Cable Wrapping Phenomenon in Cable-driven Parallel Manipulators

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In the study of cable-driven parallel manipulators, the interference between the cable and the rigid link(s) of the mechanisms have generally been excluded when considering the workspace. This leads to the loss of perfectly feasible and useful regions of workspace. In this paper, such phenomenon is modelled by allowing the cable to wrap around the rigid link and the resulting pose to be included in the workspace consideration. A kinematic model of the path of a cable in a cable-driven parallel manipulator is constructed to include the segment of the cable wrapped over the surface of the rigid link, in addition to the conventional modelling of the straight (unwrapped) segments of the actuation cables. The path described by the cable wrapping about the rigid link is a function of the displacement of the rigid link(s). When wrapping occurs, the contact between the cable and the rigid link is no longer restricted to a stationary point with respect to the body attached frame, as in the case of a conventional cable-driven parallel manipulator, but is now a function of the pose of the rigid link. It is assumed that the cable remains taut at all times, therefore finding the cable configuration is equivalent to finding a geodesic solution on a convex hull of a rigid body. In this paper, the analysis is first presented for the basic case of finding the path of a single cable wrapped over an arbitrary (convex) rigid body, with specific illustration performed for cylindrical-shaped rigid body. Modelling and analysis is then applied to the case of a selected cable-driven parallel manipulator that allows wrapped cable segments in its operation.

Keywords: cable to rigid link interference, cable wrapping, cable-driven parallel manipulator, cable robotics.

1. Introduction

A cable-driven parallel manipulator (CDPM) is a type of parallel manipulators in which transmission of mechanical motion is carried out by the use of cables. It carries with it the advantages associated with the use of cables over rigid transmission, such as low effective inertia, high reconfigurability and portability. It also brings with it the additional complexity associated with the use of cables: its unilateral nature in the direction of actuation (a cable can be used only to pull and not push), the sagging of cables due to their weight and the cable elasticity. Since the first CDPM was reported over 20 years ago, they have been developed across many applications, from manufacturing (the Robocrane project at the NIST [1]) to rehabilitation (such as the CAREX [2] and the Cable-Driven Locomotion Interface (CDLI) for rehabilitation support [3]). Fundamental studies are also found in the literature, including the (wrench closure) workspace analysis of such mechanisms [4–8], trajectory planning [9, 10] and resolution of actuation redundancy [11].

During the operation of a CDPM, physical interference is a common issue in its operation, referring to the collision between cables (cable to cable) or between the cable and a rigid link. Of course, external interference with objects in the environment can also happen. In most of the conventional studies, the operation of a cable robot was assumed to be interference free, therefore such collisions are often avoid, resulting in a reduced workspace. When interference occurs during the operation of the robot, it potentially breaks the workspace of the robot into discontinuous regions within which the interference free assumption holds.

As a result, the operational strategy of a CDPM up to this point have been done by either excluding the interference from consideration or by including it with an appropriate modelling and strategy. The former approach is often adopted, with rigorous formulation of conditions for detecting the interference, such as the algorithm constructed to exclude the collision between actuating cables reported in [12]. The resulting condition to detect interference is useful whether one is attempting to exclude or to accommodate the interference. There are also algorithms to detect both self collisions and environment interferences in the 6-DoF workspace of a cable robot [13]. The latter approach includes the interference affected workspace, introducing an appropriate strategy to accommodate the resulting effect or even to utilize the phenomenon with appropriate motion regulation strategy. Studies reported in [14, 15] present algorithms to determine and to manage the interference in the Cable-Driven Locomotion Interface (CDLI) by minimizing the tension discontinuity in the affected cable(s). In [16] a method was proposed to expand the workspace of the cable robot by permitting collisions between two cables. The inverse kinematics of the manipulator cable lengths was derived and solved numerically.

In all these investigations, however, the collision between cable and the articulated rigid bodies forming the manipulator has not been addressed. This is a justifiable assumption in the conventional cable robotics where the only rigid body in the robot is a single piece of the end-effector, thus occupies a very small portion of the workspace volume (as shown in the schematic drawing in Fig. 1a). The interference between the single (moving) rigid body with the actuating cables therefore forms only a very small portion of the feasible workspace, if at all. Recent developments in cable-driven parallel manipulators involving articulated chain(s) of rigid bodies in a tensegrity setup yield a large range of motion in a compact design due to the serial chain rigid body kinematics while enjoying the benefits of parallel cable-driven actuation [2,4,17]. An example of CDPM with articulated endeffector forming by a kinematic chain of two rigid links is shown in the schematic drawing in Fig. 1b. In these cases, interference between the actuating cables and rigid body affects a significant portion of the wrench-closure workspace. With this view, the work presented in this paper was motivated and thus pursued.



Fig. 1. : CDPMs: (a)conventional; (b)multi-link with articulated end-effector

The interference between cable and the rigid link does not always affect the operation of a cable robot in an adverse manner. In fact, in addition to the recovery of useful workspace, allowing the interference to take place will result in segments of the actuating cables wrapping over the rigid bodies which assists in producing actuation moment (and motion) about the rigid link which would not have existed without the wrapping phenomenon.

In [18], the authors reported the initial work in this study of a single cable wrapping about a single rigid body. It is demonstrated the shortest path of the segment of a cable over the surface of a cylinder which was derived using the Lagrange's method. The boundary condition between the wrapping segment of the cable and its non-wrapping segment was enforced using a condition requiring continuous path and matching tangential gradient which allowed us to obtain a unique solution for the path of the cable. The analysis was performed on a specific shape of the rigid body, which was the cylindrical shape (as it is one of the most common shapes in the design of manipulator links).

In [19], the authors presented the incorporation of initial conditions: ie. the *cable wrapping direction* and the *number of revolutions* of the cable wrapping, to the cable wrapping phenomenon on a cylindrical rigid link. This further narrows the kinematic solution of the cable path presented in [18] to unique solutions, eliminating multiple possibilities introduced by these key factors in a cable wrapping phenomenon. For the practical application in the kinematics relationship in a manipulator, a unique solution is essential. It should be stated that overlapping wrapped cables were not considered in the study.

In this paper, a differentiable curve is found and proved to be a necessary and sufficient condition towards a unique inverse kinematic solution of the cable lengths subject to the pose of the rigid link that it wraps on, respecting the no-slack condition on each cable during the operation of the cable robot. In other words, any point along the cable has to obey both C_0 (non-breakable) and C_1 (smooth) continuity. Then, an overall algorithm of incorporating the wrapping phenomenon between cables and rigid links into the cable-driven parallel manipulator is presented. The complete framework for the kinematic modelling of the cable wrapping phenomenon is formulated and presented, as well as the proof that the shortest path is obtained once the appropriate conditions are imposed. The role of the initial conditions in determining the resulting cable path is presented as part of the complete formulation. Furthermore, the kinematic modelling is extended from that of a single cable path wrapping over a rigid body to the formulation of kinematics of a cabledriven parallel manipulator which permits wrapping of its (multiple) cables on its rigid body.

The rest of the paper is organized as follows: Section 2 introduces the modelling of a cable wrapping on a rigid body with a convex surface profile and the problem description. The smoothness along the entire cable is also proved to be a necessary and sufficient condition towards a unique solution of the cable configuration respecting the "no slack" condition (the shortest path). Section 3 provides the kinematic model and the parametric expressions of the wrapping segment of the cable on a cylindrical rigid link. This is a brief summary of findings reported in the initial work in [18] but necessary for the readability of this paper. Section 4 extends the modelling with geometric analysis to take into account the direction of wrapping and multiple revolutions of the cable around the rigid link. Section 5 takes into account the transition from straight to wrapped and vice versa. Section 6 validates the kinematic models against a physical robot.

2. Modelling of a single cable wrapping on a rigid body

In a conventional cable robot, a cable is fixed rigidly to a rigid link, thus the cable applies a force onto the rigid link always on the same point with respect to a reference frame attached to the rigid link. If the rigid link in a cable robot is allowed to collide with an actuating cable such that a segment of the cable wraps on the rigid link, then the cable applies its actuating force on the point where the cable leaves the rigid link, whose location with respect to a reference frame attached to the body varies as a function of manipulator pose. In the study of cable-driven parallel manipulators, the relationship between the motion of the end-effector and the forces of the cables are critical variables of interest. When wrapping is considered, the direction of the force of each cable cannot be calculated simply as the vector from one end of the cable to the other since it depends on the path taken by the cable, which can vary with the displacement of the rigid link(s) of the robot. Therefore, the identification of the cable configuration becomes necessary and important.

2.1. Problem Description

As the cable is assumed to remain taut during the operation of the cable-driven mechanism, finding the cable configuration is equivalent to finding the shortest path between the two ends of a cable. A general description of the problem can be defined as follow:

Given the displacement of the two points (points A_i and P_i) representing the two ends of a cable (cable *i*) in a 3-dimensional Cartesian space and the convex surface profile of a rigid body, and assuming that one end of the cable is attached on the rigid body (point A_i), what is the curve with the shortest path connecting these two points when part of the cable is wrapped on the rigid body?



Fig. 2. : Schematic illustration of cable wrapping

Fig. 2 graphically illustrates the situation in the problem description. In order to consider the use of multiple cables in a CDPM, the subscript i is used to

Journal of Robotics and Mechatronics Vol.0 No.0, 200x

denote the properties or features of cable *i* in the system. The two end points of cable *i* are denoted as P_i and A_i and the point separating the wrapping and non-wrapping segments of the cable is denoted as B_i . Note that point B_i is the point where the cable "leaves" the surface of the rigid body. The proposed solution to this problem is subject to some constraints, such as the convexity of the rigid body and the slack-free assumption of the cable at all time. Hence, a spatial curve, which comprises two parts, is a geometric representation of cable *i*:

- B_iP_i: non-wrapping segment, a spatial curve connecting one end of cable i (P_i) to a point on the surface of the rigid body (B_i);
- 2. A_iB_i : wrapping segment, a spatial curve on the convex hull of the rigid body.

As slack is not allowed in the cable, the resulting curves are the shortest paths between each of these two points. It is apparent that the non-wrapping segment B_iP_i is a straight line. Hence the problem reduces to solving for the curve B_iA_i .

Let l_{s_i} and l_{w_i} be the lengths of segments B_iP_i and A_iB_i . This results in an optimization problem which find the locations of B_i and the length of the wrapping segment l_{w_i} such that the length of the entire curve $l_i = l_{s_i} + l_{w_i}$ is the shortest from P_i to A_i under the additional constraint of a given number of complete wraps.

2.2. Parametric expression of the curve

This problem is posed in a 3-dimensional Cartesian space where a global coordinate frame $\{O\}$ is defined. The frame $\{G\}$ is attached to the center of gravity of the rigid body, as defined in Fig. (2). The wrapping segment A_iB_i is a curve on the convex hull of the rigid body and can be expressed parametrically as (1).

$$\alpha_o(r) = [x_o(r), y_o(r), z_o(r)]^T \in \mathbb{R}^3, \ r \in [r_{A_i}, r_{B_i}], (1)$$

where $\alpha_o(r)$ is the parametric expression of the curve $A_i B_i$ defined in frame $\{G\}$. The positions of A_i and B_i are $\alpha_o(r_{A_i})$ and $\alpha_o(r_{B_i})$, respectively. The location of point A_i is known because it is the cable end attached on the rigid body. The location of B_i is an unknown until both $\alpha_o(r)$ and the domain of r are found.

We expect to find the parametric expression of a curve representing the path taken by the cable such that its length is the shortest under the assumption that it is wrapped around the convex hull of the rigid body. In order to derive that expression, let us consider a small variation of any part of that curve:

$$\|\Delta \alpha_o(r)\| \simeq \sqrt{\Delta x_o^2 + \Delta y_o^2 + \Delta z_o^2} \Delta r = \|\alpha_o'(r)\| \Delta r$$
(2)

where (') denotes the differential operator $\frac{d}{dr}$. Hence, the length of the wrapping segment A_iB_i can be calculated

as

$$l_{wi} = \int_{r_{A_i}}^{r_{B_i}} \left\| \alpha'_o(r) \right\| dr = \int_{r_{A_i}}^{r_{B_i}} L(x'_o(r), y'_o(r), z'_o(r)) dr \quad (3)$$

where $L(x'_o(r), y'_o(r), z'_o(r))$ is the length function and is defined by:

$$L(x'_o(r), y'_o(r), z'_o(r)) = \sqrt{x'_o(r)^2 + y'_o(r)^2 + z'_o(r)^2}.$$
 (4)

It has been demonstrated that $\alpha_o(r)$ is the geodesic solution between the points A_i and B_i on the convex hull of the rigid body [18]. The expression of $\alpha_o(r)$ can be found by solving the following Lagrangian Equations (5)-(7).

$$\frac{\partial L(r,\alpha_o,\alpha'_o)}{\partial x_o} - \frac{d}{dr} \left(\frac{\partial L(r,\alpha_o,\alpha'_o)}{\partial x'_o} \right) = 0. \quad . \quad (5)$$

$$\frac{\partial L(r, \alpha_o, \alpha'_o)}{\partial y_o} - \frac{d}{dr} \left(\frac{\partial L(r, \alpha_o, \alpha'_o)}{\partial y'_o} \right) = 0, \quad . \quad (6)$$

$$\frac{\partial L(r, \alpha_o, \alpha'_o)}{\partial z_o} - \frac{d}{dr} \left(\frac{\partial L(r, \alpha_o, \alpha'_o)}{\partial z'_o} \right) = 0. \quad . \quad (7)$$

There may be multiple solutions to $\alpha_o(r)$. Each solution of $\alpha_o(r)$ describes a different shape of the cable path. Finding the location of B_i is equivalent to finding the optimal solution to the problem (8), which is expected to result in the optimal values for $\alpha_o(r)$ and r_{B_i} such that the length of a cable is the shortest.

$$\left(\alpha_o(r_{B_i})^*, r_{B_i}^*\right) = \underset{\alpha_o(r) \in \mathbf{R}^3, r \in \mathbf{R}}{\operatorname{arg\,min}} l_i \quad \dots \quad \dots \quad (8)$$

2.3. Solving for the point B_i

Given the convex surface hull of the rigid body, the rigid body pose (position and orientation) and the location of the two end points of the cable, the problem of solving for the location of point B_i can be solved based on the *quasi-static* assumption.

As $B_i P_i$ is known as a straight line, $\overrightarrow{B_i P_i}$ can be found when the position of B_i is known. In practical cases, the C^0 continuity along the entire curve (the entire length of the cable) is a necessary condition to ensure the validity of the mathematical model of the cable. Physically, it means that we assume the cable is continuous, i.e. the cable is not broken part way through its length. This additional constraint of maintaining C^0 continuity provides us with the necessary but not sufficient condition to obtain a unique solution to the location of point B_i . An example of two configurations satisfying C^0 continuity is shown in Figure 3, where the one on the right hand does not represent a typical behaviour of a physical cable. The smoothness of the curve is therefore taken into account such that the curve with the shortest path from A_i to P_i is not only $\mathbf{C}^{\mathbf{0}}$ but also C^1 continuous at B_i . The C^1 continuity condition can be described mathe-

The C^1 continuity condition can be described mathematically as equation (9).



Fig. 3. : Curves satisfy C⁰ continuity

$$\left\langle \overrightarrow{B_iP_i}, \alpha_o'(r_{B_i}) \right\rangle = \left\| \overrightarrow{B_iP_i} \right\| \left\| \alpha_o'(r_{B_i}) \right\| \dots \dots (9)$$

such that the length of cable i (l_i) is the shortest where $\alpha_o(r)$ is defined as equation (1). The proof of the equivalence between the shortest path statement and the tangency condition (9) is shown as below.

Proof:

Firstly, l_{s_i} , l_{w_i} can be calculated respectively as follows:

$$\begin{split} l_{s_{i}} &= \left\| \overline{B_{i}} \overrightarrow{P_{i}} \right\| \\ &= \sqrt{\left(x_{P_{i}} - x_{o}(r_{B_{i}})\right)^{2} + \left(y_{P_{i}} - y_{o}(r_{B_{i}})\right)^{2} + \left(z_{P_{i}} - z_{o}(r_{B_{i}})\right)^{2}} \\ l_{w_{i}} &= \int_{r_{A_{i}}}^{r_{B_{i}}} \left\| \alpha_{o}'(r) \right\| dr \\ &= \int_{r_{A_{i}}}^{r_{B_{i}}} \sqrt{\left(x'_{o}(r)\right)^{2} + \left(y'_{o}(r)\right)^{2} + \left(z'_{o}(r)\right)^{2}} dr \end{split}$$

Then the length of the entire curve is

$$l_i(r_{B_i}) = l_{s_i}(r_{B_i}) + l_{w_i}(r_{B_i}).$$
 (10)

The minimum l_i occurs when $\nabla l_i = 0$. That is,

$$\frac{dl_i}{dr_{B_i}} = \frac{dl_{s_i}}{dr_{B_i}} + \frac{dl_{w_i}}{dr_{B_i}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From equation (11):

$$\begin{aligned} \frac{\partial l_{s_i}}{\partial r_{B_i}} &= \frac{1}{l_{s_i}} \left[-(x_{P_1} - x_o(r_{B_i})) x'_o(r_{B_i}) \right. \\ &\quad -(y_{P_1} - y_o(r_{B_i})) y'_o(r_{B_i}) - (z_{P_1} - z_o(r_{B_i})) z'_o(r_{B_i}) \right] \\ &= -\frac{1}{l_{s_i}} \left\langle \overrightarrow{B_i} \overrightarrow{P_i}, \ \alpha'_o(r_{B_i}) \right\rangle \\ &\quad \frac{\partial l_{w_i}}{\partial r_{B_i}} &= \frac{\partial}{\partial r_{B_i}} \int_{r_{A_i}}^{r_{B_i}} \left\| \alpha_o'(r) \right\| dr = \left\| \alpha_o'(r_{B_i}) \right\| \end{aligned}$$
Note that $l_{s_i}(r_{B_i}) = \left\| \overrightarrow{B_i} \overrightarrow{P_i} \right\|,$

Journal of Robotics and Mechatronics Vol.0 No.0, 200x

$$0 = \frac{\partial l_{s_i}}{\partial r_{B_i}} + \frac{\partial l_{w_i}}{\partial r_{B_i}}$$

$$0 = \frac{1}{\left\| \overline{B_i} \overrightarrow{P_i} \right\|} \left\langle \overline{B_i} \overrightarrow{P_i}, \alpha_o'(r_{B_i}) \right\rangle - \left\| \alpha_o'(r_{B_i}) \right\|$$

$$\therefore \left\langle \overline{B_i} \overrightarrow{P_i}, \alpha_o'(r_{B_i}) \right\rangle = \left\| \overline{B_i} \overrightarrow{P_i} \right\| \left\| \alpha_o'(r_{B_i}) \right\|$$

The above proof demonstrated that the C^1 continuity also applies at the connection points of the wrapping segment and non-wrapping segments. Thus the entire path is C^1 continuous. Geometrically it means that $\overrightarrow{B_iP_i}$ is tangential to the surface of the rigid body at B_i . It can be expressed as (12)

$$\frac{\overline{B_i}\overline{P_i}}{\left\|\overline{B_i}\overline{P_i}\right\|} = \frac{\alpha_o'(r_{B_i})}{\left\|\alpha_o'(r_{B_i})\right\|}.$$
 (12)

To this end, given the configuration of the cable and the rigid body, point B_i can be found by solving the equation (12).

3. Modelling of a cable wrapping on a cylinder

The general case of a cable wrapping on a rigid body with a convex shape has been given in the previous section. In the rest of this paper, we focus on the modelling of the cable wrapping phenomenon for a shape commonly used for a manipulator link: a cylinder. Fig.4 shows a cable wrapping around a cylindrical bar. One end of the cable *i* is attached on the rigid link at point A_i (which is assumed to be stationary with respect to the coordinate frame attached to the rigid link) while the other end passes through point P_i , which is assumed to be stationary in the inertial frame. Each cable is actuated by the adjustment of its length.

In order to find the configuration of the wrapping segment A_iB_i , three coordinate frames are defined as follows: Frame $\{O\}$ which is the inertial frame with origin at the base of the cylinder; Frame $\{O_c\}$ which is attached to the cylindrical rigid body with the same origin as Frame $\{O\}$; and Frame $\{O_i\}$ which translates and rotates with the cylinder. The z axis of Frame $\{O_c\}$ is aligned with the axis of the cylinder in the positive direction. Frame $\{O_i\}$ is obtained by translating Frame $\{O_c\}$ along the axis of the cylinder and rotating about its z-axis by a constant angle r_{A_i} such that point A_i always lies on the x-axis of frame $\{O_i\}$ as shown in Fig. 4.

The optimal solution of $\alpha(r)$ can be found by solving the Lagrangian equation of the function *L* with respect to $\alpha(r)$. In this case, we know that such curve on the cylindrical surface with radius *a* can be expressed parametrically as (13) defined in frame $\{O_c\}$, that is,

$$\alpha(r) = [a\cos u(r), \ a\sin u(r), \ z(r)], \ r \in [r_A, r_B].$$
(13)

Note that $\alpha(r)$ only describes the shape of any curve wrapping on the cylindrical surface. Each curve will have



Fig. 4. : The coordinate frames of the single cable wrapping system

its own coefficients and boundary conditions. With the expression of $\alpha(r)$ in (13), the Lagrangian equations become:

where L is the arc-length function in (4):

Hence, the solution of $\alpha_i(r)$ for the wrapping segment A_iB_i of cable *i* can be expressed as

with $r \in [r_{A_i}, r_{B_i}]$. r_{A_i} is a known constant because the position of A_i is given but the values of r_{B_i} and *b* will need to be solved by using the constraint in (12). The expression in (17) represents a helix (which captures the shape of a cable path wrapped tautly around cylindrical body) and *r* represents its the angular displacement. The details of this solutions can be found in [18].

4. Enhanced Modelling by Geometric Analysis

The segment of a cable wrapping on a cylindrical link has been found to be in the form of (17). To better capture the phenomenon of a cable wrapping on the cylindrical surface, two parameters, namely, the wrapping direction (λ_i) and the number of revolutions (n_i) of cable wrapping are introduced in the parametric expression. Thus the expression of the wrapping segment of cable *i* becomes:

$$\alpha_i(r) = [a\cos(\lambda_i r), a\sin(\lambda_i r), b_i r], \ r \in [r_{A_i}, r_{B_i}].$$
(18)

Here $\lambda_i = \pm 1$ represents the cable wrapping about the positive and negative z_i direction, respectively, as shown in Fig.5. In (18), the total angular displacement of r (de-

noted as $r_{A_iB_i}$) and the coefficient b_i will need to be determined subject to the pose of the cylindrical link and the setting of the mechanism. The coordinate of P_i is known through the rigid body motion of the cylinder in frame $\{O_i\}$, i.e. ${}^iP_i = [{}^ix_{P_i}, {}^iy_{P_i}, {}^iz_{P_i}]$.



Fig. 5. : The wrapping direction (λ_i) and the number of revolution (n_i)

4.1. Wrapping Direction

At the initialisation of the problem, the wrapping direction (λ_i) and the number of revolutions (n_i) of the path of each cable in the cable driven parallel manipulator need to be set to match the initial conditions of the physical system. Given these initial conditions, these parameters can then be determined, and kept track throughout the operation of the manipulator, as presented in Section 4.2.

To find the unique solution of $r_{A_iB_i}$ with initially specified or calculated λ_i and n_i , we can consider the projection of the cable on the $x_i y_i$ -plane because $r_{A_i B_i}$ is directly proportional to n_i . The analysis is conducted mainly based on the wrapping direction of the cable and the position of point P_i . The wrapping phenomenon can be analyzed in each quadrant of $x_i y_i$ -plane. Here I_i , II_i , III_i and IV_i denote the four quadrants of $x_i y_i$ -plane where A_i is always located at (a,0,0) in $\{O_i\}$. Q_{Pi} and Q_{Bi} are the quadrants that P_i and B_i are located in, respectively. As $B_i P_i$ is tangent to the cylinder, so is the projection of the vector on the plane to the corresponding section. This implies that the position of point B_i for the case of cable wrapping over a rigid cylindrical body, will only appear in the same or the adjacent quadrant of point P_i 's. The operator "-" and "+" in the first columns of Table 1 and Table 2 are adopted to illustrate the quadrant before or after Q_{Pi} while I_i also means one "unit" of quadrant in a quantitative manner.

The angles θ_i and γ_i are graphically shown in Table 1 and Table 2 and are defined as $\theta_i = \angle O_i P_i B_i$ and $\gamma_i = \angle O_i P_i C_i$. Both θ_i and γ_i can be calculated by using (19) and (20). Their ranges are defined as $\theta_i \in [0, \frac{\pi}{2}]$ and $\gamma_i \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. By defining the ranges of the two angles, there exists a unique pair of solutions to (19) and (20). Hence, $r_{A_iB_i}$ for each case in Table 1 and Table 2 can be determined in terms of θ_i , γ_i and n_i .

$$\boldsymbol{\theta}_{i} = \sin^{-1} \left(\frac{a}{\sqrt{\left(i x_{P_{i}}\right)^{2} + \left(i y_{P_{i}}\right)^{2}}} \right), \ \boldsymbol{\theta}_{i} \in \left[0, \frac{\pi}{2}\right] \ (19)$$

$$\gamma_i = \tan^{-1} \left(\frac{i_{X_{P_i}}}{i_{Y_{P_i}}} \right), \ \gamma_i \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \quad . \quad . \quad (20)$$

Table 1 illustrates the cases where the cable wraps about the positive z_i -axis, i.e. $\lambda_i = 1$. When P_i is in quadrant I_i and II_i , i.e. ${}^i y_{P_i} \ge 0$, $r_{A_iB_i}$ can be expressed as

When P_i is in quadrant III_i and IV_i , i.e. ${}^iy_{P_i} < 0$, $r_{A_iB_i}$ can be expressed as

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Table 2 illustrates the cases that the cable wraps about the negative z_i -axis, i.e. $\lambda_i = -1$. When P_i is in quadrant I_i and II_i , i.e. ${}^i y_{P_i} \ge 0$, $r_{A_iB_i}$ can be expressed as

When P_i is in quadrant III_i and IV_i , i.e. ${}^iy_{P_i} < 0$, $r_{A_iB_i}$ can be expressed as

Note that γ_i calculated by (20) is negative in quadrant II and IV for both wrapping directions as ${}^i x_{P_i} / {}^i y_{P_i}$ is negative. Therefore, from (21)-(24), it can be shown that for any λ_i ,

$$r_{A_iB_i} = (\theta_i - \lambda_i \gamma_i) + 2n_i \pi; \text{ for } \lambda_i^{\ i} y_{P_i} \ge 0 \quad . \quad . \quad (25)$$

$$r_{A_iB_i} = \pi + (\theta_i - \lambda_i\gamma_i) + 2n_i\pi; \text{ for } \lambda_i^{\ i}y_{P_i} < 0.$$
 (26)

(25) and (26) can be expressed in a compact expression that accommodates both cases ($\lambda_i^{\ i} y_{P_i} \ge 0$ and $\lambda_i^{\ i} y_{P_i} < 0$),

$$\phi_i = \frac{\pi}{2} \left(1 - \lambda_i \frac{^{i} y_{P_i}}{|^{i} y_{P_i}|} \right) + (\theta_i - \lambda_i \gamma_i). \quad . \quad . \quad . \quad (28)$$

One last case is the coincidence of the projections of A_i and P_i . In this situation, multiple cable configurations can be obtained with different values of $r_{A_iB_i}$. Hence, $r_{A_iB_i}$ needs to be recomputed subject to the change of the wrapping state. This case is discussed in Section 4.2.

4.2. Number of Revolutions of Cable Wrapping

The number of revolutions of the cable *i* changes when point B_i crosses the positive x_i - axis. This can be indicated by the value of $r_{A_iB_i}$ as it represents the location of point B_i : the ϕ_i part of $r_{A_iB_i}$ calculated from (28) determines Q_{Bi} , for $r_{A_iB_i} \in [-2\pi, 2\pi]$ while the $2n_i\pi$ part of



Table 1. Positive wrapping $\lambda_i = 1$

Table 2. : Negative wrapping $\lambda_i = -1$



 $r_{A_iB_i}$ relates to the number of revolutions. Moreover, two situations trigger the update of n_i and they both require the comparison of values of $r_{A_iB_i}$ for two subsequent sampling instances. $r_{A_iB_i}$ and n_i of the previous sampling instance are denoted as r_{ip} and n_{ip} , respectively. The first update condition occurs when $r_{A_iB_i}$ changes from quadrant IV_i to quadrant I_i . The number of revolution then changes by λ_i , i.e. $n_i = n_{ip} + \lambda_i$. The second condition is when $r_{A_iB_i}$ changes from quadrant I_i to quadrant IV_i , where the number of revolution changes by $-\lambda_i$, i.e. $n_i = n_{ip} - \lambda_i$. When n_i is determined, $r_{A_iB_i}$ can be calculated by (27). The algorithm can be summarized in the flowchart in Fig 6.



Fig. 6. : Steps to calculate $r_{A_iB_i}$

5. Switching between wrapping and nonwrapping states

The wrapping phenomenon is one of the circumstances that may occur when a cable interacts with one or more rigid bodies during the operation of the mechanism. Therefore, the general interaction between a cable and a rigid body should be treated as a combination of wrapping and non-wrapping phenomenon. Moreover, from the point of view of the wrapping direction, there are three possible states, namely, *positive wrapping* ($\lambda_i = 1$), *negative wrapping* ($\lambda_i = -1$) and *non-wrapping* ($\lambda_i = 0$). Transitions occur between wrapping and non-wrapping states. Each change of wrapping direction from $\lambda_i = 1$ to $\lambda_i = -1$ and vice versa has to pass through the nonwrapping state $\lambda_i = 0$. The following expression summarizes the transitions among the three states:

$$(\lambda_i = 1) \rightleftharpoons (\lambda_i = 0) \rightleftharpoons (\lambda_i = -1).$$

The operator " \Rightarrow " indicates the bidirectional transition between the wrapping and the non-wrapping states of the cables. In this section, a method is introduced to determine the "trigger" condition of the switch among these three states. Such condition is defined by the intersection of A_i and B_i . When the two points intersect, the switch will be triggered. This circumstance happens when the length of the wrapping segment becomes zero and the non-wrapping segment is tangential to the surface at point A_i .



Fig. 7. : Determination of the wrapping state

An algorithm is constructed to determine the occurrence of wrapping as well as its direction. Firstly, $r_{A_iB_i}$ is calculated by (27). When the transition occurs, the number of revolutions of the cable must be zero ($n_i = 0$). In order to judge whether the cable is under positive wrapping, negative wrapping or non-wrapping, different ranges of $r_{A_iB_i}$ will need to be considered. From (27) and the domains of θ_i and γ_i (given in (19) and (20), respectively), the full range of $r_{A_iB_i}$ is equal to the domain of ϕ_i which is $\left[-\frac{\pi}{2}, 2\pi\right]$ when state transition occurs (i.e. $n_i = 0$). Here the range is divided into three parts, namely, $\left[-\frac{\pi}{2}, 0\right]$, $\left[0, \frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2}, 2\pi\right]$. Each of the first two sub-ranges results in one case each while the last sub-range results in two cases. The values of $r_{A_iB_i}$ and λ_i in the previous sampling instance (denoted as r_{ip} and λ_{ip} , respectively) are employed in making the decision. The flowchart in Fig. 7 illustrates how the algorithm determines the appropriate $r_{A_iB_i}$ with respect to the pose of the cylinder when wrapping condition is considered. Note that $r_{A_iB_i} = 0$ in the non-wrapping state since the wrapping segment does not exist. The four cases are given as follows:

Case 1. a $r_{A_iB_i}$ value within $\left[-\frac{\pi}{2}, 0\right]$ represents a cable configuration such as shown in Fig. 8(a) which cannot happen in a practical situation. In fact, it implies that in this circumstance the cable does not wrap. Furthermore, the cable is assumed to be always under tension(taut). In practice, Fig. 8(b) would result from this case. Thus $r_{A_iB_i} = 0$.



Fig. 8. : Entering non-wrapping state when $\lambda_i = 1$: (a) invalid configuration, (b) actual configuration

- *Case* 2. if $r_{A_iB_i}$ is within $[0, \frac{\pi}{2}]$, the cable wraps without changing its direction, i.e. the value of $r_{A_iB_i}$ remains. Otherwise, $r_{A_iB_i}$ must be within the range of $[\frac{\pi}{2}, 2\pi]$ and r_{ip} will need to be used to classify *Cases* 3 and 4.
- *Case* 3. if $r_{ip} \neq 0$, it implies that the cable was wrapping in the previous sampling instance. Thus the cable continues to wrap in the same direction and $r_{A_iB_i}$ remains unchanged.
- *Case* 4. if $r_{ip} = 0$, it indicates that the cable was not wrapping originally but it has a wrapping segment in II_i , III_i , or IV_i in the current sampling instance. However, with a practical assumption that the cable should always start to wrap from I_i , $r_{A_iB_i} \in \left[\frac{\pi}{2}, 2\pi\right]$ implies the need of a change in the wrapping direction in reality. Thus the wrapping direction reverses, i.e. $\lambda_i = -\lambda_{ip}$, and $r_{A_iB_i}$ will need to be recomputed by (27) with the updated λ_i according to (27). Fig. 9 illustrates this circumstance.

Journal of Robotics and Mechatronics Vol.0 No.0, 200x



Fig. 9. : Changing wrapping direction (λ_i) : (a) from nonwrapping state: $r_{ip} = 0$, (b) invalid wrapping: $\lambda_i = \lambda_{i_p}$, (c) reverse wrapping: $\lambda_i = -\lambda_{i_p}, r_{A_iB_i} \in (0, \frac{\pi}{2})$

At this stage, the values of λ_i , n_i and $r_{A_iB_i}$ are finalised. The coefficient b_i , in (18) can be calculated by (29) such that the position of point B_i is found.

$$b_i = {^i z_{p_i}}/\left[r_{A_i B_i} + \cot\left(\lambda_i r_{A_i B_i} + \gamma_i\right)\right]. \quad (29)$$

After b_i is calculated, the unique configuration of cable *i* is found.

6. Validation of kinematic model for Cabledriven Parallel Manipulatorwith wrapping

A 3-DOF (1 moving rigid link) manipulator with 4 cables, such as shown in Fig. 10, is considered for the experimental validation. The cylindrical rigid link with 2cmdiameter is articulated by a spherical joint at its base, with its centre located at the origin of Frame $\{O_c\}$.



Fig. 10. : A 3DoF manipulator with 4 cables

The cylinder is considered to be long enough such that the cables would never leave the surface during wrapping. Here cooridinate of P_i is given in the inertial frame $\{O\}$. (a, r_{A_i}, h_i) denotes the polar coordinate of the location of point A_i with respect to the frame attached to the cylindrical rigid link $\{O_c\}$. $\{O_i\}$ is defined by rotating $\{O_c\}$ about z_c by the angle of r_{A_i} and is originated at $(0, 0, h_i)$ defined in $\{O_c\}$. Then the coordinate of A_i is (a, 0, 0) in O_i . ${}^{O}P_i$ and h_i are constants and are given in Table 3. The initial conditions of the operation are listed in Table 4.

Cable <i>i</i>	1	2	3	4
$^{o}P_{i}$	[39,0,0]	[0, 39, 0]	[-39, 0, 0]	[0, -39, 0]
$r_{Ai}(rad)$	0	$\frac{\pi}{3}$	π	$\frac{3\pi}{2}$
h_i (cm)	14.2	16.2	20.2	24.2

Table 3. : The parameters of the 3DoF-4-cable-1-cylinder mechanism in the example

Cable <i>i</i>	1	2	3	4
λ_i	-1	0	1	0
n_i (rev)	0	0	1	0
r_{Bi} (rad)	4.7380	0	4.7380	0

Table 4. : Initial cable robot configuration ($\theta_z = 0^\circ$)



Fig. 11. : The cylinder rotates for 540° about Z-axis

Fig. 11(a) shows the initial cable configuration of the cable-driven parallel manipulator. The rigid-link then undergoes a motion trajectory that sees it rotated about Z-axis from $\theta_z = 0^{\circ}$ (initial) to $\theta_z = 540^{\circ}$. The inverse kinematics of the mechanism was calculated to



Fig. 12. : Cable lengths of the manipulator for $0^{\circ} \le \theta_z \le 540^{\circ}$ rotatiing about *Z*-axis.

obtain the resulting cable configurations over the duration of the rotation. Fig. 11(b) shows the configuration at the completion of the 540° rotation. The properties of the wrapping segment at $\theta_z = 540^\circ$ are listed in Table 5. Fig. 12 shows the resulting kinematic relationship between cable length and one of the robot's end-effector coordinates (θ_z). It can also be seen that the cable lengths calculated from the kinematic model matches the cable length measurements from the physical robot for the range of values of θ_z .

Cable <i>i</i>	1	2	3	4
λ_i	-1	-1	-1	-1
n_i (rev)	2	1	0	1
r_{Bi} (rad)	14.1628	7.3560	1.5964	7.3560

Table 5. : Final cable robot configurations ($\theta_z = 540^\circ$)

It can be observed in this example that:

1. Initially at $\theta_z = 0^\circ$, both cable 2 and 4 are in the nonwrapping state while cable 1 and 3 are already wrapping about the cylinder in opposite directions.

2. When $\theta_z = 120^\circ$, cable 2 and 4 enter the wrapping state and start to wrap till the end of the motion.

3. When $\theta_z = 300^\circ$, cable 3 enters the non-wrapping state and wraps again in an opposite direction when $\theta_z = 420^\circ$.

4. During the motion, cable 1 remained in negative wrapping ($\lambda_1 = -1$) for $0^\circ \le \theta_z \le 540^\circ$ while cable 3 experienced all the three states.

5. For this trajectory of the end-effector, cable lengths vary linearly with respect to θ_z when wrapping is in effect.



Fig. 13. : Cable lengths of the manipulator for $0^{\circ} \le \theta_z \le 270^{\circ}$ rotating about Z-axis.



Fig. 14. : Cable lengths of the manipulator for $0^{\circ} \le \theta_z \le 270^{\circ}$ rotating about *Z*-axis.

In the previous example, the axis of the cylinder remains vertical during the operation. Another example could be the axis of cylinder with a constant deflection by 30° from Z - axis and it rotates about Z - axis at the center of the ball joint from 0° to 270° as shown in Fig.

Journal of Robotics and Mechatronics Vol.0 No.0, 200x

13, the cable lengths change during the motion as shown in Fig. 14.

7. Conclusion and Future Work

The inverse kinematic modelling for a cable driven parallel manipulator which accounts for its cables wrapping (about its rigid link) is presented. It includes a procedure to find the cable configuration (wrapped and/or nonwrapped) which results in the shortest cable path, given the no-slack condition at all time. The conditions required to find such a configuration are provided and justified with a proof to ensure the shortest path is found. The algorithm also takes into consideration the wrapping direction and number of revolutions of cable wrapping about the rigid link to account for the initial conditions when matching with a physical scenario of a manipulator. The validity of the proposed algorithms was also demonstrated with measurements against a physical cable robot.

Future work will consider the incorporation of this modelling strategy into the motion control strategy of a cable driven manipulator of such characteristics. Inspired by the biomechanics of human (and animal) bodies, the framework can be utilised to provide a more accurate kinematics estimation for some musculoskeletal biomechanics that involve muscle wrapping about skeletal systems.

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