# Motion Planning with Continuous Contact for Dribbling in RoboCup

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### Abstract

A set of constraints for motion planning whilst maintaining contact between a robot and a passive object is considered. It is demonstrated that a set of kinematic limits for the trajectory can be determined based upon the available force range and system response to perturbations. The proposed constraints are discretised and reformulated for a 2-D nonholonomic robot before being utilised by a modified fluid based motion planner capable of kinematic parameter control. The effect of the constraints and the performance of the motion planner is demonstrated on a dribbling scenario for soccer playing robots. The results show the flexibility and effectiveness of the constraints in generating a suitable trajectory and the capability of the developed motion planner to observe the determined constraints.

## 1 Introduction

The motion planning of a mobile robot in a dynamic environment is a heavily researched topic within the field of robotics. Consideration of kinematic constraints such as actuation limitations and contact constraints allow for the safe application of these techniques onto a wide range of practical systems including nonholonomic robots, manipulators and robotic soccer. Specifically, in the case of robotic dribbling, motion planning must consider both the construction of trajectories that avoid other robots and respect field boundaries as well as the methodology with which to maintain possession of the ball throughout the generated motion. Evaluation of contact constraints can therefore allow the robot to both maintain possession and direct the ball's such that the dribbling problem reduces to a constrained motion planning problem.

The control of non constrained objects via the manipulation of robot dynamics consists of a range of tasks such as grasping [Montana, 1992; Abel *et al.*, 1985] and soccer dribbling [Altinger *et al.*, 2010; Damas *et al.*, 2002; de Best and van de Molengraft, 2008]. Specifically in the case of robotic dribbling there are two main approaches based on the ball handling mechanism utilised by the robot: Active mechanisms [de Best and van de Molengraft, 2008; Tang et al., 2012] utilise an actuator that prevents the robot from losing the ball such that the dribbling problem becomes a mechanism design problem. This approach guarantees that possession of the ball will be maintained however the application of an active mechanism requires significant hardware design and results in unnatural dribbling technique. In contrast, passive mechanisms [Altinger et al., 2010] typically utilise concavities that hold a portion of the ball throughout motion. This means that passive ball handling techniques transform the dribbling problem into a problem of impulse momentum based collision generation [Flores et al., 2012] or into a constrained kinodynamic motion planning problem whereby continuous contact [Li et al., 2007; Liu and Wang, 2005; Alouges et al., 2010 between the ball and robot is required.

In the motion planning of kinodynamically constrained mobile robots, several approaches have been studied. Search based methods Scheur and Fraichard, 1996; Sprunk et al., 2011] search through a sampled representation of the environment with the constraints formulated into the searching process. The primary drawback is the significant computational resources required making these methods unsuited for systems operating in dynamic environments. Spline based methods [Borenstein and Koren, 1991; Filippis et al., 2012] utilise geometric splines that satisfy desired constraints to construct the robot trajectory. These techniques require intermediate goal locations to be determined and can struggle in complicated environments. The Artificial Potential Field (APF) [Khatib, 1986] approach constructs a potential field function for the operating environment. Fluid motion planners are a subset of the APF approach that makes use of Harmonic Potential field functions that do not possess local minimum. These techniques can be modified for the control of kinematic

variables [Damas *et al.*, 2002; Desai and Kumar, 1991; Owen *et al.*, 2011] and provide a closed loop expression for the velocities at each time step resulting in high computational efficiency.

In this paper, continuous contact kinematic constraints are determined and applied to a modified fluid motion planner. By examining the interaction dynamics between a robot and a passive object, a set of kinematic constraints are formulated. These constraints are discretised and applied to a nonholonomic 2-D mobile robot. From the resulting constraints a modified fluid motion planner is proposed and demonstrated on a RoboCup dribbling example.

The remainder of the paper is organised as follows: Section 2 presents the constraints on the robot states necessary to maintain continuous contact between a robot and a passive rigid body. The resulting motion planning constraints for a 2-D nonholonomic robot are formulated in Section 3. Section 4 formulates a modification to fluid motion planners for the application of the determined constraints. The motion planning of a 2-D direct drive robot under continuous contact constraints in simulated and the results are shown is Section 5. Finally, Section 6 concludes the paper and presents areas of future work.

## 2 Continuous Contact Constraint

In this section, the robot kinematics necessary for maintaining continuous contact between two rigid bodies is considered. Two types of constraints are evaluated: the allowable force range of the interaction dynamics and the stability of the configuration in response to an applied perturbation.

The rigid body system model is shown in Figure 1. The pose of the robot can be defined by the absolute position of the centre  $\mathbf{x}_{\mathbf{A}} = [x_A, y_A]^T$ , expressed in Cartesian coordinates and its orientation  $\theta$  which is the Euler angle representing a rotation around the inertial Z-axis. Similarly the pose of the passive body is given by absolute position of its centre  $\mathbf{x}_{\mathbf{B}} = [x_B, y_B]^T$  where this body is considered symmetrical such that its orientation is not considered.

In addition to the absolute coordinate system  $\mathbf{x} = [x, y]^T$ , a moving robot fixed frame  ${}^1\mathbf{x} = [{}^1x, {}^1y]^T$  is defined to align with the robot orientation such that

$$\begin{bmatrix} 1 \\ 1 \\ y \end{bmatrix} = {}_0^1 R \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

where  ${}^{1}_{0}R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$  is defined as the transformation between the absolute coordinate system and the moving frame.



Figure 1: The Rigid Body System Model

### 2.1 Actuation Range

Force range limits are applicable to the system due to the requirement of actuating the passive object without losing contact with the robot. Provided that the passive object is assumed to be located at the desired point of contact relative to the robot centre  $\mathbf{x}_{AB} = [x_{AB}, y_{AB}]$ with angular velocity  $\omega = [0, 0, \dot{\theta}]^T$ , the acceleration of the object can be considered as the superposition of the translational robot acceleration  $\ddot{\mathbf{x}}_A$ , the rotational robot acceleration  $\omega \times (\omega \times \mathbf{x}_{AB}) + \dot{\omega} \times \mathbf{x}_{AB}$ , and the acceleration resulting from external forces acting upon the system  $\ddot{\mathbf{x}}_{B,ext}$ . This means that the acceleration of the object can be expressed in the robot fixed frame as

$${}^{1}\ddot{\mathbf{x}}_{\mathbf{B}} = {}^{1}_{0}R\ddot{\mathbf{x}}_{\mathbf{A}} + \omega \times (\omega \times {}^{1}\mathbf{x}_{\mathbf{AB}}) + \dot{\omega} \times {}^{1}\mathbf{x}_{\mathbf{AB}} + {}^{1}\ddot{\mathbf{x}}_{\mathbf{B},\mathbf{ext}}$$
$$= \begin{bmatrix} \ddot{x}_{A}\cos(\theta) + \ddot{y}_{A}\sin(\theta) - {}^{1}x_{AB}\dot{\theta}^{2} - {}^{1}y_{AB}\ddot{\theta} + {}^{1}\ddot{x}_{B,ext} \\ -\ddot{x}_{A}\sin(\theta) + \ddot{y}_{A}\cos(\theta) - {}^{1}y_{AB}\dot{\theta}^{2} + {}^{1}x_{AB}\ddot{\theta} + {}^{1}\ddot{y}_{B,ext} \end{bmatrix}$$
(2)

Continuous contact between the rigid bodies can only be maintained provided that the acceleration of the passive body lie within the range defined by the positive spanning vectors normal to the points of contact  $\mathbf{v_1}$  and  $\mathbf{v_2}$  as is shown in Figure 1. Assuming that the robot contact mechanism possesses the capability to actuate with sufficiently large contact forces, this means that for the positive spanning vector range  $(\mathbf{v_1}, \mathbf{v_2})$ , the requirements for maintaining continuous contact can be formulated in the form of the inequality

$$\hat{\mathbf{v}}_1 \le {}^1 \hat{\ddot{\mathbf{x}}}_{\mathbf{B}} \le \hat{\mathbf{v}}_2. \tag{3}$$

This inequality imparts constraints on the resulting trajectory at which the robot can manoeuver whilst preserving continuous contact with the passive rigid body.

## 2.2 Perturbation Analysis

The response of the configuration to an applied perturbation must be considered to determine whether the two bodies are able to return to the optimal contact position without significant change in robot behaviour. This capability is necessary such that the robot is able to consistently perform the same manoeuvres independent of sensor noise, uneven surfaces and model uncertainty.

The ability for the passive object to return to the optimal point of contact  $\mathbf{x}_{\mathbf{C}} = \mathbf{x}_{\mathbf{A}} + \mathbf{x}_{\mathbf{AB}}$  requires that that there exists a time t such that,

$$\int_0^t (\mathbf{^i} \dot{\mathbf{x}}_{\mathbf{B}} - \dot{\mathbf{x}}_{\mathbf{C}}) \mathbf{d}\tau = \mathbf{0}, \qquad (4)$$

without the passive object leaving the operating actuation range of the robot  $\mathcal{X}$  at any time  $t^* \leq t$ , such that

$$\int_0^{t^+} (\dot{\mathbf{x}}_{\mathbf{B}} - \dot{\mathbf{x}}_{\mathbf{C}}) d\tau = \mathcal{X}.$$
 (5)

For (4) to be achievable it is necessary that there must be an acceleration cancelling out the undesirable component of the passive object velocity acting normal to the velocity of the desired contact point. This restricts the resulting motion of the robot, where provided that the acceleration imparted onto the passive object by the robot satisfies the inequality

$${}^{1}\ddot{\mathbf{x}}_{B} \times {}^{1}\dot{\mathbf{x}}_{C} \le \mathbf{0},\tag{6}$$

there is the potential for passive object to return to the desired contact location.

It should be noted that satisfaction of (6) does not guarantee consistent stability for all possible perturbations as a result of (5). Instead stability can be guaranteed using (6) only for those perturbations for which the initial velocity of the ball relative to the desired contact point remains small enough such that the applied acceleration is able to cancel out the undesired velocity before it leaves the operating region  $\mathcal{X}$ .

#### 3 Trajectories to Dribble

The continuous contact constraints formulated in Section 2 consider a generalised robot model independent of actuation limits. In this section, the formulated constraints are applied to a RoboCup soccer playing robot and discretised such that a range of suitable accelerations can be determined at each instant in time resulting in a set of kinodynamic motion planning constraints necessary for maintaining continuous contact.

The model robot used in this application corresponds to the robotic system shown in Figure 1. This robot is a direct drive 2-D nonholonomic mobile robot with an omnidirectional dribbling device such that the velocity components are a function of the robot heading direction  $\phi$  and speed V where

$$\begin{bmatrix} \dot{x}_A \\ \dot{y}_A \end{bmatrix} = \begin{bmatrix} V \cos \phi \\ V \sin \phi \end{bmatrix}.$$
 (7)

Substitution of (7) and its derivative into the determined acceleration (2) under the assumption of negligible external forces such as friction, results in the robot specific acceleration

$${}^{1}\ddot{\mathbf{x}}_{\mathbf{B}} = \begin{bmatrix} V\cos(\alpha) + V\dot{\phi}\sin(\alpha) - {}^{1}x_{AB}\dot{\theta}^{2} - {}^{1}y_{AB}\ddot{\theta}\\ -V\sin(\alpha) + V\dot{\phi}\cos(\alpha) - {}^{1}y_{AB}\dot{\theta}^{2} + {}^{1}x_{AB}\ddot{\theta} \end{bmatrix}, \quad (8)$$

where  $\alpha = \theta - \phi$  represents the angle of the dribbler relative to the heading direction. This acceleration must satisfy the force range inequality (3), with unit vectors normal to the point of contact that provide a force range  $({}^{1}\hat{\mathbf{x}}_{AB}, {}^{1}\hat{\mathbf{x}})$  such that

$$\mathbf{\hat{x}_{AB}} \le \mathbf{\hat{x}_B} \le \mathbf{\hat{x}}.$$
 (9)

To allow for application onto a discrete motion planning system, the linear and angular velocity can be discretised such that

$$V_{k+1} = V_k + V\Delta t$$
  
$$\dot{\theta}_{k+1} = \dot{\theta}_k + \ddot{\theta}\Delta t.$$
 (10)

Applying the system discretisation (10) to the passive object acceleration (8), holding the dribbler angle constant and assuming that the time step chosen  $\Delta t \ll 1$ such that  $\Delta t^n \approx 0 \ \forall n > 1$  then results in an allowable acceleration range of the form

$$\frac{\ddot{A\phi} \leq B}{E} \leq \dot{V} \leq \frac{\ddot{F\phi} + G}{H},$$
(11)

where A, B, C, D, E, F, G and H represent constants at each timestep that are a function of the system instantaneous velocities  $V_k$  and  $\dot{\phi}_k$ , the time step  $\Delta t$ , the dribbler angle  $\alpha$  and the relative contact position  ${}^1\mathbf{x}_{AB}$  for that time step.

Similarly substitution of (7) into the stability condition (4) results in the robot specific stability requirement

$$\dot{x}_B = \dot{x}_A - {}^1 y_{AB} \dot{\theta} = V \cos \alpha - {}^1 y_{AB} \dot{\theta} \ge 0$$
  
$$\dot{y}_B = \dot{y}_A + {}^1 x_{AB} \dot{\theta} = -V \sin \alpha + {}^1 x_{AB} \dot{\theta} \ge 0$$
 (12)

where this result has made use of the available unit force range  $({}^{1}\hat{\mathbf{x}}_{AB}, {}^{1}\hat{\mathbf{x}})$ .

This equation can also be discretised via the substitution of the discretised velocity (10) into the stability region (12) resulting to an equivalent functional form to the accelerations allowed by the force range (11).

It should be noted that in addition to the determined acceleration ranges, the robot's actuation and saturation limits act to limit the allowable accelerations and velocities, where the actuation limits are defined as  $V \leq V_{max}$  and  $\dot{\theta} \leq |\dot{\theta}|$ .

Figure 2 illustrates the possible values of robot states, V and  $\dot{\theta}$  for the stability range (12), force range (9) and actuation limitations of the robot. The region  $X_1$  represents the intersection of all the stability, force and actuation limit ranges. This region therefore corresponds to the region for which all motion planning algorithms should seek to remain within when navigating a desired trajectory. In contrast the regions  $X_2$  lies outside of the stability range and hence consistent performance cannot be guaranteed in the event of a perturbation. Finally the regions  $X_3$  and  $X_4$  lies outside of the actuation limitations and force range of the robot respectively. As such these ranges cannot be actuated whilst maintaining possession of the ball.



Figure 2: Allowable Velocity Ranges

## 4 Modified Fluid Motion Planning Method

The fluid motion planning method is an APF approach that is inspired by natural fluid streamline flow [Waydo and Murray, 2003; Owen *et al.*, 2011]. The technique makes use of the established differential equations of fluid mechanics where the robot is driven to target locations modelled as sink elements, while avoiding obstacles modelled as source elements. Closed form expressions for the robot velocity are determined at each instant in time making fluid motion planning highly computationally efficient and therefore applicable to robot motion planning in dynamic environments such as robot soccer playing.

The fluid flow model for the motion planner is shown in Figure 3, where the robot is located at position  $\mathbf{x}_{\mathbf{A}}$ , the passive object is located at position  $\mathbf{x}_{\mathbf{B}}$  and the target destination is modelled as a sink (attraction) element located at position  $\mathbf{x}_{\mathbf{d}}$ . To aid in ensuring a path suitable for non-holonomic systems, a source (repulsion) element is located directly behind the passive object in the heading direction  $\mathbf{d}$  such that

$$\mathbf{x}_{\mathbf{s}} = \mathbf{x}_{\mathbf{B}} - \Delta \mathbf{d} \cdot \mathbf{d} \tag{13}$$

where  $\mathbf{d} = [\cos \phi, \sin \phi]^T$  and  $\Delta d > 0$  represents a constant distance between the robot and the source location



Figure 3: Fluid Motion Planning Model

behind the robot position.

The potential functions for the described model can be expressed as

$$\phi(\mathbf{x}_{\mathbf{B}}) = \frac{\mathbf{Q}_{\mathbf{s}}}{2\pi} \log ||\mathbf{x}_{\mathbf{B}} - \mathbf{x}_{\mathbf{s}}|| - \frac{\mathbf{Q}_{\mathbf{d}}}{2\pi} \log ||\mathbf{x}_{\mathbf{B}} - \mathbf{x}_{\mathbf{d}}|| \quad (14)$$

where  $Q_s, Q_d > 0$  represents the strengths of the source and sink elements, respectively. The absolute frame velocity vector  $\dot{\mathbf{x}}$  for the desired trajectory can be determined by taking the gradient of the potential function from (14). Figure 3 illustrates the robot trajectory genterated by the fluid motion planner. The geneated trajectory is a streamline from the fluid potential field that provides the allowable velocity commands for the robot to execute.

Obstacles can be avoided utilising the circle theorem [Owen *et al.*, 2011] which can be represented as

$$\phi_{circle}(\mathbf{x}_{\mathbf{B}}) = \phi(\mathbf{x}_{\mathbf{B}}) + \phi\left(\frac{\mathbf{R}^{2}\mathbf{x}_{\mathbf{B}}}{\left|\left|\mathbf{x}_{\mathbf{B}}\right|\right|^{2}}\right).$$
 (15)

Multiple obstacles avoidance has also been implemented [Waydo and Murray, 2003], where the resulting trajectory is determined by applying a distance based weighted superposition on the trajectories for each individual obstacle. The velocity of the trajectory in an n obstacle environment can be expressed a

$$\dot{\mathbf{x}}_{\mathbf{B}} = \sum_{i=1}^{n} \alpha_i \dot{\mathbf{x}}_i, \qquad (16)$$

where  $\alpha_i$  and  $\dot{\mathbf{x}}_i$  represents the weight and trajectory velocity for a single obstacle i, respectively.

### 4.1 Kinematic Constraints

With respect to a fluid motion planner, kinematic constraints may be satisfied through the appropriate selection of source and sink strength,  $Q_s$  and  $Q_d$ , respectively. From the potential functions (14) and (15), the velocity can be expressed in the form

$$\dot{\mathbf{x}}_B = Q_s \dot{\widehat{\mathbf{x}}}_s - Q_d \dot{\widehat{\mathbf{x}}}_d , \qquad (17)$$

where  $\hat{\mathbf{x}}_s$  and  $\hat{\mathbf{x}}_d$  represent the components of the velocity due to the source and sink elements, respectively, with strengths  $Q_s = Q_d = 1$ .

For the system model presented in Section 3 the velocity of the passive object can be expressed in an intermediate frame t oriented with the robot's heading direction as

$$\begin{aligned} \mathbf{t} \dot{\mathbf{x}}_{B} &= \begin{bmatrix} V - \dot{\phi}(^{1}y_{AB}\cos\alpha + ^{1}x_{AB}\sin\alpha) \\ \dot{\phi}(^{-1}y_{AB}\sin\alpha + ^{1}x_{AB}\cos\alpha) \end{bmatrix} & (18) \\ &= \begin{bmatrix} 1 & -(^{1}y_{AB}\cos\alpha + ^{1}x_{AB}\sin\alpha) \\ 0 & (-^{1}y_{AB}\sin\alpha + ^{1}x_{AB}\cos\alpha) \end{bmatrix} \begin{bmatrix} V \\ \dot{\phi} \end{bmatrix} \\ &= T_{v} \begin{bmatrix} V \\ \dot{\phi} \end{bmatrix}, \end{aligned}$$

where  $T_v$  is the matrix transformation from the linear and angular velocities to the ball velocity.

Similarly the desired fluid motion planning velocity can be expressed in this intermediate frame as

where  $R_z(-\phi)$  is the rotation matrix to the intermediate frame and  $T_f$  is the unit source vector matrix.

Equating the ball velocity (18) with the desired fluid motion planning velocity (19) and rearranging for the robot linear and angular velocity therefore results in the expression

$$\begin{bmatrix} V\\ \dot{\phi} \end{bmatrix} = T \begin{bmatrix} Q_s\\ Q_d \end{bmatrix} \tag{20}$$

where  $T = T_v^{-1} R_z(-\phi) T_f$  is the transformation matrix from the source and sink vectors to the velocity vectors.

As a result, the problem of generating a trajectory that satisfies the constraints determined in Section 3 can be solved by the selection of an appropriate desired speed and angular velocity. This speed and velocity must intersect with the region defined by the positive vector span of the columns of the transformed unit source vector matrix.

To maximise the natural stream line portion of the trajectory, it is desired that the ratio constant  $Q_r = \frac{Q_s}{Q_d}$  is maintained as a constant value. To achieve this in the motion planning strategy, a new  $Q_r$  is selected only if the  $Q_r$  for the previous time sample violates the continuous contact constraint.

## 5 Simulation and Results

To demonstrate the proposed fluid motion planning strategy presented in Section 4 and the effect of the trajectory constraints from Section 3, the motion planning for a nonholomic robot dribbling a soccer ball is shown in this section.

The nonholonomic robot is considered to possess a radius of 0.2m and actuation limitations of  $V_{max} = 4$ m/s and  $\left|\dot{\phi}\right|_{max} = \pi$  rad/s constraining its resulting motion.

In the first scenario, the continuous contact trajectory constraints were not considered and the general fluid motion planer was utilised with a constant linear acceleration of  $\dot{V} = 0.1 \text{ m/s}^2$  such that force was consistently applied to the ball. Figure 4 shows the resulting trajectory of the system, from initial position  $\mathbf{x}_{\mathbf{B}} = [0.3, 0]^{\mathbf{T}}$  to destination  $\mathbf{x}_{\mathbf{B}} = [0, -2]^{\mathbf{T}}$ , where it can be observed that this trajectory proved incapable of maintaining possession throughout the motion. The complete trajectory of the robot involves traversing the entire path of the semi-circle trajectory. Only the portion of the path before the displayed red cross may be completed whilst in contact with the ball as the additional constraints required for continuous contact are not satisfied beyond this point.

Figures 5(a) and 5(b) show the linear and angular velocities for the trajectory shown in Figure 4. For each velocity, the corresponding range of angular velocities for which the continuous contact constraints can be achieved is shown in Figure 5(b). It can be seen that there is a limited available range of angular velocities at each instant in time and that the failure to maintain continuous contact coincides with the instant at which the fluid motion planner provides an angular velocity outside of the allowable range.



Figure 4: Robot Trajectory Without Continuous Contact Constraint

In the second scenario, the continuous contact trajectory constraints were considered in the motion planning solution. For the same initial and final position as the previous scenario, the resulting trajectory is depicted in Figure 6.

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Figure 5: Speed and Velocity Profiles for Scenario 1

It can be seen that the resulting motion is significantly different to the motion without consideration of the continuous contact constraint. Figures 7(a) and 7(b) show the linear and angular velocities for this trajectory. It can be observed that to maintain contact with the ball, the robot has increased the acceleration of its motion thereby providing a larger range for the allowable angular velocities. This has resulted in the robot travelling from initial position to destination without subsequent loss of the ball. The resulting robot behaviour indicates the capability of the motion planner to generate a suitable trajectory for the dribbling problem.



Figure 6: Robot Trajectory With Continuous Contact Constraint

To demonstrate the ability of the constraints to be applied to a wider range of scenarios, Figure 8 shows the performance of the motion planner under different initial conditions and with an obstacle located between the robot and its destination. It can observed that the



Figure 7: Speed and Angular Velocity Profiles for Sce-

resulting trajectory is continuous and smooth and that the ball was able to be manoeuvred without violation of the determined constraints. The corresponding velocity and angular velocity profile for this trajectory are shown in Figures 9(a) and 9(b), respectively.



Figure 8: Robot Trajectory with Obstacle Avoidance

From the result displayed, it is shown that the proposed continuous contact constraints enable suitable selection of accelerations such that the fluid motion planner is able to generate collision free trajectories for the dribbling of a ball. The proposed algorithm makes use of closed form solutions for the robot state at each time step, thereby preserving the high computational efficiency of fluid motion planning techniques. The presented results do not show the effect of external forces when applied to the continuous contact constraint equation (3). The most common external force that would occur in practice, corresponds to the friction force be-



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Figure 9: Speed and Angular Velocity Profiles for Scenario 3

tween the ball, robot and ground. This acts to resist a change in relative position between the ball and robot such that its inclusion would result in a larger range of available trajectories, making the presented results a conservative estimate that does not require explicit friction modelling.

The use of the fluid motion planner is however limited by the selection of suitable accelerations within the determined continuous contact acceleration range. A strategy of allowing the passive object to manoeuvre under its own inertia could be used to allow for a greater range of possible trajectories for this motion planner. As such, the motion planning strategy and kinodynamic constraint formulation can be effective in real time applications where the continuous contact between a passive object and a robot is required.

## 6 Conclusion

In this paper, a set of trajectory coefficients necessary for maintaining continuous contact throughout robot motion has been formulated. The formulation of these constraints considered the available force range and the system response to perturbation such that consistent behaviour of the system could be achieved. To allow for application on a discrete nonholonomic motion planning system, the constraints were discretised and reformulated in terms of nonholonomic kinematic parameters. The constraints were then applied to a modified fluid motion planner in which control of the kinematic variables was formulated. The results were simulated for a 2-D direct drive nonholonomic soccer playing robot, where the robot was able to maintain contact with the ball from an initial pose to a final position. Future work will focus on the application of the formulated constraints to a wider range of continuous contact problems, the extension of the formulated constraints to include friction and the integration of the motion planning technique with spline based methods.

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