Force Observer for an Upper Limb Rehabilitation Robotic Device using Iterative Learning Control

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Abstract— **This paper focuses on detecting the forces exerted by stroke patients in the repetitive exercises using the rehabilitation robotic system: EMU, without using any external force sensor. As the model of such a system is hard to identify precisely, a simple feedback-based iterative learning control algorithm is proposed to identify such forces. The convergence analysis of such force estimators is provided in this paper. The simulation and experimental results illustrates the effectiveness of the proposed force observers.**

I. INTRODUCTION

Each year, more than 15 million people in the world experience their first stroke, with approximately two in three surviving with permanent disability [1]. Recently, various robotic devices have been developed as promising tools that can extend the physiotherapists ability: to care for more people, to follow up over more extended periods of time, and to assist people at a distance, in the comfort of their own homes with more information that can be measured using sensors of robotic devices [2], [3].

In order to fully utilize rehabilitation robotic devices, measuring the forces exerted by the patient in the exercises are useful to quantify the patients motion capabilities, to ensure safety movements of patients, to provide the tactile feedback to the patients, to encourage and modulate neural plasticity [4], and to modulate the assistance appropriate to the patient's conditions [5]. It can also be used to estimate the motor control dynamics of the human user, such as muscle stiffness and limb spasticity [6]. Adding force sensors to rehabilitation robotic device is not always preferred in a clinical setting due to the cost and extra efforts of integration in the mechanical design of the manipulator. Under such a situation, force observers are generally preferred.

Most of the force observers used for robotic manipulators are model-based [5], [7], [8]. Such observers require a relatively accurate nominal model for an external force estimation. This paper focuses on designing a force observer for an end-effector based rehabilitation robotic device: EMU [9]. Although this device is completely backdrivable, it is observed that the accurate model parameters are difficult to identify for a wide range of operating conditions. Hence a model free force observer is preferred. On the other hand, the model-free techniques such as the extended state observer (ESO) cannot be directly used [10], [11], as it usually estimates the lumped uncertainties and is difficult to identify the external forces from the lumped uncertainties.

It is well-accepted that "consistent repetition that re-establishes communication between the damaged parts of the brain and the body is crucial in stroke rehabilitation" [3]. Hence it is very natural to link the rehabilitation processes with iterative learning control (ILC), which uses the repetitive nature of a task to learn an unknown control signal [12], [13]. In particular, when the external forces from the patient do not change much over repetition, the unknown forces can be treated as iteration-invariant disturbances. Some adaptive-like ILC algorithms might be useful [14] for this purpose.

It is worthwhile to highlight that the ultimate goal of this work is to estimate unknown time-varying but iteration-invariant external forces in a rehabilitation robotic device. This is different from a standard ILC setting, where the standard control objective is to track a desired reference signal. A careful design is thus needed to formulate the force estimation in the setting of ILC.

Although the external forces can be treated as iteration-invariant matched uncertainties, due to the existence of modeling uncertainties, they cannot be treated as parametric uncertainties that can be factorized as unknown parameters and known functions or basis functions. On the other hand, the EMU system or the robotic manipulator with position and velocity measurements has the relative degree 1, if a pure feed-forward ILC is used to learn the unknown forces, the derivatives of measurements are needed to ensure convergence by using contraction mapping (CM) techniques [15], [16]. Due to possible high frequency noises from measurements, the feed-forward ILC needs a careful design of filters in implementations [17].

With the consideration of implementation, this paper proposes a feedback-based ILC algorithm that can estimate the forces exerted by patients when using an upper limb rehabilitation robotic device. The role of the feedback is to ensure the uniform boundedness of the trajectories for a given time interval over any iteration. A new composite energy function is used to show that the force observer converges in $\mathcal{L}_2$ norm sense and the trajectories of the robot manipulator are stabilized. Simulation and experimental results support the theoretical findings.

II. PRELIMINARIES AND MOTIVATION

The set of real numbers is denoted by $\mathcal{R}$ and the set of natural numbers are represented by $\mathcal{N}$. The Euclidean norm of any vector $\mathbf{x} \in \mathcal{R}^n$ is calculated as $|\mathbf{x}|^2 \triangleq \mathbf{x}^\intercal \mathbf{x}$. For any matrix $A \in \mathcal{R}^{n\times m}$, $|A|$ represents the induced matrix norm. For a square matrix $A \in \mathcal{R}^{n\times n}$, $A > 0$ indicates A is a positive definite matrix. $(\cdot)^\intercal$ represents the transpose of a vector or a matrix. I_n denotes the identity matrix of dimension n. The compact set B_Δ is defined as $B_\Delta :\triangleq \{\mathbf{x} \in \mathcal{R}^n \mid \ |\mathbf{x}| \leq \Delta\}$.

Definition 1: For any signal in $\mathcal{L}^2[0, T_f]$[1], its $\mathcal{L}^2$ norm is defined as $\|\mathbf{x}\|_{\mathcal{L}^2} \triangleq \left(\int_0^{T_f} |\mathbf{x}(\tau)|^2 d\tau\right)^{\frac{1}{2}}$. For any signal

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[1]The space of $\mathcal{L}^2[0, T_f]$ contains all square-integrable functions over a finite interval $[0, T_f]$

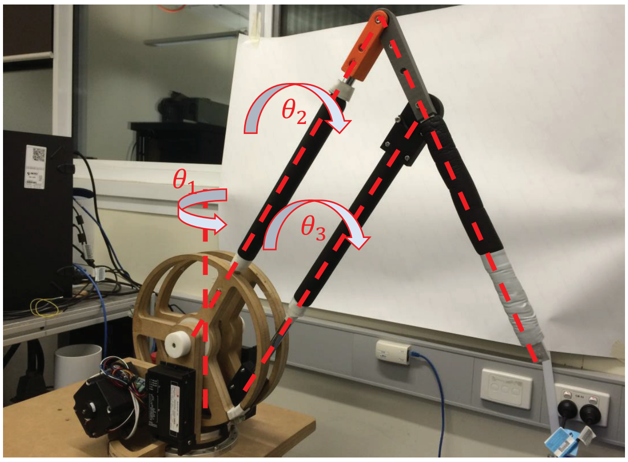

Fig. 1. Prototype of a rehabilitation robot, EMU.

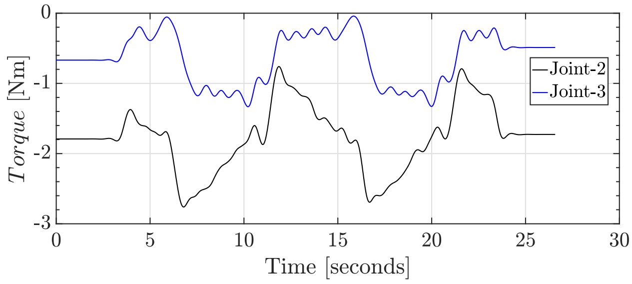

Fig. 2. Applied torque

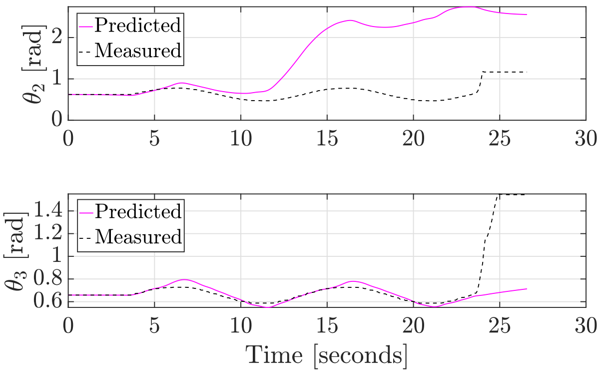

Fig. 3. Comparison of predicted joint angles using identified model and measured joint angles

in $\mathcal{L}_\infty[0, T_f]^2$, the supremum norm is defined as $\|\mathbf{x}\|_s \triangleq \max_{t\in[0,T_f]} |\mathbf{x}(t)|_\infty$, $\circ$

Definition 2: A continuous function $\alpha : [0,\ a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [18, Definition 3.3]. $\circ$

Next a motivating example for this paper is discussed.

A. A Motivating Example

The robotic manipulator can be modelled using Lagrangian formulation with an appropriate parameterization. Once the equations of motion is derived, standard techniques such as parameter identifications can be used estimate the unknown parameters. The unknown parameters may include effective lumped masses, positions of centre of mass, friction coefficients etc. Details on modelling and identification of parameter in robotic manipulators can be found in [19] and references therein.

The prototype of a rehabilitation robot, EMU [9] (shown in Fig. 1) is used in this motivating example. Standard model identification procedures are followed to obtain the nominal model parameters of the system. Torque input that generate output trajectories with increasing frequencies, ranging from approximately 0.1 Hz to 2 Hz is used for parameter identification.

To demonstrate that the identified parameters may not always yield accurate predictions of joint positions when the torques applied that are quite different from the torque used in identification. The torque shown in Fig. 2 is applied to the robot. The variation of estimated output and measured output for two joints: θ_2 and θ_3 are shown in Fig 3. However, it is observed that better predictions can be obtained if comparatively high frequencies in output trajectory are used. Clearly, the model is not accurate in predicting the measured values for a wide range of frequencies. As it is well known that, the external force observers in literature [5], [7], [8] are model-based, accurate model information is needed for a successful force estimation. But in reality, it is hard to obtain an accurate model that can work for a wide range of excitation frequencies. Hence, such model based observers has limited applications. This motivates this work to explore model-free force observers where the repetitions in a task can be used to learn the external disturbance using a suitable ILC.

[2]The space of $\mathcal{L}_\infty[0, T_f]$ contains all essentially bounded functions over a finite interval $[0, T_f]$.

III. PROBLEM FORMULATION

The dynamic model of a m link rigid robotic manipulator can be represented by the following model

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{f}(\dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta}) \quad = \quad \mathbf{u} + \mathbf{d} \qquad (1)$$

where $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}} \in \mathcal{R}^m$ are joint angles, velocities and accelerations. The notion of $M(\cdot) \in \mathcal{R}^{m\times m}$ represents the inertia matrix, $C(\cdot,\cdot) \in \mathcal{R}^{m\times m}$ represents the total Coriolis and Centripetal terms, $\mathbf{f}(\cdot) \in \mathcal{R}^m$ is the friction component, and $\mathbf{g}(\cdot) \in \mathcal{R}^m$ is the gravity force. The input $\mathbf{u}$ is the applied torque and $\mathbf{d}$ is the unknown time-varying, but iteration-invariant input disturbance. It is assumed that the robot performs a given repetitive task with a finite time interval $t \in [0,\ T_f]$.

The following properties hold for the robotic manipulator systems (1):

Property 1: The inertia matrix $M(\cdot)$ in (1) is symmetric and positive definite in the domain of interests. More precisely, , there exist two positive constants μ_1 and μ_2, s.t $0 < \mu_1 I_n \leq M(\cdot) \leq \mu_2 I_n$. $\square$

Property 2: The matrix $(\dot{M} - 2C)$ in (1) is a skew symmetric matrix. Hence for any $\mathbf{x} \in \mathcal{R}^m$, the following relation hold: $\mathbf{x}^\intercal(\dot{M} - 2C)\mathbf{x} = 0$. $\square$

Property 3: Let Δ be given. For any $\begin{bmatrix} \boldsymbol{\theta}^\intercal & \dot{\boldsymbol{\theta}}^\intercal \end{bmatrix}^\intercal \in B_\Delta$, there exist three positive constants C_b, F_b, and G_b such that: $\left|C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\right| \leq C_b \left|\dot{\boldsymbol{\theta}}\right|$, $\left|\mathbf{f}(\dot{\boldsymbol{\theta}})\right| \leq F_b \left|\dot{\boldsymbol{\theta}}\right|$ and $|G(\boldsymbol{\theta})| \leq G_b$, $\square$

Assumption 1: The external disturbance $\mathbf{d}(t)$ is invariant in iteration and is uniformly bounded for all $t \in [0, T_f]$, i.e there exists a positive constant d_b such that $\|\mathbf{d}\|_s < d_b < \infty$. □

Remark 1: Generally, the forces exerted by patients are not iteration-invariant as large human variations can be observed during the training session. It is usually assumed that the human variation is a white noise with zero mean [20]. Hence the role of this force estimator is to estimate the averaged force from the patient, which can represent the patient's ability during the training sessions. ○

The control objective is to identify the iteration-invariant input disturbance $\mathbf{d}(t)$ when the control task is repeated with a finite time interval $t \in [0,\ T_f]$.

Remark 2: It is noted that the role of the ILC based force estimator is to learn an unknown time-varying disturbance which is different from a standard tracking objective in an ILC setting. Any disturbance rejection techniques such as ESO [11] will not work for this control objective. ○

IV. MAIN RESULT

Let $\mathbf{x}_1 = \theta$ and $\mathbf{x}_2 = \dot{\theta}$ and $\mathbf{x} = \left[\ \mathbf{x}_1^\intercal,\ \mathbf{x}_2^\intercal\ \right]^\intercal \in \mathcal{R}^{2m}$. For convenience, the following notations are used.

$$\mathbf{b}(\mathbf{x}_1, \mathbf{x}_2) \triangleq -M^{-1}(\mathbf{x}_1)\left[C(\mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 + \mathbf{f}(\mathbf{x}_2) + \mathbf{g}(\mathbf{x}_1)\right]. \tag{2}$$

Without any loss of generality, any force estimation problem in (1) can be converted to a stabilization problem for the nonlinear dynamics, represented in state-space as:

$$\begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{x}_2 \\ \mathbf{b}(\mathbf{x}_1, \mathbf{x}_2) \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}(\mathbf{x}_1) \end{pmatrix} (\mathbf{u} + \mathbf{d}), \tag{3}$$

equivalently (1) can be represented as:

$$\dot{\mathbf{x}}_i = \boldsymbol{\phi}(\mathbf{x}_i) + G(\mathbf{x}_i)(\mathbf{u}_i + \mathbf{d}) \tag{4}$$

where $\boldsymbol{\phi}(\mathbf{x})^\intercal = \left[\mathbf{x}_2^\intercal,\ \mathbf{b}(\mathbf{x}_1, \mathbf{x}_2)^\intercal\right]$ and $G(\mathbf{x}) = \begin{bmatrix} 0_{m\times m} \\ M^{-1}(\mathbf{x}_1) \end{bmatrix}$.

Remark 3: This problem formulation is quite similar to unknown parametric uncertainties presented in [15, Chapter 7]. As $G(\mathbf{x})$ is unknown, in the context of identifying the unknown forces $\mathbf{d}(t)$, the standard adaptive techniques [15], [21] cannot be directly applied. ○

Remark 4: The problem formulation is also similar to standard tracking problem used in ILC [15], [22], [23]. The system has a relative degree 1, if a contraction mapping based method is used, the derivative signal $\dot{\mathbf{x}}(t)$ is needed to design the pure feed-forward updating law with some convergence condition. This paper is motivated from the idea presented in [22]. The energy function used in [22] is exploited with careful analysis of the uniform boundedness of the trajectories. ○

It is assumed that the nonlinear mappings $\boldsymbol{\phi}(\cdot)$ and $G(\mathbf{x})$ are unknown. Due to Property 1, the $G(\cdot)$ is bounded globally.

The following assumptions are needed.

Assumption 2: For a given compact set B_Δ, there exists a continuous feedback: $\mathbf{h}(\mathbf{x}) : \mathcal{R}^{2m} \to \mathcal{R}^m$ and a continuously differentiable Lyapunov function $V(\mathbf{x}) : \mathcal{R}^{2m} \to \mathcal{R}_{\geq 0}$ and class $\mathcal{K}$ functions α_1, α_2 and α_3 which satisfies:

$$\begin{aligned} \alpha_1(|\mathbf{x}|) \ &\leq\ V(\mathbf{x})\ \leq\ \alpha_2(|\mathbf{x}|) \\ \frac{\partial V}{\partial \mathbf{x}}\left[\boldsymbol{\phi}(\mathbf{x}) + G(\mathbf{x})\mathbf{h}(\mathbf{x})\right] \ &\leq\ -\alpha_3(|\mathbf{x}|) \\ \left|\frac{\partial V}{\partial \mathbf{x}}\right| \ &\leq\ \alpha_4(|\mathbf{x}|) \end{aligned} \tag{5}$$

□

Remark 5: This assumption indicates that there is a feedback controller that can stabilize the system semi-globally. This assumption always holds for robotic systems. A simple PD controller with a sufficiently large gain usually satisfies this assumption. ○

Assumption 3: The system (4) satisfies an identical initial condition in all iterations, i.e. $\mathbf{x}_i(0) = \mathbf{x}^0$. □

Remark 6: Assumption 3 is a standard assumption in an ILC setting. This assumption can be relaxed. For example, refer [24], [25]. ○

The proposed control is given by

$$\begin{aligned} \mathbf{u}_i(t) &= \mathbf{h}(\mathbf{x}_i(t)) - \mathcal{P}[\widehat{\mathbf{d}}_i(t)] \\ \widehat{\mathbf{d}}_i(t) &= \mathcal{P}[\widehat{\mathbf{d}}_{i-1}(t)] + \Gamma_1 \mathbf{x}_{1,i} + \Gamma_2 \mathbf{x}_{2,i} \\ & \widehat{\mathbf{d}}_0(t) = 0,\ i = 1, 2, 3, ... \end{aligned} \tag{6}$$

where $\Gamma_1 \in \mathcal{R}^{m\times m}$ and $\Gamma_2 \in \mathcal{R}^{m\times m}$ are symmetric positive definite matrices, $\widehat{\mathbf{d}}_i(t)$ is an estimate of the disturbance vector $\mathbf{d}(t)$, $\mathbf{h}(\mathbf{x}, t)$ is chosen based on Assumption 2, $\mathcal{P}[\cdot]$ represent a projection operator. The projection operator for a scalar d is defined as $\mathcal{P}[d] \triangleq sign\,(d)\, min\{d_b, |d|\}$. For any $\mathbf{d} \in \mathcal{R}^m$ and the projection function is defined as $\mathcal{P}[\mathbf{d}] \triangleq \left[\mathcal{P}[d^1], \cdots, \mathcal{P}[d^m]\right]^\intercal$.

Remark 7: The projection operator is widely used in many adaptive control laws [26], [27]. It can be treated as saturation operator when input saturation is considered [28]. ○

Theorem 1 states that the proposed force observer can work.

Theorem 1: Assume that Assumptions 1, 2 and 3 holds. The trajectories of the closed- loop system, which consists of the plant, the control law and the updating law, satisfy

1) $\lim_{i\to\infty} |\mathbf{x}_i| = 0$ uniformly.
2) $\widehat{\mathbf{d}}_i$ converges to $\mathbf{d}(t)$ in $\mathcal{L}^2$ norm.

■

Proof. The proof of Theorem 1 can be found in Appendix.

V. SIMULATION AND EXPERIMENTAL VALIDATION

The effectiveness of the proposed force estimator is illustrated by using simulation examples and implementation on the prototype of an end effector based rehabilitation robot, EMU [9]. It is a fully actuated, highly backdrivable, three degrees of freedom robot with revolute joints. The joint-links are highlighted in Fig. 1 which shows the structure of the prototype model. For the ease of presentation, only two degrees of freedom: θ_2 and θ_3, are used in simulations as well as in experimental validations where control input and disturbance that excites θ_1 is taken as zero. For simulation, the nominal model of EMU from [29] is used.

The feedback control, that can stabilize the robot at a specified joint configuration $\mathbf{x}^0 \in \mathcal{R}^m$ is given by:

$$\mathbf{h}(\mathbf{x}_i) = -K_1\left(\mathbf{x}_{1,i} - \mathbf{x}^0\right) - K_2\mathbf{x}_{2,i} + \mathbf{g}_0(\mathbf{x}_{1,i}) \tag{7}$$

where K_1 and K_2 are symmetric positive definite matrices and $\mathbf{g}_0(\mathbf{x}_1)$ is the gravity compensation based on a nominal model. Assume that the initial joint configuration is given by $\mathbf{x}^0$, i.e $\mathbf{x}_{1,i}(0) = \mathbf{x}^0$ for all iteration i. Hence, the ILC update law in (6) takes the form:

$$\widehat{\mathbf{d}}_i(t) = \mathcal{P}[\widehat{\mathbf{d}}_{i-1}(t)] + \Gamma_1(\mathbf{x}_{1,i} - \mathbf{x}^0) + \Gamma_2\mathbf{x}_{2,i} \tag{8}$$

For the three degree of freedom manipulator, $\mathbf{x}_1 = [\theta_1,\ \theta_2,\ \theta_3]^\intercal$, $\mathbf{x}_2 = [\dot{\theta}_1,\ \dot{\theta}_2,\ \dot{\theta}_3]^\intercal$.

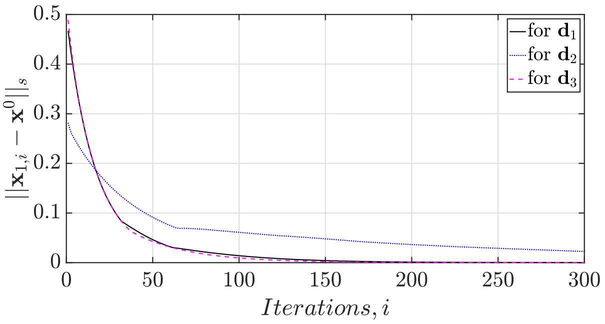

Fig. 4. Simulation Results: Supremum error of output error (joint position).

The disturbance torque has been applied as a feed-forward torque input in the controller to ensure an iteration invariant disturbance in experimental validation.

In simulation, three different disturbances are considered for illustration in simulations, which are as follows:
$\mathbf{d}_1(t) = [0, \ -1.2\sin(2\pi\frac{t}{T_f}), \ -\sin(2\pi\frac{t}{T_f})]^\intercal$, $\mathbf{d}_2(t) = [0, \ -6\sin(10\pi\frac{t}{T_f}), \ -5\sin(10\pi\frac{t}{T_f})]^\intercal$, and $\mathbf{d}_3(t) = \begin{cases} \mathbf{d}_1(t) & 0 \le t \le \frac{1}{2}T_f \\ \mathbf{0} & \frac{1}{2}T_f < t \le T_f \end{cases}$.

In experiments, only $\mathbf{d}_3(t)$ is tested. The disturbance bound is taken as $d_b = 1.6$ which is needed for the projection operator $\mathcal{P}[\cdot]$ in the proposed control law. The following parameters are taken for simulation and experiments: $K_1 = 0.1I_3$, $K_2 = 0.05I_3$, $\Gamma_1 = 0.05I_3$ and $\Gamma_2 = 0.1I_3$, where I_3 is an identity matrix of order 3. The initial joint configuration is taken as $\mathbf{x}_1(0) = \mathbf{x}^0 = [0, \ 0.9, \ 0.8]^\intercal$. The task is executed for a finite time interval $T_f = 2.5s$ and data are recorded with a sampling time of $0.001s$.

A. Simulation Results

In order to demonstrate the performance of the proposed control, the supremum norm of the error in joint angles $\left\|\mathbf{x}_{1,i} - \mathbf{x}^0\right\|_s$ is shown in Fig. 4. The convergence in Fig.4 indicates that the learning control is effective in compensating the unknown disturbances and therefore yielded a good estimate of input disturbance.

B. Experimental Results

As mentioned earlier, a known input disturbance $\mathbf{d}_3(t)$ is added from the control module in robotic manipulator shown in Fig. 1. It has to be noted that the effective disturbance torque at the the joint configuration will be different due to the nonlinearities in the electro-mechanical couplings of the manipulator configuration. However, adding disturbance torque with the control input to the actuators will generate an iteration-invariant disturbance torque at the joint level. Hence Assumption 1 is satisfied in experiments. The variation of supremum norm of error in joint position, $\left\|\mathbf{x}_{1,i} - \mathbf{x}^0\right\|_s$ is shown in Fig. 5. The convergence of $\left\|\mathbf{x}_{1,i} - \mathbf{x}^0\right\|_s$ indicates that the estimated values of disturbance, $\widehat{\mathbf{d}}_i$ converges to the disturbance torque at the joints. A butter-worth filter with a cut-off frequency of $20Hz$ is used to filter out noises in the disturbance estimation, $\widehat{\mathbf{d}}(t)$. The variation of joint positions at iteration number at $i = 1, \ 5, \ 18$ and corresponding estimated disturbances are shown in Fig. 6 and Fig. 7, respectively. Note that $\widehat{d}_{3,2}(t)$ and $\widehat{d}_{3,3}(t)$ in Fig. 7 indicates the second and third element of vector $\widehat{\mathbf{d}}_3(t)$ which is corresponds to the estimated disturbance at joints θ_2 and θ_3. The performance in experiments indicates the effectiveness of the proposed model-free estimation technique.

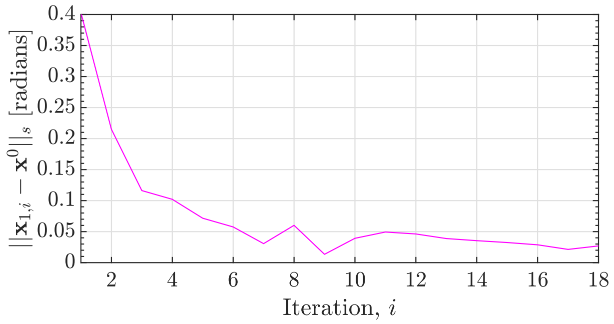

Fig. 5. Experimental Results: The variation of supremum norm of output error.

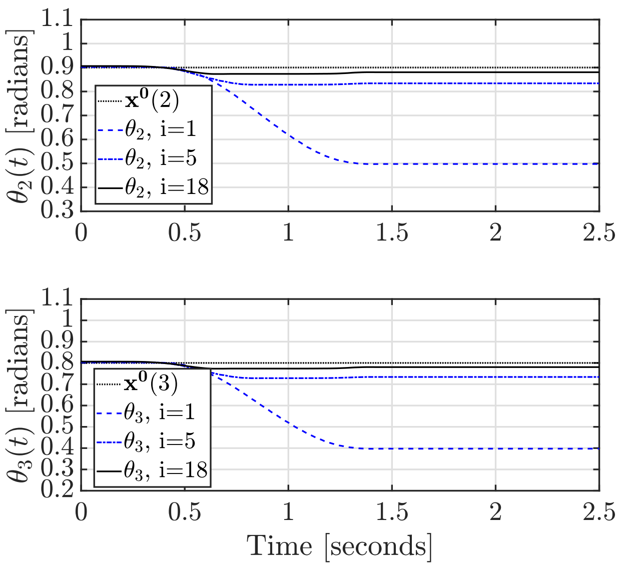

Fig. 6. Experimental Results: The output trajectories for $i = 1, 5, 18$

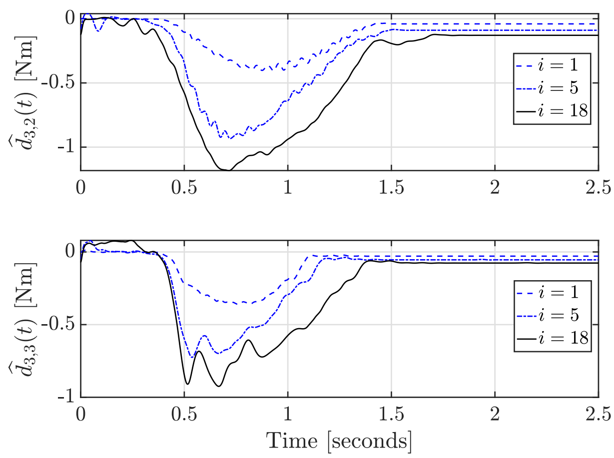

Fig. 7. Experimental Results: Estimated disturbance torque for $i = 1, 5, 18$

Our future work will validate the proposed force estimator with measurement from force sensors, for healthy subjects and stroke patients with appropriate Ethics approval.

VI. CONCLUSION

This paper proposed an iterative learning control algorithm to estimate external forces coming from stroke patients when repeating tasks with the help of a rehabilitation robot device. Without using the information of the model, the proposed ILC algorithm can identify time-varying but iteration-invariant forces. Simulation results and experimental validations show the effectiveness of the proposed method. Future work will test this force observer on stroke patients.

APPENDIX

Proof of Theorem 1.

Proof: In the first part of the proof, a few notations and relevant relations are established to facilitate the proof.

A new fictitious velocity, $\boldsymbol{\xi}$ is introduced for the ease of presentation, which is defined as:

$$\boldsymbol{\xi} \triangleq \mathbf{x}_2 + \Gamma_2^{-1}\Gamma_1\mathbf{x}_1 \tag{9}$$

The ILC law in (6) can be written in terms of $\boldsymbol{\xi}$ as:

$$\widehat{\mathbf{d}}_i = \mathcal{P}[\widehat{\mathbf{d}}_{i-1}] + \Gamma_2\boldsymbol{\xi}_i \tag{10}$$

For ease of presentation, denote $\Delta\mathbf{d}_i \triangleq \mathbf{d} - \widehat{\mathbf{d}}_i$, $\Delta\widetilde{\mathbf{d}}_i \triangleq \mathbf{d} - \mathcal{P}[\widehat{\mathbf{d}}_i]$, $M_i \triangleq M(\mathbf{x}_{1,i})$, $C_i \triangleq C(\mathbf{x}_{1,i}, \mathbf{x}_{2,i})$.

Using Property 3 from [28] and the boundedness of $\mathbf{d}$, it is possible to show that the following relation holds:

$$\left|\mathbf{d}(t) - \mathcal{P}[\widehat{\mathbf{d}}(t)]\right|^2 \leq \left|\mathbf{d}(t) - \widehat{\mathbf{d}}(t)\right|^2, \text{ i.e. } \left|\Delta\widetilde{\mathbf{d}}_i\right|^2 \leq |\Delta\mathbf{d}_i|^2 \tag{11}$$

In addition, using Property 4 from [28] and the boundedness of $\mathbf{d}$, it is possible to show that:

$$|\mathcal{P}[\mathbf{d}_i] - \mathbf{d}_i| \leq |\Gamma_2\boldsymbol{\xi}_i| \tag{12}$$

Multiplying with inertia matrix M on the time derivative of (9), followed by substituting (6) and (10) yields:

$$\begin{aligned} M_i\dot{\boldsymbol{\xi}}_i =& M_i\dot{\mathbf{x}}_{2,i} + M_i\Gamma_2^{-1}\Gamma_1\dot{\mathbf{x}}_{1,2} \\ =& M_i\mathbf{b}_i + \mathbf{u}_i + \mathbf{d} + M_i\Gamma_2^{-1}\Gamma_1\mathbf{x}_{2,i} \\ =& \Delta\widetilde{\mathbf{d}}_i - C_i\Gamma_2\boldsymbol{\xi}_i + \boldsymbol{\zeta}_i \end{aligned} \tag{13}$$

where $\boldsymbol{\zeta}_i \triangleq M_i\mathbf{b} + C_i\Gamma_2\boldsymbol{\xi}_i + M_i\Gamma_2^{-1}\Gamma_1\mathbf{x}_{2,i} + \mathbf{h}_i$. Therefore by rearranging the terms in (13), it is possible to show that:

$$\Delta\widetilde{\mathbf{d}}_i = M_i\dot{\boldsymbol{\xi}}_i + C_i\Gamma_2\boldsymbol{\xi}_i - \boldsymbol{\zeta}_i \tag{14}$$

A. Boundedness of trajectories

The boundedness of trajectories at any iteration can be achieved as follows. Substituting (6) back into system dynamics (4) yields,

$$\dot{\mathbf{x}}_i = \quad \boldsymbol{\phi}(\mathbf{x}_i) + G(\mathbf{x}_i)\left(\mathbf{h}(\mathbf{x}) - \mathcal{P}[\widehat{\mathbf{d}}_i(t)] + \mathbf{d}\right) \tag{15}$$

This means that the Lyapunov function,V from Assumption 2 satisfies

$$\begin{aligned} \dot{V}(\mathbf{x}_i) \;=\; \frac{\partial V}{\partial \mathbf{x}}\dot{\mathbf{x}}_i \;=&\; \frac{\partial V}{\partial \mathbf{x}}\left[\boldsymbol{\phi}(\mathbf{x}_i) + G(\mathbf{x}_i)\mathbf{h}(\mathbf{x})\right] \\ &+ \frac{\partial V}{\partial \mathbf{x}}G(\mathbf{x}_i)\left(-\mathcal{P}[\widehat{\mathbf{d}}_i] + \mathbf{d}\right) \\ \leq&\; -\alpha_3(|\mathbf{x}_i|) + C_r\alpha_4(|\mathbf{x}_i|) \end{aligned} \tag{16}$$

where $C_r = \max_{\mathbf{x}_i \in \mathcal{R}^{2m},\ t\in[0,T_f]} |G(\mathbf{x}_i)| \left|-\mathcal{P}[\widehat{\mathbf{d}}_i(t)] + \mathbf{d}(t)\right|$. Due to Property 1, $G(\cdot)$ is bounded. Due to Assumption 1, $\mathbf{d}(t)$ is bounded for all $t \in [0, T_f]$. By definition, projection function $\mathcal{P}[\cdot]$ is also bounded by the boundedness of $\mathbf{d}(t)$. This shows that C_r is also bounded. Hence, based on the properties of $\alpha_3(\cdot)$ and $\alpha_4(\cdot)$, it is possible to find a compact set $\mathcal{D}$ such that $\dot{V} \leq 0$. This leads to the boundedness of trajectories in all iterations.

B. Non-increasing Energy Function

Consider the energy function,$J_i(t)$ for some positive λ, $\forall t \in [0, T_f],\ i \in \mathcal{N}$:

$$J_i(t) \;=\; \int_0^t e^{-\lambda\tau}\Delta\mathbf{d}_i{}^{\intercal}(\tau)\Delta\mathbf{d}_i(\tau)d\tau, \tag{17}$$

The difference of energy function between two iterations, $\Delta J_i = J_i - J_{i-1}$. Using (11) and (10), the following inequality realtion can be obtained:

$$\begin{aligned} \Delta J_i =& \int_0^t e^{-\lambda\tau}\left(|\Delta\mathbf{d}_i|^2 - |\Delta\mathbf{d}_{i-1}|^2\right)d\tau \\ \leq& \int_0^t e^{-\lambda\tau}\left(|\Delta\mathbf{d}_i|^2 - \left|\Delta\widetilde{\mathbf{d}}_{i-1}\right|^2\right)d\tau \\ =& \int_0^t e^{-\lambda\tau}\left(\Delta\mathbf{d}_i - \Delta\widetilde{\mathbf{d}}_{i-1}\right)^{\intercal}\left(\Delta\mathbf{d}_i + \Delta\widetilde{\mathbf{d}}_{i-1}\right)d\tau \\ =& \int_0^t e^{-\lambda\tau}\left(\mathcal{P}[\widehat{\mathbf{d}}_{i-1}] - \widehat{\mathbf{d}}_i\right)^{\intercal}\left(2\mathbf{d} - \widehat{\mathbf{d}}_i - \mathcal{P}[\widehat{\mathbf{d}}_{i-1}]\right)d\tau \\ =& \int_0^t e^{-\lambda\tau}\left(-\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\left(2\Delta\mathbf{d}_i + \Gamma_2\boldsymbol{\xi}_i\right)d\tau \end{aligned} \tag{18}$$

Note that $\Delta\mathbf{d}_i = \Delta\widetilde{\mathbf{d}}_i + (\mathcal{P}[\mathbf{d}_i] - \mathbf{d}_i)$. Substituting this equality back into (18), followed by substituting (12) yields

$$\begin{aligned} \Delta J_i \leq& -2\int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\Delta\widetilde{\mathbf{d}}_i d\tau - \int_0^t e^{-\lambda\tau}\left|\Gamma_2\boldsymbol{\xi}_i\right|^2 d\tau \\ &- 2\int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\left(\mathcal{P}[\mathbf{d}_i] - \mathbf{d}_i\right)d\tau \\ \leq& -2\int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\Delta\widetilde{\mathbf{d}}_i d\tau + \int_0^t e^{-\lambda\tau}\left|\Gamma_2\boldsymbol{\xi}_i\right|^2 d\tau \end{aligned} \tag{19}$$

Substituting for $\Delta\widetilde{\mathbf{d}}_i$ from (14) into (19) yields:

$$\begin{aligned} \Delta J_i =& -2\int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\left(M_i\dot{\boldsymbol{\xi}}_i + C_i\Gamma_2\boldsymbol{\xi}_i\right)d\tau \\ &+ \int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\left(\Gamma_2\boldsymbol{\xi}_i + 2\boldsymbol{\zeta}_i\right)d\tau \end{aligned} \tag{20}$$

The following relation holds:

$$\begin{aligned} &-2e^{-\lambda t}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}M\dot{\boldsymbol{\xi}}_i \\ &= -\frac{d}{dt}\left(e^{-\lambda t}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}M_i\boldsymbol{\xi}_i\right) + e^{-\lambda t}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}\dot{M}_i\boldsymbol{\xi}_i \\ &\quad - \lambda e^{-\lambda t}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}M_i\boldsymbol{\xi}_i. \end{aligned} \tag{21}$$

Substituting (21) back to (20) and the invoking Property 2 results in

$$\begin{aligned} \Delta J_i \leq& -e^{-\lambda t}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}M_i\boldsymbol{\xi}_i - \lambda\int_0^t e^{-\lambda\tau}(\Gamma_2\boldsymbol{\xi}_i)^{\intercal}M_i\boldsymbol{\xi}_i d\tau \\ &+ \int_0^t e^{-\lambda\tau}\left(\Gamma_2\boldsymbol{\xi}_i\right)^{\intercal}\left(\Gamma_2\boldsymbol{\xi}_i + 2\boldsymbol{\zeta}_i\right)d\tau \end{aligned} \tag{22}$$

Using Property 1 and 3, it is possible to show that there exists two positive constants c_1 and c_2 such that, $\boldsymbol{\zeta}$ can be bounded as:

$$\boldsymbol{\zeta}_i \leq c_1\left|\boldsymbol{\xi}_i\right| + c_2\left|\boldsymbol{\xi}_i\right|^2. \tag{23}$$

Because of Property 1, there exists a positive constant $\gamma > \Gamma_2^{\mathsf{T}} M_i > 0$ for any symmetric positive definite matrix Γ_2. This leads to

$$\Delta J_i \leq -\gamma e^{-\lambda\tau} |\boldsymbol{\xi}_i|^2 d\tau - (\lambda\gamma) \int_0^t e^{-\lambda\tau} |\boldsymbol{\xi}_i|^2 d\tau + \int_0^t e^{-\lambda\tau} R(|\boldsymbol{\xi}_i|) d\tau \tag{24}$$

where $R(|\boldsymbol{\xi}_i|) = 2|\Gamma_2| \left(c_3 |\boldsymbol{\xi}_i|^2 + c_2 |\boldsymbol{\xi}_i|^3\right)$ is a polynomial function in $|\boldsymbol{\xi}|$ and $c_3 = c_1 + |\Gamma|$. For any compact set $\mathcal{D}$, there exists a constant $c_4 = c_4(\mathcal{D})$ such that

$$R(|\boldsymbol{\xi}_i|) \leq c_4 |\boldsymbol{\xi}_i|^2$$

By selecting $\lambda > \frac{1}{\gamma} c_4$, it follows that

$$\Delta J_{i+1} \leq \quad -\gamma e^{-\lambda\tau} |\boldsymbol{\xi}_i|^2 d\tau - \lambda\gamma \int_0^t e^{-\lambda\tau} |\boldsymbol{\xi}_i|^2 d\tau \leq 0$$

This shows that, by choosing a suitable λ results in a non-increasing energy function.

C. Convergence Property

As the energy function is bounded and non-increasing in the iteration-domain, the pointwise convergence of input signal in terms of $\mathcal{L}^2$ norm can be obtained. This indicates that the tracking error converges point-wisely. Using Barbalet Lemma, the uniform boundedness of $\boldsymbol{\xi}_i$ can be ensured, which leads to the uniform convergence of $\mathbf{x}_i$ when $i \to \infty$.

This completes the proof. ■

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