapping methods. These methods require the consideration and implementation of certain parameters, musical or otherwise.

Effective sonifications may rely on synaesthesia or crossmodal correspondences,[2] and—as will be discussed shortly—they satisfy the criterion of gestural similarity, based on the

[1] Mike Blow, 'On the Simultaneous Perception of Sound and Three-dimensional Objects' (PhD thesis, Oxford Brookes University, 2014); Radoslaw Niewiadomski et al., 'Does Embodied Training Improve the Recognition of Mid-level Expressive Movement Qualities Sonification?,' *Journal on Multimodal User Interfaces* 13 (2018): 191–203.

[2] Charles Spence, 'Crossmodal Correspondences: A Tutorial Review,' *Attention, Perception and Psychophysics* 73, no. 4 (2011): 971–95.

perception of identity between gesture, music production, and image production. In a nutshell, a simple musical sequence and a short visual sketch are seen as 'similar' if they appear as being produced by the same gesture. As an example, a sequence of pencil-drawn points on a piece of paper produced with a detached movement of the hand can be perceived as similar to a sequence of staccato musical notes produced with a similar detached movement (of the hand on the piano keyboard, or of the bow on a violin string). These ideas, which have been experimentally verified,[3] are connected with cognitive hypothesis (for example, the studies of Rosenblum and coauthors on the existence of a 'supramodal brain') and the pioneering work of scholars such as Alexander Truslit, as described in the paper by Bruno Repp.[4] In one of Truslit's experiments, there are sketched drawings; some violinists improvise being inspired by these lines; other people draw lines while listening to them. Truslit found that there are striking similarities between the initial and the final drawings. This article focuses on what might be called—to borrow a mathematical term—'equivalence classes' of musical renditions.

In mathematics, an equivalence class is a set equipped with an equivalence relation, that is, a relation verifying 1) reflexivity, 2) symmetry, and 3) transitivity. In the framework of this article, 'equivalence class' indicates the set of (in principle infinite) possible musical renditions of the same initial visual form, which have something in common. As an example, it is possible to consider a 'similarity relation' between musical pieces derived from the same image, whose articulation, melodic profiles, and overall structure is created to be gesturally similar to the image. The properties of an equivalence relation can be verified as follows:

1) a piece P1 derived from a drawing is similar to itself;
2) if a piece P1 is similar to a piece P2, then P2 is similar to P1;
3) if a piece P1 is similar to a piece P2, and P2 is similar to P3, then P1 is similar to P3, although the degree of similarity could be slightly decreased.

In principle, this approach can use algorithms to create a rough structure of the musical piece (as described in a pseudocode)[5] or it can simply inform the general structure of a piece by guiding the 'paper and pencil' compositional process.

Pae White's *Qwalala*

The relationship between mathematics, visual arts, and contemporary music explored in this article begins with a glass wall created by the American artist Pae White for the 57th International Venice Biennale. Titled *Qwalala*, it was part of the exhibition 'Le Stanze del Vetro' on the Island of San Giorgio in Venice, where it was shown from 2017 to 2019 (see Figures 1 and 2). The installation consists of a curved wall made up of several types of bricks, each of which contained inner coloured lines (Figure 1, left), and was inspired by a river (Figure 2 shows its

[3] Maria Mannone and Dimitri Papageorgiou, 'Gestural Similarity, Mathematics, Psychology. Hints from a First Experiment and Some Applications between Pedagogy and Research,' *Zbornik radova Akademije umetnosti* [*Collected Papers of the Academy of Arts of Novi Sad*] 8 (2020): 137–53; Maria Mannone, Dimitri Papageorgiou and Tom Collins, 'Absent Gestures Influence the Interpretation of Audio-visual Correspondences: A Psychological Validation of the Mathematical Theory of Musical Gestures,' (unpublished paper, submitted for review 2020).

[4] Lawrence D. Rosenblum, James W. Dias and Josh Dorsi, 'The Supramodal Brain: Implications for Auditory Perception,' *Journal of Cognitive Psychology* 29, no. 1 (2016): 65–87; Bruno Repp, 'Music as Motion: A Synopsis of Alexander Truslit's (1938) *Gestaltung und Bewegung in der Musik*,' *Psychology of Music* 21. 1 (1993): 48–72.

[5] Maria Mannone et al., 'Quantum GestART: Identifying and Applying Correlations between Mathematics, Art, and Perceptual Organization,' *Journal of Mathematics and Music* (online, 11 March 2020): 1–33.

winding, snake-like form).[6] In an installation such as *Qwalala*, visual elements are arranged and displaced in three dimensions, allowing the observer to interact with and 'explore' the artwork by walking alongside it, through it, and observing it from different points of view. Thus, the experience of an installation occurs through both space and time.

Figure 1. Side view of *Qwalala*, from the Manica Lunga Library on the Island of San Giorgio. Image credit: F. Favali (left); Close-up photograph of the detail in the wall. Image credit: M. Mannone (right).

Figure 2. The installation as seen from the bell tower of the San Giorgio Cathedral in the Island of San Giorgio, Venice. Image credit: M. Mannone.

In response to the installation, and to Mannone's mathematical investigation of *Qwalala*, the Italian composer Federico Favali composed *Qwalala* (2019) for harp and ensemble (11 instruments). The purpose of this article is to consider some aspects of Favali's composition in light of a mathematical technique developed by Mannone. In this essay, some basic elements of the score will be compared with their corresponding visual elements, in order to envisage, broadly speaking, an abstract artistic 'idea' that can be embodied in different artistic media, as a common origin of music and images. The score of *Qwalala* will be published by Donemus, and the refinement of the piece is in progress.

There are many examples of music inspired by architectural forms and vice versa, including *Metastasis* by Xenakis and *Verticals Ascending* by Henry Brandt. *Metastasis*—written in graphic notation as a sketch by Iannis Xenakis between 1953 and 1954—features graphs of mass motion

[6] The title of the installation comes from the Pomo word *qwalala*, which translates loosely to 'place from where water descends.' Its winding form references the Gualala river in Northern California.

with pitch on one axis and time on the other, an adaptation of the Le Corbusier design for the Philips Pavilion at Expo ’58 in Brussels. The 1967 piece by Brandt, on the other hand, is conceptually based on the architecture of the Watts Towers in Los Angeles, and reflects in sound the visual counterpoint of the buildings. In this sense, a link can be drawn between *Verticals Ascending* and *Qwalala*, as in both of these cases, the architectural characteristics of the building generates the musical piece. Another example is the Sagrada Família, whose form inspired the homonymous piece in the suite *Tridimensional Music* (2011) by Maria Mannone, based on the mapping of the image in a tridimensional space: pitch, time, and volume.[7] In our study, however, the starting point for *Qwalala* was provided by an installation, halfway between sculpture and architecture. To compare artworks and their musical renditions, the mathematical concepts used in this article are: category theory, pattern/envelope formalism, and the concept of ‘attractors’.

Category Theory

Category theory is an abstract branch of mathematics that was born in the 1940s to formalise the concept of transformations of transformations. A category is constituted by objects, visually represented as points, and morphisms, that is, transformations between objects, visually represented as arrows.[8] The morphisms have to verify compositionality, associativity, and identity properties. A category itself can be transformed into another category; thus, it is possible to define transformations between points of a category and points of another category, and between arrows of a category into arrows of another category. In this way, a category is seen as a ‘point’, and it is possible to define arrows between arrows, creating nested structures. The visual idea of ‘arrows between arrows’ corresponds to the concept of transformations between transformations. As an example, in music, a crescendo can be seen as an arrow mapping; for example, a volume change of *piano* into *forte*. A comparison between a fast crescendo and a slow crescendo can then be visualised as an arrow between these two arrows. The comparison between speed on a violin and speed on a clarinet can be visualised as yet another arrow between these arrows, and so on. It is easy to understand that music is full of complex and nested relations, and diagrammatic thinking can help shed light on some of its fundamental mechanisms. Categories have often been applied to music,[9] in particular, to musical gestures.[10] In addition to category theory, this research can exploit homotopy theory. In fact, the analysis of

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[7] See Maria Mannone, *Dalla Musica all’Immagine, dall’Immagine alla Musica* (Palermo: Compostampa, 2011) and Maria Mannone, ‘Networks of Music and Images,’ *Gli Spazi della Musica* 6, no. 2 (2017): 38–52. There are also examples of architects working with sound, such as Sanford Kwinter and Marcos Novak. Kwinter explains his vision that music and concept art can result in effective innovation in architecture in ‘Concepts: The Architecture of Hope; On Difficulty and Innovation,’ *Harvard Design Magazine* 19 (2003): http://www.harvarddesignmagazine.org/issues/19/concepts-the-architecture-of-hope-on-difficulty-and-innovation. In his essays in *Architecture as a Translation of Music*, Novak coined the term ‘archimusic’ to describe the result of the mix between architecture and music. See *Architecture as a Translation of Music*, ed. Elizabeth Martin (New York: Princeton Architectural Press, 1994), 64–72.

[8] Saunders Mac Lane, *Categories for the Working Mathematician* (New York: Springer, 1978).

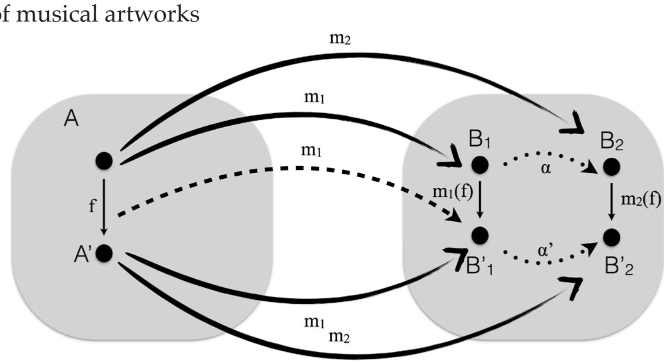
[9] Guerino Mazzola, *The Topos of Music: Geometric Logic of Concepts, Theory, and Performance* (Base: Birkhäuser, 2002). However, the simple proposed application appears in ‘Introduction to Gestural Similarity’.

[10] Guerino Mazzola and Moreno Andreatta, ‘Diagrams, Gestures and Formulae in Music,’ *Journal of Mathematics and Music* 1, no. 1 (2010): 23–46; Juan Sebastián Arias, ‘Spaces of Gestures are Function Spaces,’ *Journal of Mathematics and Music* 12, no. 2 (2018): 89–105.

curves as paths and the deformation of paths can exploit the formal tools of homotopy theory.[11] In the framework of category theory, one can use arrows to represent associations of visual elements with sound elements. Arrows can also help connect simple visual elements to complex visual elements, and also to their musical renditions. While considering possible musical renditions of the same image, the comparison between them can be formally investigated with arrows between arrows, or more precisely, 'natural transformations'. Categories have also been used to formally represent the transition from artwork creation to the artistic critique, as a process of investigation from the idea to the artwork to its impression over a listener/observer, as described by Akihiro Kubota and his co-authors.[12]

The research presented here does not focus on audience reactions, even if it can be supposed that, if an artwork has been translated from one domain to another according to gestural similarity, the audience may acknowledge some similarities. In the present research, artworks in different sensorial domains are considered. Starting from a visual artwork, it is possible to obtain different musical renditions, which have something in common. Figure 3 shows how this can be formalised using the language of category theory (see also Figure 14). The arrow f represents the transformation of the visual artwork into a slightly different one (we suppose that this f is associative).[13] The arrow $m_1(f)$ represents a transformation from the musical piece B_1 into the musical piece B'_1; the same for $m_2(f)$. One can suppose that $m_1(f)$ and $m_2(f)$ are associative. The arrows m_1 and m_2 are functors, meaning they map objects from a category—in this instance, visual artworks—into objects from another category—musical pieces—and visual transformations (f) into musical transformations, here $m_1(f)$ and $m_2(f)$. The dashed arrow represents the transformation from the arrow f to the arrow $m_1(f)$. The dotted arrow α represents the variation from a musical rendition B_1 to a musical rendition B_2. The dotted arrow α' represents the variation between B'_1 and B'_2. It is possible to indicate α' and α with α_B and $\alpha_{B'}$, respectively, and with Nat : $m_1 \rightarrow m_2$ the natural transformation from the functor m_1 to the functor m_2, an arrow between arrows. To distill this information into formulas: $B_1 = m_1(A)$, $B_2 = m_2(A)$, $B'_1 = m_1(A')$, $B'_2 = m_2(A')$.

Figure 3. Visual representation of the transition from the category of visual artworks to the category of musical artworks



[11] In mathematics, a curve is said to be homotopic to another one if it can be continuously deformed into the other, without any cuts or gluing. Homotopy theory is a branch of mathematics whose starting point is the formal definition of this idea. See Allen Hatcher, *Algebraic Topology* (Cambridge: CUP, 2000).

[12] Akihiro Kubota et al., 'A New Kind of Aesthetics—The Mathematical Structure of the Aesthetic,' *Philosophies* 2, no. 3 (2017): 1–10.

[13] Homotopy is not associative, but homotopy equivalence class is. See Hatcher, *Algebraic Topology*.

Envelopes and Patterns

Translating a complex visual form into a complex musical form is a non-trivial process between ontologically diverse objects, as the initial idea of the visual form can generally not be translated literally into the musical form. However, if special attention is paid to the choice of simple, constitutive visual elements, and if some general elements of the complex form are associated with appropriate musical elements—for example, an enlargement in the visual form might correspond to an increase in volume, or an empty space to a sudden silence—such a translation can be effective.[14] As a development of this idea, it is possible to distinguish between what are called 'patterns' and 'envelopes'.[15] The patterns are basic and recognisable elements that can be modified according to an overall form, or that are simply distributed along that form, which is the envelope. For example, the patterns of a wall are its bricks; where the wall curves, the bricks must follow that same profile. Musically, if the patterns are melodic cells, and the envelope is a rising sequence, the melodic cells are gradually transposed higher in pitch. If the envelope is an overall diminuendo, the melodic cells become progressively softer, and so on.

It is possible to mathematically describe *Qwalala*'s structure considering several factors. In a first simplification, there are three elements: an overall line of the wall, the bricks, and their decorations as inner lines. The overall shape of the installation can be seen as a curved envelope made up of bricks as patterns, which have different coloured shadows, and one side of each is curved. Because of the inner decorations, each decorated brick can itself be seen as an envelope containing an inner pattern; that is, the coloured lines (see Figure 4).

The schematisation of a visual form into patterns and envelopes is used in a recent study that investigates musical forms, visual forms, and their 'translations'.[16] Furthermore, it is possible to find similarities between the structure of the bricks and their inner lines and musical elements. In this case, the bricks and their lines have been respectively linked to simple musical chords and melodic lines, according to Mannone's theoretical approach and Favali's aim in composing *Qwalala*, which was to create a musical piece starting from the form and the characteristics of the installation.

A glass block is, of course, completely different physically from a musical chord, and a glass decoration is not a melody at all. However, the main geometric property of the blocks in *Qwalala* are their parallelepiped-like form, which is homogeneously extended through space. In music, an element that is homogeneously extended through time is a 'block' of simultaneous sound, that is, a chord or a cluster. Likewise, a simple image—like the brick decorations—and a short musical sequence may be called 'similar' if they appear as being generated by the same physical gesture. For example, a sequence of dots on canvas can be created with the same detached movement necessary to play staccato on a violin or piano,[17] an idea that also applies

[14] See Maria Mannone, 'Introduction to Gestural Similarity in Music. An Application of Category Theory to the Orchestra,' *Journal of Mathematics and Music* 12, no. 2 (2018): 63–87.

[15] Mannone et al., 'Quantum GestART.'

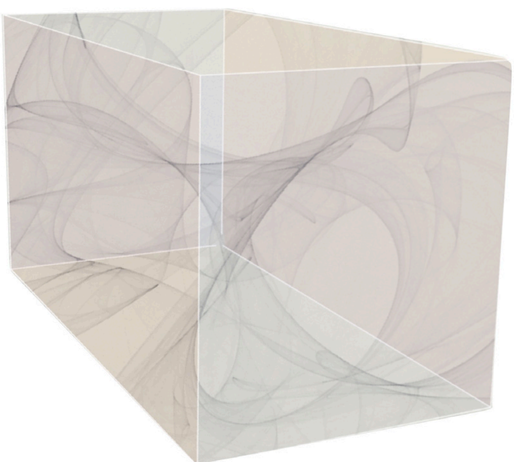
[16] Maria Mannone, 'A Musical Reading of a Contemporary Installation and Back: Mathematical Investigations of Patterns in *Qwalala*,' *Journal of Mathematics and Music* (forthcoming, 2021). The article is also focused on visual/musical *Qwalala*, but it targets mathematicians and contains theoretical developments. See also Mannone et al., 'Quantum GestART.'

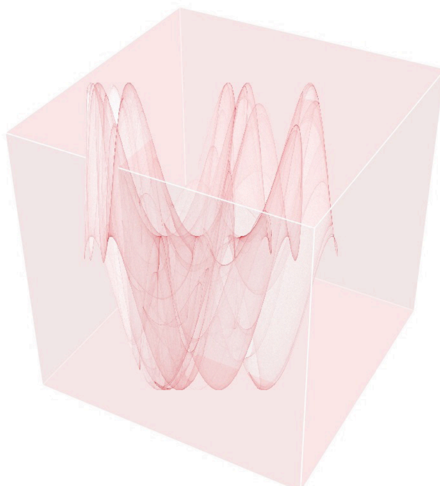
[17] Of course, such movements exist within different parameters on each instrument. For example, the parameters for the violin involve the position and pressure of the bow; for piano, the horizontal and vertical distance from the keyboard; and for flute, the intensity and direction of the player's airstream. See Mannone, 'Introduction to Gestural Similarity in Music.'

to simple images and musical sequences. In general, in the visual arts, there are complex forms made up of combinations of simple shapes, lines, brush strokes, and so on, while in music there are complex forms made up by combinations of simple elements (together forming a composition). Yet, in the case of the installation *Qwalala*, rather than brush strokes we have the movements and techniques of the glass artists.

Each brick is different. However, some have inner decorations, while others are transparent with no decoration. Although the bricks' inner decorations are random, they share some characteristics that are approximately repeated (for example, colour, curves and curls). The repetition of these irregular items makes them recognisable as patterns; thus, they can be identified as irregular patterns. As a musical equivalent, one can consider polyphonic music, where an irregular melodic line appears here and there, each time with the same duration. As it continues this element becomes recognisable as a motif. In contrast with irregular patterns, a regular pattern is seen here as an entity, or a geometric form, which is repeated identically, or with some rotations and inversions, as within a frieze or a wallpaper decoration. For example, a regular pattern can be a musical element (such as a melodic cell or a chord sequence), which is almost always repeated identically or with some transformations that do not change its form. In *Qwalala*, there are irregular patterns (the lines), but also regular patterns (the form of blocks). It is also possible to find regular patterns at the level of the bricks, and not just the lines inside them. The forms of a parallelepiped and a cube (the latter of which is found at each end of the installation) can be seen as 'patterns' of the wall. Bricks are compared to 'regular' chords, while lines within the bricks are compared to 'irregular' melodic lines. Musically, one could think of regular and static elements, in contrast with dynamic and irregular lines.

Figure 4. Possible modellings of *Qwalala* glass blocks made with the software Mathematica.[18] The inner patterns are modelled via de Jong attractors.[19]





[18] Maria Mannone, 'Venice: *Qwalala* and Other Contemporary Glassworks in Light of Categories,' *Journal of Mathematics and the Arts* (unpublished paper, submitted for review 2019).

[19] Paul Bourke, 'Peter de Jong Attractors,' *Paul Bourke*, http://paulbourke.net/fractals/peterdejong; Kenny Colnago, 'Smooth Peter De Jong Attractor,' *Mathematica Stack Exchange* (forum), 15 June 2014, https://mathematica.stackexchange.com/questions/50839/smooth-peter-de-jong-attractor.

Irregular musical sequences are therefore linked to the irregular lines of the bricks. An almost random and irregular line can be obtained with an ideal creative gesture, which can analogously create an irregular melodic line. A collection of random lines (such as the ones in Figure 5a) can be rendered musically as a polyphony of random melodic elements, with the precise pitches chosen at the discretion of the composer. Thus, it is clear that there is a certain amount of freedom in creating these links. Nonetheless, it is mathematically possible to model the random lines in the bricks by using the de Jong attractors (see Figure 5b). These are examples of 'strange attractors'.[20] In physics, an 'attractor' is a configuration that is approached but never reached, just as a line can never be sketched perfectly by hand. More precisely, an attractor is a set of numbers, points, configurations, or curves towards which a system trends. An attractor can be 'chaotic' when a small change of the initial conditions leads to a change of the overall form. The behaviour of a strange attractor is complex, and often fractal. Visually, the attractor is a curve that is never reached, and there are (in principle) infinite lines around it, which never exactly reach the target. In this study, the de Jong attractor has been chosen because the irregularity and elegance of the visual outcomes are similar to some of the lines in *Qwalala* (see Figure 5a). In Favali's piece, melodic lines have been freely composed thinking of random visual lines. They can be compared with the sonification of attractor-generated lines.

Figure 5. Inner lines within a brick of *Qwalala* (left); An approximation of these lines made with a strange attractor (right). By changing the parameters and colour functions, one can obtain different lines and colours, approximating the variety of lines in *Qwalala.*

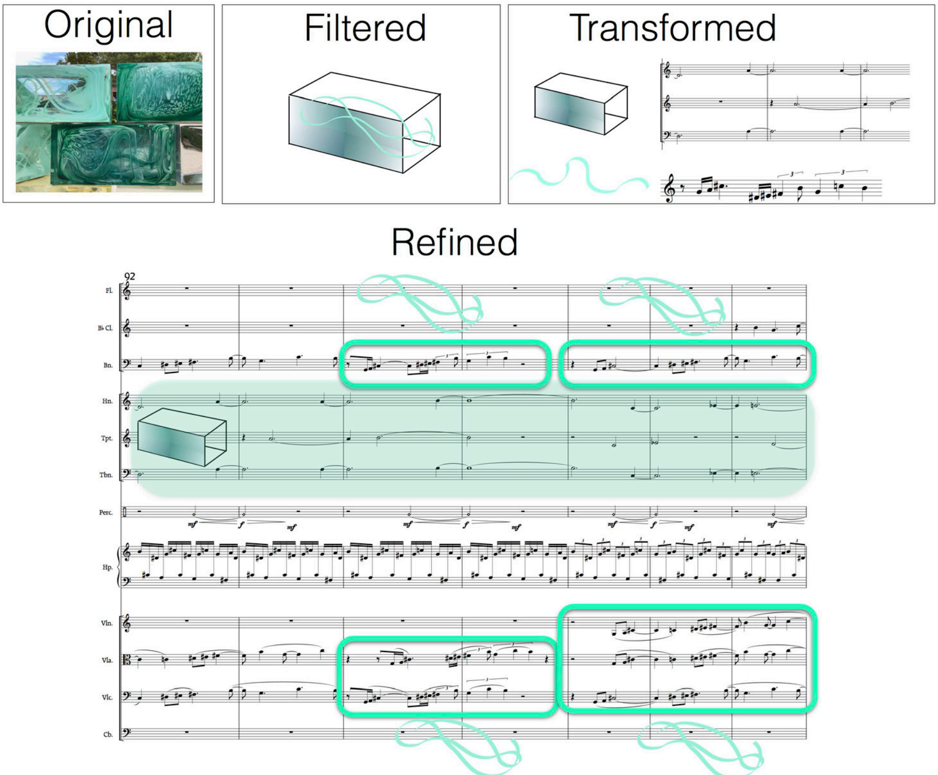
In *Qwalala*, lines within bricks are rendered musically as melodic lines superimposed over chords, or within them, correlating to the bricks of the installation. This idea points towards a compositional method known as RTF,[21] whose steps are: 'filtering' (deconstructing a complex visual form in terms of simple shapes); 'transforming' (associating visual and musical elements to obtain a rough musical piece); and 'refining' (enriching the composition with detail and

[20] A further example of a strange attractor is the Lorenz attractor, which is used in weather forecasts and to explain the so-called 'butterfly effect'.

[21] Mannone et al., 'Quantum GestART.'

other finishing touches).[22] Figure 6 shows how these ideas were applied to *Qwalala*. While the organisation of a simple visual structure and a musical form can be trickier to compare, both can be deconstructed into simple, constitutive elements. Thus, the idea of gestural similarity can lead to a 'translation' method from images to music (and vice versa). This is not a deterministic method but a search for analogies of structures that can, in principle, be perceived by the listener.

Figure 6. An example of application of the RTF method to *Qwalala*, from the glass bricks to an excerpt of Favali's score *Donemus* [forthcoming], bb. 92–98[23]



Architectural and Musical Structures of *Qwalala*

The musical themes of *Qwalala* are (almost) freely chosen: they imitate the movements of the inner lines on each brick. In fact, there is a gestural similarity between the line creation within the bricks and the melodic movements of the cells. Bricks are represented with chord blocks, containing the lines. The overall shape of the piece contains variations in volume and pitch, in accordance with the structure of the installation. In addition, the duration of the piece and the temporal distribution of musical elements is in proportion with the alignment

[22] The Filter, Transform, and Refine process is known in the literature as the RTF method, as the letters are applied from right to left. The mathematical formalism of patterns and envelopes, alongside RTF, is described in detail in Mannone et al., 'Quantum GestART,' and is also contextualised in both categorical language and Dirac notation, used for Quantum Mechanics. Technical details have been omitted for the sake of simplicity. This process leaves room for the creativity of composers, because more attention is devoted to the 'class of possible renditions', as opposed to a direct, one-to-one relation.

[23] Maria Mannone, 'A Musical Reading of a Contemporary Installation and Back: Mathematical Investigations of Patterns in *Qwalala*,' *Journal of Mathematics and Music* (forthcoming 2021).

of the installation's visual elements (including blocks, colours, height variations, and the two openings used as doors).

The association between colours, musical timbres, and musical harmonies is also roughly based on an extension of the idea of gestural similarity into the realm of colour and timbre perception. Although there are no definitive correlations between sound and colour, one might perceive red as 'violent', and subsequently associate it with middle to low-register brass instruments for their ability to generate high-intensity and high-volume sounds. On the other hand, hearing a flute playing softly and looking at a light blue colour may evoke the same feeling of calm. The installation *Qwalala* features four principal colours: blue, yellow, green, and red. Figure 7 shows the themes that constitute the piece's musical structure and their corresponding colours.

Figure 7. The main thematic elements of *Qwalala* alongside their corresponding colours (from Federico Favali, *Qwalala*, Donemus [forthcoming])

The colours of the installation are fundamental to the piece's musical structure. Each has its own sonic system and unique 'character', which was determined by a feeling, sensation or mood evoked by each colour (as perceived by the composer). Different people may also perceive different colours, but these are often from an identifiable range. As such, the idea of gestural similarity discussed in this paper can be extended to colours. This was explored in a recent experiment, and, theoretically, this is part of the extension of 'gestural similarity' to colours, the 'ChromoGestural Similarity'.[24]

The association of colours with music deals with orchestral timbres but also with pitch-sets, according to the artistic choice of the composer. In *Qwalala*, the colour blue corresponds to the whole-tone scale (F, G, A, B, C♯, D♯), which—in addition to being the main colour of a river—has been taken to represent a calm, or 'floating' sensation. The colour yellow is related to the octatonic scale (C, D♭, E♭, E, F♯, G, A, B), corresponding here with feelings of (psychological) tension. Green is linked to a transposition of Messiaen's sixth mode (G, A, B, C, C♯, D♯, E♯, F♯), also taken to symbolise calm. Lastly, red is represented by the Phrygian mode (D, E♭, F, G, A, B♭, C), which here evokes a heightened emotional state. Furthermore, each of the four colours has corresponding instruments; thus, visual colours and tone colours

[24] Maria Mannone and Giovanni Santini, 'Perceived Similarities between Classes of Colors and Classes of Timbres' (unpublished paper, submitted for review, 2020).

become interrelated. Throughout the piece, the instrumentation of the theme changes several times, to allow for variation in dynamics. Table 1 lists the instruments that play the first iteration of each colour's theme.

Table 1. Relationship between colour and instrumentation in *Qwalala*

Colour	Instruments
Red	Brass, percussion
Green	Strings
Yellow	Violin, horn, bassoon
Blue	Flute, clarinet, horn

White is the initial (alongside red) and final colour seen in the installation. In the piece, white functions as the sum of the other themes, and is represented by the harp. As explained earlier, visual bricks correspond with musical chords, whereas the internal lines of the bricks accord with these musical themes (see Figure 6 above).

Regarding structure, the piece begins with a thin texture (one instrument), which becomes thicker as the installation increases in size, returning to the original texture at the end. The drawing in Figure 8 illustrates the structure of the installation and indicates the duration of each section of the composition as planned at the initial stage of composition. Of course, some of these durations were modified during the compositional process.

Figure 8. A sketch of *Qwalala* with Federico Favali's original timings for the piece. Drawing by Maria Mannone.

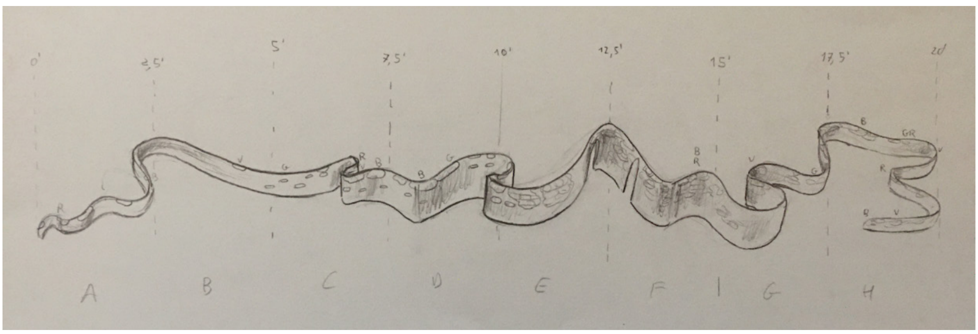


Figure 9 shows the 'beginning' of the installation, musically rendered with solo harp (white) playing fragments of the red theme. The colour red is then represented with its corresponding theme, performed by its associated instruments (brass and percussion). Figure 10 shows the corresponding passage of the score, and Figure 11 is a superimposition of the score onto the image.

In the installation *Qwalala*, there are two places where the wall is broken. Each of these functions as a kind of door, through which observers can pass. These breaks are represented by a thinned texture in the score; in other words, the texture represents these two openings. Figure 12 shows one of these breaks, and Figure 13 shows the superimposition of the score onto the image.

Figure 9. The 'beginning' of *Qwalala*, as seen as a path from right to left. Image credit: M. Mannone, 2019.

Figure 10. Favali, *Qwalala*, bb. 1–6

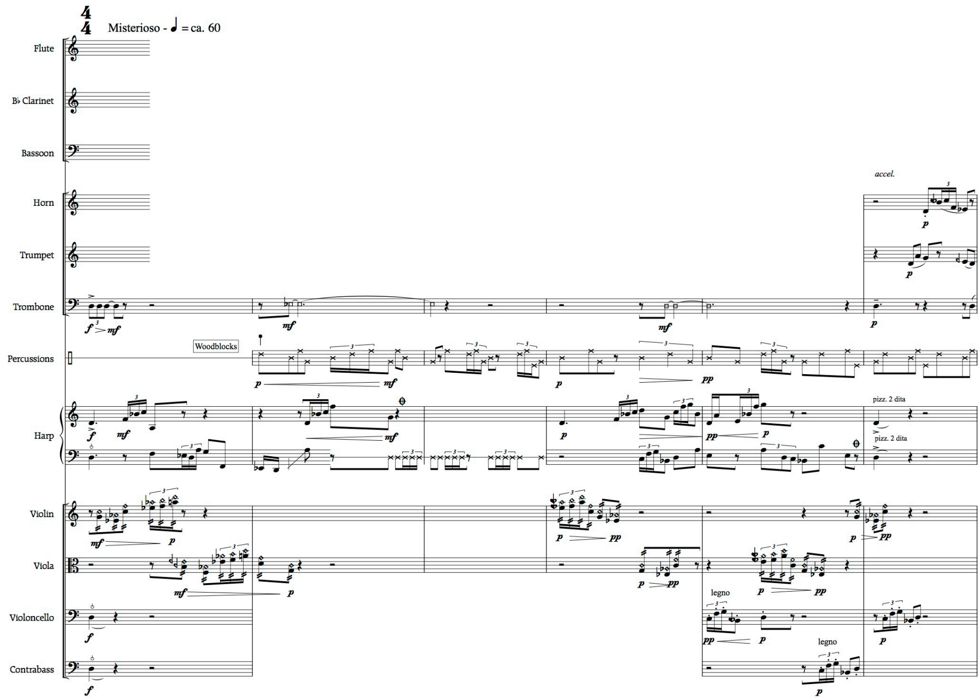


Figure 11. Opening passage of *Qwalala* approximately superimposed onto an image of the installation

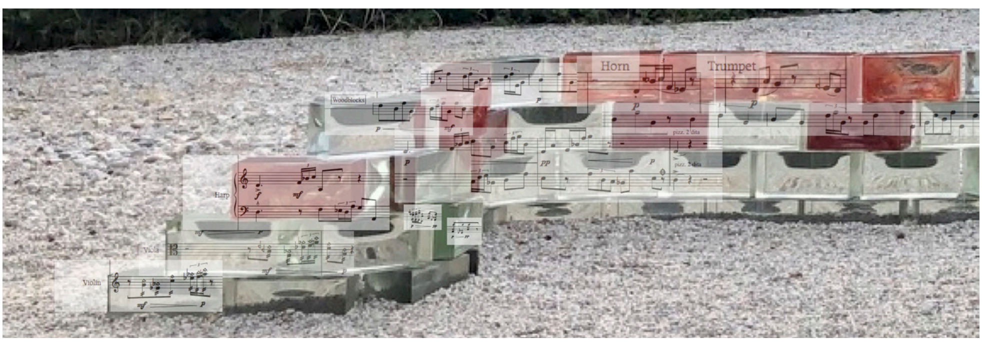
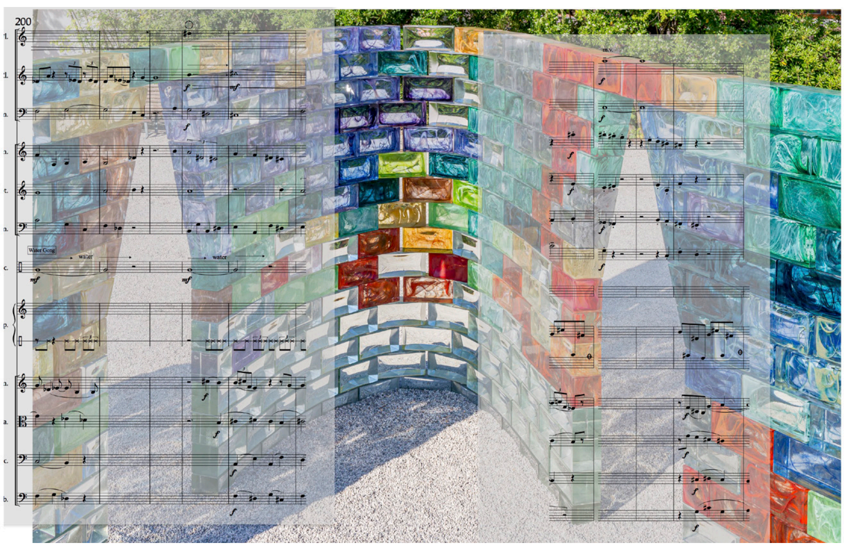


Figure 12. Favali's musical representation of one of the 'doors' in *Qwalala*, bb. 200–205

Figure 13. The two doors and the corresponding fragments of score. Courtesy of 'Le stanze del vetro,' © Ph. Enrico Fiorese.



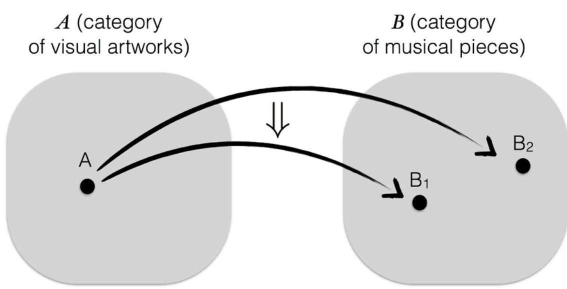
From the last beat of bar 200 to bar 204, both the cello and double bass have a pause. Similarly, the violin and viola have pauses from bar 201 to bar 203. Looking at the score (in Figure 12 above), it is possible to see a sort of opening, representative of one of the passageways in the installation. Thus, what one sees with their eyes aligns with what they hear while listening to the piece. The second door is represented between bars 212 and 215.

Conclusion

If another composer was to create a new piece based on *Qwalala*, it would of course be different to Favali's. However, they would likely follow a similar compositional process of selecting musical parameters that followed the overall waved structure with the blocks, their colours, and inner lines. Mathematical transformations can be used to compare these different musical renditions, finding their similarities and differences. Differences would be mostly contingent on individual techniques and personal compositional styles, while similarities might be found in the use of gesture and the interpretation of listener perception, thus (potentially) creating a recognisable sonic representation of the installation's overall form.

These processes of scientific and artistic investigation can be used to study the transition from a visual to a musical artwork, to compare different musical transpositions, and, finally, they allow for a generalisation of ideas. Rendering an existing artwork in a different medium (in our case, a physical installation as a piece of music), allows for a deeper understanding of the artistic concept behind the original work itself. Figure 14 aims to restate this intuition within a categorical framework. Intuitively, given an artwork A, the set (theoretically infinite, practically finite) of all possible translations of A into other media contributes to a deeper understanding of A itself, because each transposition B_i catches and highlights a specific aspect of A, allowing us to see it in a new light. These transpositions (from A to B_1, A to B_2, and so on), and the comparison between them, are represented with arrows and double arrows respectively. We can imagine a collection of arrows, starting from the artworks and pointing to the reader, that indicate the same artistic idea.[25]

Figure 14. A simplification of the conjecture relating to the transformation of artworks from one medium to another. Let A be an artwork belonging to the artistic domain represented by the category *A*. Let B_1, …, B_n be a set of n artworks belonging to the artistic domain represented by the category *B*. The artworks B_i, with i = {1, …, n} are obtained as transpositions of A in the domain *B* and we suppose they verify gestural similarity. The existence of B_1, …, B_n and the analysis of their natural transformations contributes to the understanding of the artistic idea underlying A.



[25] This idea, labelled the 'Art Conjecture', is described in more mathematical detail in Mannone, 'A Musical Reading.'

This article has shown how the processes of scientific investigation and artistic creation are worthy of regular and rigorous exploration, and that mathematical analysis is central to this endeavour. In particular, we focused on the musical patterns and symmetries in *Qwalala*, investigating the possibility of retrieving patterns within the original installation starting from the analysis of the score. Further questions concern the technical and philosophical issues behind the creation of specific artworks, the uniqueness and variety of artistic concepts, and the relationship of these concepts to the specific medium chosen by the artist. Even if it does not answer them, mathematics allows us to state these questions in a clear way. Furthermore, the mathematical investigation itself and the development of new models can stimulate additional questions, continuously connecting mathematical thinking with artistic creation, and artistic thinking with mathematical creation.

About the Authors

Maria Mannone is an Italian theoretical physicist and composer who is currently a postdoctoral researcher at Ca' Foscari University of Venice. In Italy she attained Masters degrees in Theoretical Physics, Piano, Composition and Conducting, and also gained a Masters in Acoustics, Informatics and Signal Processing applied to Music (ATIAM) from IRCAM-UPMC Paris VI Sorbonne. Her PhD is from the University of Minnesota.

Federico Favali graduated in piano from Lucca and musicology from the University of Bologna before undertaking studies in composition at the Conservatory of La Spezia, King's College London, University of Birmingham, New York University and Universidad Nacional de Tres de Febrero (Buenos Aires). His music has been performed worldwide by ensembles such as the Birmingham Contemporary Music Group and Lontano Ensemble. As a musicologist, his focus is on analysis of contemporary music.