Conditions on the Cable-Routing Matrix for Wrench Closure of Multilink Cable-Driven Manipulators

Darwin Lau
Institut des Systèmes Intelligents et de Robotique
Université Pierre et Marie Curie
Paris, France, F-75005
Email: laudt@unimelb.edu.au

Denny Oetomo
Department of Mechanical Engineering
The University of Melbourne
Victoria, Australia, 3010
Email: doetomo@unimelb.edu.au

*Wrench-closure is an important property of cable-driven parallel manipulators (*CDPMs*) representing the ability to generate wrench in any direction by positive cable forces. For single link CDPMs, it is well known that $m \geq n+1$ cables are necessary for an n degree-of-freedom CDPM to achieve wrench-closure. However, for multilink cable-driven manipulators (*MCDMs*), the single link condition is not enough and the cable routing should also be considered. In this paper, necessary conditions to achieve wrench-closure for MCDMs based solely on the cable routing arrangements are mathematically derived. Since the approach is independent on the exact attachment locations, the proposed necessary conditions can be efficiently validated during the design and synthesis of MCDMs. Analysis is performed on a range of different MCDM structures to identify cable arrangements that do not satisfy wrench-closure for an MCDM.*

1 Introduction

Multilink cable-driven manipulators (*MCDMs*) are a class of cable-driven parallel manipulators (*CDPMs*) that possesses a serial multi-body rigid link structure and parallel arrangement of cables to drive the mechanism. MCDMs [1, 2] have been studied in recent years due to their anthropomorphic structure and unique advantages [3–6].

As with CDPMs, one unique characteristic of MCDMs is that cables can only be driven unilaterally in tension and not in compression (*positive cable force*). This constraint results in challenges in control [7–9], cable arrangement optimisation [10–12] and workspace analysis [13–18] of CDPMs. One key challenge in the study of MCDMs is the large number of cable routing for increasing number of cables and links [4]. *Cable routing* refers to the path in which a cable is connected to the links of the manipulator.

The ability to achieve a particular workspace criterion is an important property that describes the feasibility of cables for the CDPM to produce wrench and motion. Various types of CDPM specific workspace conditions have been studied, such as cable-interference workspace [13], static workspace [14], wrench-feasible workspace [15] and wrench-closure workspace [16–18].

Amongst the workspace conditions, the wrench-closure condition (*WCC*), defined as the ability for the manipulator in a particular pose to sustain any external wrench or to produce any velocity and acceleration by a set of positive cable forces, has been of significance. The wrench-closure workspace (*WCW*) represents the set of poses in which the manipulator satisfies the WCC. As a result, the WCW contains the poses in which the manipulator is able to generate motion in any generalised coordinates direction.

The WCC condition is dependent on both the pose of the manipulator and the attachment locations of the cables. For a given set of attachment locations, the generation of poses that satisfy the WCC is computationally complex, particulary for systems with a high number of degrees-of-freedom with many cables. As such, the analysis of WCW during the design process of the manipulator would be computationally impractical. An alternative approach to understand the WCW is to study the *wrench-closure validity* of the system, where a CDPM is referred to as wrench-closure valid if the WCC can be achieved in at least some pose. As such, wrench-closure validity is a pose independent criterion that can be used as a requirement in designing CDPMs where its WCW is not empty.

However, the classification of wrench-closure validity is still dependent on the design of cable attachments. As such, it is possible to consider *necessary conditions* on the wrench-closure validity that form a more basic set of criteria that must be satisfied in the design of an MCDM. Any cable arrangement that do not satisfy the necessary conditions will result in a manipulator with an empty WCW and hence a potentially very restricted range of motion. For single link CDPMs with n degree-of-freedom (*DoF*) actuated by m cables, one well-known necessary condition to achieve WCC is that a minimum of $m = n+1$ cables is required [19, 20]. For MCDMs, it was shown that a minimum requirement of $m = n+1$ cables is also required to achieve wrench-closure validity for MCDMs [21]. In [22], the consideration of the distributions of cables (cable routing) was incorporated. However, the analysis only considered single segment cable routings where cables were connected from the base to one of the manipulator links.

In this paper, necessary conditions on the cable routing required to achieve wrench-closure for MCDMs with arbitrary cable routing are mathematically studied. By deriving these conditions with respect to the Cable-Routing Matrix (CRM), it is shown that stricter conditions on the cable routing than $m \geq n+1$ should be considered to achieve wrench-closure. The wrench-closure validity analysis is demonstrated on different MCDM examples, showing that the proposed conditions result in a reduction in possible valid cable arrangements that have to be considered when designing MCDMs. The primary advantages of the proposed method are that the WCW does not need to be generated and the conditions are independent to both pose and attachment locations, resulting in an intuitive and computationally efficient method that is beneficial to determining invalid cable arrangements for MCDMs.

The remainder of the paper is organised as follows: Section 2 presents the CRM representation for cable routing. Section 3 formulates conditions on the Jacobian matrix of MCDMs to achieve wrench-closure. These conditions are then expressed with respect to the CRM in Section 4, forming the necessary conditions for wrench-closure with respect to the cable routing. Section 5 illustrates the formulated conditions on several example manipulators. Finally, Section 6 concludes and presents areas of future work.

2 Cable-Routing Matrix Representation

To represent and model the cable routing for MCDMs, a Cable-Routing Matrix (*CRM*) was introduced in [4] to allow arbitrary cable routing to be described. The CRM is a 3-D matrix representation that describes the path in which cables route within the MCDM, where the term $c_{ij(k+1)}$ within the CRM describes the cable routing relationship between segment j of cable i and link k, where:

1. $c_{ij(k+1)} = -1$: segment j of cable i begins from link k
2. $c_{ij(k+1)} = 1$: segment j of cable i ends at link k
3. $c_{ij(k+1)} = 0$: segment j of cable i not connected to link k

For an m cable system, each cable has a maximum of s segments and each cable segment can be attached onto a maximum of $p+1$ bodies (inertial base and p links). Defining $\mathcal{C} = \{-1, 0, 1\}$, the CRM for an MCDM system C can be regarded as a 3-D matrix with dimensions $m \times s \times (p+1)$ and $C \in \mathcal{C}^{m\times s\times(p+1)}$. Properties 1 to 4 were defined to mathematically ensure that the CRM cable routings are valid.

Property 1. $\sum_{k=1}^{p+1} c_{ijk} = 0 \ \forall i \in \{1,\dots,m\} \ \forall j \in \{1,\dots,s\}$. *Cable segments have beginning and ending attachments.*

Property 2. $\sum_{k=1}^{p+1} |c_{ijk}| = 2 \ \forall i, \ \forall j \in \{1,\dots,s_i\}$. *Segments* 1 *to* s_i *are attached to different links.*

Property 3. $c_{ijk} = 0 \ \forall i, \ \forall j \in \{s_i+1,\dots,s\}, \ \forall k$. *Segments* $j > s_i$ *must not have any attachment points to any links.*

Property 4. $\forall i, \ \forall j \in \{1,\dots,s_i-1\} \ \exists k \in \{1,\dots,p+1\} : c_{ijk} - c_{i(j+1)k} = 2$. *Consecutive segments are connected.*

The cable segment vector $\mathbf{l}_{ij}$ for segment j of cable i can be expressed as

$$\mathbf{l}_{ij} = \sum_{k=0}^{p} \left[c_{ij(k+1)} \mathbf{r}_{OA_{ijk}} \right] , \tag{1}$$

where O is the origin of the inertial frame and A_{ijk} is the attachment location of segment j of cable i on link k. For an n DoF CDPM, the equations of motion can be expressed as

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = -J^T(\mathbf{q})\mathbf{f} , \tag{2}$$

where M, $\mathbf{C}$ and $\mathbf{G}$ represent the mass-inertia matrix, centrifugal and Coriolis vector, and gravitational vector, respectively. The generalised coordinates and set of cable forces are denoted by $\mathbf{q} \in \mathbb{R}^n$ and $\mathbf{f} \in \mathbb{R}^m$, respectively. The transpose of the Jacobian matrix $J^T \in \mathbb{R}^{n\times m}$ can be expressed as

$$J^T = \begin{bmatrix} J^T_{1\mathbf{q}_1} & \dots & J^T_{m\mathbf{q}_1} \\ \vdots & & \vdots \\ J^T_{1\mathbf{q}_p} & \dots & J^T_{m\mathbf{q}_p} \end{bmatrix}, \ J^T_{i\mathbf{q}_a} = \sum_{k=a}^{p} S_a^T \begin{bmatrix} I_{3\times3} \\ \left[\mathbf{r}_{P_aA_{ijk}}\right]^\times \end{bmatrix} V_{ik}^T , \tag{3}$$

where $I_{3\times3}$ is a 3-by-3 identity matrix, $S_a^T \in \mathbb{R}^{n_a\times6}$ represents the projection of forces of link a to its degrees of freedom (n_a) and defines the type of joint, $\mathbf{r}_{P_aA_{ijk}}$ is the position vector from P_a to A_{ijk}, and $[\cdot]^\times$ represents the skew symmetric matrix. The term V_{ik}^T can be expressed as

$$V_{ik}^T = \sum_{j=1}^{s} \left[c_{ij(k+1)} \, \hat{\mathbf{l}}_{ij} \right] . \tag{4}$$

3 Jacobian Matrix Analysis of Wrench-Closure

For an m cable n DoF CDPM system in pose $\mathbf{q} \in \mathbb{R}^n$, the WCC is satisfied if any arbitrary wrench can be produced by a set of positive cable forces. From the equations of motion (2), the WCC condition is mathematically equivalent to

$$WCC(\mathbf{q}) \Leftrightarrow \forall \mathbf{w}\ \exists \mathbf{f} \geq \mathbf{0} : \mathbf{w} = J^T(\mathbf{q})\mathbf{f}\,, \quad (5)$$

where $\mathbf{w} \in \mathbb{R}^n$ represents the wrench produced on the manipulator by the cable forces $\mathbf{f} \in \mathbb{R}^m$ under the positive force constraint $\mathbf{f} \geq \mathbf{0}$. The transpose of the Jacobian matrix $J^T \in \mathbb{R}^{n\times m}$ for MCDMs is that defined in (3).

A CDPM is defined as being *wrench-closure valid (WCV)* if the WCC from (5) can be achieved in at least some pose

$$WCV \Leftrightarrow \exists \mathbf{q} : WCC(\mathbf{q})\ . \quad (6)$$

Wrench-closure validity analysis can be performed by studying the transpose of the Jacobian matrix J^T from (5). For single link CDPMs to achieve wrench-closure, $J^T \in \mathbb{R}^{n\times m}$ must be of full rank and contain more columns than rows. The wrench-closure validity for single link CDPMs can be mathematically expressed as

$$\exists \mathbf{q} : WCC(\mathbf{q}) \Rightarrow m \geq n+1. \quad (7)$$

MCDMs differ from single link CDPM systems in that it is possible for cables to provide actuation forces to only a subset of the manipulator's links. As introduced in Section 2, $J^T_{i\mathbf{q}_a}(\mathbf{q})$ from (3) represents the effect of the force of cable i on the wrench exerted on link a in pose $\mathbf{q}$. Property 5 describes the physical interpretation of zero elements in J^T.

Property 5. *If the force of cable i does not contribute to the generalised force of link a in pose* $\mathbf{q}$ *then* $J^T_{i\mathbf{q}_a}(\mathbf{q}) = \mathbf{0}$. *Hence, if* $J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}$ *then force in cable i contributes to wrench exerted on link a in pose* $\mathbf{q}$.

Necessary conditions for wrench-closure validity can be derived by considering and analysing a p link MCDM system as p individual subsystems.

Theorem 1. *Each link of an MCDM requires a minimum number of* $n_a + 1$ *cables that can be used to actuate the link, where* n_a *denotes the number of degrees of freedom of link a relative to link* $a-1$. *Hence, the wrench-closure validity for an MCDM system can be expressed as*

$$\exists \mathbf{q} : WCC(\mathbf{q}) \Rightarrow m_a(\mathbf{q}) \geq n_a + 1\ \forall a \in \{1,\ldots,p\}, \quad (8)$$

where $m_a(\mathbf{q})$ *refers to the number of cables that can be used to actuate link a in pose* $\mathbf{q}$.

Proof. *The definition of WCC from* (5) *for MCDMs requires that any wrench can be produced by every link simultaneously. Hence, if any of the links are unable to independently generate an arbitrary wrench, the WCC for the entire system cannot be satisfied at pose* $\mathbf{q}$. *Extending from the single link condition from* (7), *if any link has less than* n_a+1 *cables that can actuate link a, then the manipulator cannot satisfy WCC*

$$\exists \mathbf{q}\ \exists a : m_a(\mathbf{q}) < n_a + 1 \Rightarrow \neg WCC(\mathbf{q}). \quad (9)$$

The negation of (9) *completes the proof.*

For a single link system the resulting redundancy is $m-n$. Property 6 extends this concept to each link for a multilink manipulator.

Property 6. *Given that the system satisfies the WCC from Theorem 1, the maximum redundancy for link a is* $m_a - n_a$, *where* m_a *is the number of cables that can be used to actuate link a.*

Although the requirement of (8) is a simple extension of the well-known single link condition, it is important to note that $m_a(\mathbf{q})$ for MCDMs is not simply the number of cables in the system. At pose $\mathbf{q}$, the *effective number of actuating cables* on link a can be expressed as

$$m_a(\mathbf{q}) = A_a(\mathbf{q}) + B_a(\mathbf{q}), \quad (10)$$

where in pose $\mathbf{q}$:

1. $A_a(\mathbf{q})$ refers to the number of cables that produce a wrench on link a, but not on links $b > a$ (Lemma 1).
2. $B_a(\mathbf{q})$ refers to the number of cables that produce a wrench on link a and on some link(s) $b > a$, but are not required in actuating links $b > a$ (Lemma 2).

Lemma 1. $A_a(\mathbf{q}) = \sum_i \alpha_{ia}(\mathbf{q})$ *represents the number of cables that produce a wrench on link a at pose* $\mathbf{q}$, *but not on links* $b > a$, *where*

$$\alpha_{ia}(\mathbf{q}) = \begin{cases} 1\ ,\ J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}, J^T_{i\mathbf{q}_b}(\mathbf{q}) = \mathbf{0}\ \forall b > a \\ 0\ ,\ \textit{otherwise} \end{cases}. \quad (11)$$

Proof. *The condition* $\alpha_{ia}(\mathbf{q}) = 1$ *from* (11) *can be directly obtained from Property 5.*

Property 7. *In general, it can be shown that if* $J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}$ *and* $J^T_{i\mathbf{q}_b}(\mathbf{q}) \neq \mathbf{0}$, *then* $J^T_{i\mathbf{q}_c}(\mathbf{q}) \neq \mathbf{0}$ *where* $a < c < b$. *That is, if cable i produces a resultant wrench on both joints a and b, then a resultant wrench is also produced on the joints of the links in between. This is true due to the propagation of interaction forces through the joints of the manipulator.*

Lemma 2. $B_a(\mathbf{q}) = \min\{\sum_i \beta_{ia}(\mathbf{q}), m_{a+1}(\mathbf{q}) - n_{a+1}\}$ *represents the number of cables that produce a resultant wrench on link a and on some link(s)* $b > a$, *but are not required in actuating links* $b > a$, *where*

$$\beta_{ia}(\mathbf{q}) = \begin{cases} 1\ ,\ J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}, J^T_{i\mathbf{q}_{a+1}}(\mathbf{q}) \neq \mathbf{0} \\ 0\ ,\ \textit{otherwise} \end{cases}, \quad (12)$$

with the boundary conditions $n_{p+1} = m_{p+1} = 0$ *and* $\beta_{ip}(\mathbf{q}) = 0\ \forall \mathbf{q}$ *always holds since* $J^T_{i\mathbf{q}_{p+1}}(\mathbf{q})$ *does not exist.*

Proof. *At the outermost link,* $B_p(\mathbf{q}) = 0$ *due to the fact that there exists no link* $b > p$. *For the remaining links* $a < p$, $\sum_i \beta_{ia}(\mathbf{q})$ *from* (12) *represents the number of cables that produce a resultant wrench on both links* a *and* $a+1$ *in pose* $\mathbf{q}$ *(Property 5). The number of cables* $B_a(\mathbf{q})$ *must also consider the fact that some cables may be required to actuate link* $a+1$. *From Property 6, the maximum number of redundant cables that can be inherited from link* $a+1$ *is* $m_{a+1}(\mathbf{q}) - n_{a+1}$. *From Property 7, since cable forces produce wrench over consecutive joints,* $m_{a+1}(\mathbf{q}) - n_{a+1}$ *not only represents the number of redundant cables from link* $a+1$, *but also from links* $b \geq a+1$. *Considering maximum number of redundant cables from links* $b \geq a+1$, *the number of cables* $B_a(\mathbf{q})$ *can be expressed as* $\sum_i \beta_{ia}(\mathbf{q})$ *and cannot exceed* $m_{a+1}(\mathbf{q}) - n_{a+1}$, *resulting in* $B_a(\mathbf{q}) = \min\{\sum_i \beta_{ia}(\mathbf{q}), m_{a+1}(\mathbf{q}) - n_{a+1}\}$.

From Lemma 1, Lemma 2 and (10), the condition (8) from Theorem 1 can be expressed as

$$\exists \mathbf{q} : WCC(\mathbf{q}) \Rightarrow A_a(\mathbf{q}) + B_a(\mathbf{q}) \geq n_a + 1\ \forall a. \tag{13}$$

The necessary condition (13) for the WCC at a particular pose can be evaluated from the terms of the J^T matrix. It should be noted that the necessary condition (13) is dependent on both the manipulator pose and cable attachment locations, requiring the determination of J^T at $\mathbf{q}$.

4 Conditions on the Cable-Routing Matrix

Necessary conditions independent to the exact attachment locations are now formulated in this section.

4.1 Relationship With the Jacobian Matrix

Theorem 2 presents the necessary condition for the Jacobian matrix vectors $J_{i\mathbf{q}_a}(\mathbf{q})$ are zero dependent on the CRM.

Theorem 2. $\sum_{k=a}^{p} c_{ij(k+1)} = 0\ \forall j \Rightarrow J^T_{i\mathbf{q}_a}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}$. *Cable* i *has no effect on the motion of link* a *for all manipulator poses and cable attachment locations if all segments of cable* i *begin and end on, or are not attached to, links* a *or above.*

Proof. *From the definitions* (3) *and* (4), *the term* $J^T_{i\mathbf{q}_a}(\mathbf{q})$ *can be expressed as*

$$J^T_{i\mathbf{q}_a}(\mathbf{q}) = S_a^T \sum_{j=1}^{s} \sum_{k=a}^{p} \left[\begin{bmatrix} I_{3\times 3} \\ \left[\mathbf{r}_{P_a A_{ijk}}\right]^{\times} \end{bmatrix} c_{ij(k+1)}\, \hat{\mathbf{l}}_{ij} \right], \tag{14}$$

where p *and* s *represent the number of links of the system and the maximum number of segments for a cable, respectively. Denoting*

$$J^T_{ia_j}(\mathbf{q}) = \sum_{k=a}^{p} \left[\begin{bmatrix} I_{3\times 3} \\ \left[\mathbf{r}_{P_a A_{ijk}}\right]^{\times} \end{bmatrix} c_{ij(k+1)}\, \hat{\mathbf{l}}_{ij} \right], \tag{15}$$

the statement $J^T_{i\mathbf{q}_a}(\mathbf{q}) = \mathbf{0}$ *from* (14) *for all poses* $\mathbf{q}$ *is true if*

$$J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall j\ \forall \mathbf{q} \Rightarrow J^T_{i\mathbf{q}_a}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}, \tag{16}$$

Using (1), *the expression from* (15) *can be expressed as*

$$J^T_{ia_j} = \sum_{k=a}^{p} \sum_{b=0}^{p} \begin{bmatrix} I_{3\times 3} \\ \left[\mathbf{r}_{P_a A_{ijk}}\right]^{\times} \end{bmatrix} c_{ij(k+1)} c_{ij(b+1)} \mathbf{r}_{OA_{ijb}}. \tag{17}$$

The conditions for $J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall j\ \forall \mathbf{q}$ *in* (16) *can be expressed with respect to the CRM terms for the cable segments* $j \leq s_i$ *and* $j > s_i$, *and are described in Conditions 1 and 2, respectively. The number of segments for cable* i *is denoted by* s_i.

Condition 1. *For cable segments* $j > s_i$, $J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}$, $\forall j > s_i$ *is true by Property 3 as* $c_{ijk} = 0\ \forall j > s_i$.

Condition 2. *For segments* $j \leq s_i$, $J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}$ *is true if and only if the beginning and ending attachments for segment* j *are either both below or both on and above link* a.

For segments $j \leq s_i$, *assuming that segment* j *of cable* i *begins on link* x *and ends on link* y, *then* $c_{ij(x+1)} = -1$, $c_{ij(y+1)} = 1$ *and* $c_{ij(k+1)} = 0\ \forall k \neq x, y$. *Consider the following scenarios for links* x *and* y:

1. $x, y < a$: *both beginning and ending attachments are below link* a, *hence* $c_{ij(k+1)} = 0\ \forall k \geq a$, *resulting in* $J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}$.
2. $x, y \geq a$: *both beginning and ending attachments are on link* a *or above, substituting* $c_{ij(x+1)} = -1$ *and* $c_{ij(y+1)} = 1$ *into* (17), *then* $J^T_{ia_j} = \mathbf{0}\ \forall \mathbf{q}$ *is always true.*
3. $x < a, y \geq a$ *or* $x \geq a, y < a$: *one of the attachments is below link* a *and the other is on or above link* a, *then there would always exist some* $\mathbf{q}$ *such that* $J^T_{ia_j}(\mathbf{q}) \neq \mathbf{0}$ *in general. For example, if* $x < a, y \geq a$ *then*

$$J^T_{ia_j}(\mathbf{q}) = \begin{bmatrix} I_{3\times 3} \\ \left[\mathbf{r}_{P_a A_{ijy}}\right]^{\times} \end{bmatrix} (\mathbf{r}_{OA_{ijy}}(\mathbf{q}) - \mathbf{r}_{OA_{ijx}}(\mathbf{q})).$$

It can be observed that Conditions 1 and 2 are satisfied for $\sum_{k=a}^{p} c_{ij(k+1)} = 0$, *and hence*

$$\sum_{k=a}^{p} c_{ij(k+1)} = 0 \Leftrightarrow J^T_{ia_j}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}. \tag{18}$$

Combining (18) *and* (16) *results in* $\sum_{k=a}^{p} c_{ij(k+1)} = 0\ \forall j \Leftrightarrow J^T_{i\mathbf{q}_a}(\mathbf{q}) = \mathbf{0}\ \forall \mathbf{q}$ *to complete the proof.*

The physical interpretation of $\sum_{k=a}^{p} c_{ij(k+1)} = 0\ \forall j$ is that all segments of cable i are either not attached to, or both begin and end on link a and above. When a cable begins and ends above link a, the equal and opposite forces on both ends of the cable produce a zero resultant wrench on link a. On the other hand, if $\exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0$, then there is some segment of cable i where one of its end is attached to link a or above and the other end is attached to a link below link a. In such a scenario, then cable i has the potential to produce a resultant wrench on the motion of link a.

Since from Property 4 consecutive segments of a cable must be connected, the condition $\exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0$ applies to all links between links k_{il} and k_{iu}, where k_{il} and k_{iu} refer to the lowest and highest link number that the cable i is attached to, respectively. Property 8 describes the set of links in which Theorem 2 holds for any particular cable.

Property 8. *Denoting the lowest and highest link number that cable i is attached to as k_{il} and k_{iu}, respectively, then $\exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0$ holds for links $k_{il} < a \leq k_{iu}$ and $\sum_{k=a}^{p} c_{ij(k+1)} = 0 \, \forall j$ for links $a \leq k_{il}$ and $a > k_{iu}$.*

4.2 Cable-Routing Matrix Terms

From the relationship between the CRM and Jacobian matrix terms introduced in Section 4.1, necessary conditions on the CRM to achieve wrench-closure validity can be derived in a similar manner to the analysis in Section 3.

Definition 1. *$\sum_i \alpha_{ia}^*$ represents the number of cables that are attached to link a and not to any link $b > a$,*

$$\alpha_{ia}^* = \begin{cases} 1, \exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0, \forall j \sum_{k=a+1}^{p} c_{ij(k+1)} = 0 \\ 0, \textit{ otherwise} \end{cases} . \quad (19)$$

From Property 8, $\exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0$ and $\forall j \sum_{k=a+1}^{p} c_{ijk} = 0$ imply that $a = k_{iu}$ for cable number i.

Definition 2. *$\sum_i \beta_{ia}^*$ represents the number of cables where some segments are connected to links both above and below link a, where*

$$\beta_{ia}^* = \begin{cases} 1, \exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0, \exists j : \sum_{k=a+1}^{p} c_{ij(k+1)} \neq 0 \\ 0, \textit{otherwise} \end{cases} , \quad (20)$$

with the boundary condition $\beta_{ip}^* = 0$ since at $a = p$, $c_{ij(p+2)}$ does not exist.

Definitions 1 and 2 allow necessary conditions on wrench-closure validity to be expressed in a similar manner to (13). The resulting conditions are described with respect to the CRM and are independent of both manipulator pose and attachment locations.

In the same manner as Section 3, the number of cables m_a^* that have the potential to actuate joint a consists of:

1. Cables that are connected to link a, where link a is the highest numbered link to which the cable is attached to.
2. Cables that are connected to links both above and below link a.

Lemma 3 introduces the effective number of cables m_a^* that have the potential to actuate link a based from the CRM.

Lemma 3.

$$m_a^* = \sum_i \alpha_{ia}^* + \min\left\{\sum_i \beta_{ia}^*, m_{a+1}^* - n_{a+1}\right\}, \quad (21)$$

with the boundary conditions $m_{p+1}^* = n_{p+1} = 0$.

Proof. The negation of Theorem 2 suggests that $\exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0$ is true if cable i has an effect on joint a, where

$$\exists \mathbf{q} : J_{i\mathbf{q}_a}^T(\mathbf{q}) \neq \mathbf{0} \Rightarrow \exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0. \quad (22)$$

Hence Definitions 1 and 2 not only provide knowledge on how cables are attached to link a, but also whether the cables have the potential to actuate link a.

Firstly, since $\sum_i \alpha_{ia}^*$ cables are attached to link a but not to the links above a, all of these cables can be potentially used to actuate joint a. However, from the number of cables $\sum_i \beta_{ia}^*$ that are attached to both links a and $a+1$, it is possible that only a subset of the cables can be used to actuate joint a as they may be required to actuate joint $a+1$ or above. From Property 6, the maximum number of redundant cables from link $a+1$ is hence $m_{a+1}^* - n_{a+1}$. In a similar manner to Lemma 2, the number of redundant cables that can actuate link a is hence $\min\{\sum_i \beta_{ia}^*, m_{a+1}^* - n_{a+1}\}$. From Property 8, since $\beta_{ia}^* = 1$ applies to links k_{il} to $k_{iu} - 1$, m_a^* represents the number of redundant cables from links $b > a$.

Theorem 3 represents the relationship between m_a introduced in Section 3 and m_a^* from (21). The theorem states that the number of cables that have the potential to actuate link a as observed from the cable routing must be greater than the number of cables that actually produces a resultant wrench on link a. Using Theorem 3, the necessary conditions on WCC in some pose $\mathbf{q}$ from (13) can then be extended to the necessary conditions on wrench-closure validity expressed with respect to the CRM. The following properties (Properties 9, 10 and 11) and lemmas (Lemmas 4 and 5) that relate the terms α_{ia} and β_{ia} to α_{ia}^* and β_{ia}^* will be required in proving Theorem 3.

Property 9. *From the definitions of α_{ia} and β_{ia} presented in Lemmas 1 and 2, respectively, it is not possible for $\alpha_{ia}(\mathbf{q}) = 1$ and $\beta_{ia}(\mathbf{q}) = 1$ simultaneously. Mathematically, this can be expressed as*

$$\alpha_{ia}(\mathbf{q}) = 1 \Rightarrow \beta_{ia}(\mathbf{q}) = 0, \; \beta_{ia}(\mathbf{q}) = 1 \Rightarrow \alpha_{ia}(\mathbf{q}) = 0. \quad (23)$$

Similarly, from Definitions 1 and 2, it is not possible for $\alpha_{ia}^* =$

1 and $\beta^*_{ia} = 1$ simultaneously. Hence, this can be expressed as

$$\alpha^*_{ia} = 1 \Rightarrow \beta^*_{ia} = 0,\ \beta^*_{ia} = 1 \Rightarrow \alpha^*_{ia} = 0. \tag{24}$$

However, note that it is possible for both $\alpha_{ia}(\mathbf{q}) = 0$ and $\beta_{ia}(\mathbf{q}) = 0$ simultaneously. Similarly, it is possible for $\alpha^*_{ia} = 0$ and $\beta^*_{ia} = 0$ simultaneously.

Lemma 4. $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) + \beta_{ia}(\mathbf{q}) = 1 \Rightarrow \alpha^*_{ia} + \beta^*_{ia} = 1$. *If there exists some pose such that cable i produces a wrench on link a then it implies that there is some segment of cable i where one end is attached onto link a and above, and the other end is attached to some link below link a.*

Proof. Properties 7 and 8 state that resultant wrenches are produced on consecutive links. Furthermore, since it is not possible for $\alpha_{ia}(\mathbf{q}) = 1$ and $\beta_{ia}(\mathbf{q}) = 1$ simultaneously (Property 9), then by the definitions of α_{ia} and β_{ia} in (11) *and* (12)*, respectively,*

$$\alpha_{ia}(\mathbf{q}) + \beta_{ia}(\mathbf{q}) = 1 \Leftrightarrow J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}\ . \tag{25}$$

Similarly, by using the definitions of α^*_{ia} and β^*_{ia} in (19) *and* (20)*, respectively,*

$$\alpha^*_{ia} + \beta^*_{ia} = 1 \Leftrightarrow \exists j : \sum_{k=a}^{p} c_{ij(k+1)} \neq 0\ . \tag{26}$$

Combining the relationships (25) *and* (26) *with the implication in* (22) *completes the proof.*

Property 10. $\alpha_{ia}(\mathbf{q}) = 1 \Rightarrow \alpha_{ib}(\mathbf{q}) = 0\ \forall b > a$. *By the definition of $\alpha_{ia}(\mathbf{q})$ from* (11)*, if $\alpha_{ia}(\mathbf{q}) = 1$ then $J^T_{i\mathbf{q}_b} = \mathbf{0}\ \forall b > a$ and hence $\alpha_{ib}(\mathbf{q}) = 0$.*

Property 11. $\alpha_{ia}(\mathbf{q}) = 1 \Rightarrow \beta_{ib}(\mathbf{q}) = 0\ \forall b > a$. *By the definition of $\alpha_{ia}(\mathbf{q})$ from* (11)*, if $\alpha_{ia}(\mathbf{q}) = 1$ then $J^T_{i\mathbf{q}_b} = \mathbf{0}\ \forall b > a$ and hence $\beta_{ib}(\mathbf{q}) = 0$.*

Lemma 5. $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) = 1 \Rightarrow \exists k_{iu} \geq a : \alpha^*_{ik_u} = 1, \beta^*_{ib} = 1, a \leq b < k_{iu}$. *If $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) = 1$ then it implies that cable i is attached to some link(s) below link a and also to link a or above. Note that link k_{iu} denotes the highest link number that cable i is attached to.*

Proof. By definition $\alpha_{ia}(\mathbf{q})$ from (11)*, if $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) = 1$ then it implies that $\exists \mathbf{q} : J^T_{i\mathbf{q}_a}(\mathbf{q}) \neq \mathbf{0}$. Hence by* (22) *and* (26)*, the implication can be expressed as $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) = 1 \Rightarrow \alpha^*_{ia} + \beta^*_{ia} = 1$. By Property 8, $\beta^*_{ib} = 1$ holds for consecutive links from $a \leq b < k_{iu}$ and $\alpha^*_{ik_u} = 1$.*

Theorem 3. *The number of cables m^*_a based on the CRM from* (21) *is always larger than the effective number of cables m_a from* (10) *that can be used to actuate link a*

$$\forall a\ \forall \mathbf{q} : m^*_a \geq m_a(\mathbf{q})\ . \tag{27}$$

Proof. For link a, $m_a(\mathbf{q})$ from (10) *and m^*_a from* (21) *can be expressed as*

$$m_a = \min\{\sum_i \alpha_{ia} + \beta_{ia}, \sum_i \alpha_{ia} + m_{a+1} - n_{a+1}\}$$
$$m^*_a = \min\{\sum_i \alpha^*_{ia} + \beta^*_{ia}, \sum_i \alpha^*_{ia} + m^*_{a+1} - n_{a+1}\}\ . \tag{28}$$

From (28)*, it can be observed that the implication* (27) *is satisfied if both of the following statements are true*

$$\forall a\ \forall \mathbf{q} : \sum_i \alpha^*_{ia} + \beta^*_{ia} \geq \sum_i \alpha_{ia}(\mathbf{q}) + \beta_{ia}(\mathbf{q}) \tag{29}$$
$$\forall a\ \forall \mathbf{q} : \sum_i \alpha^*_{ia} + m^*_{a+1} \geq \sum_i \alpha_{ia}(\mathbf{q}) + m_{a+1}(\mathbf{q})\ . \tag{30}$$

The relationship (29) *is true as a direct result from Lemma 4 since if there exists a pose $\mathbf{q}$ that $\alpha_{ia}(\mathbf{q}) + \beta_{ia}(\mathbf{q}) = 1$ then $\alpha^*_{ia} + \beta^*_{ia} = 1$ is also true. The implication* (30) *can be shown by induction.*

Base case: *At the outer most link $a = p$, since $m_{p+1} = m^*_{p+1} = 0$ is true by definition, then $\forall \mathbf{q} : m^*_p \geq m_p(\mathbf{q})$ is true.*

Inductive step: *By Property 10, Property 11 and Lemma 5, $\exists \mathbf{q} : \alpha_{ia}(\mathbf{q}) = 1$ implies that $\alpha_{ib}(\mathbf{q}) = \beta_{ib}(\mathbf{q}) = 0$ for links $b > a$ and $\alpha^*_{ib} + \beta^*_{ib} = 1$ for links $a \leq b \leq k_{iu}$, and the base step of $\forall \mathbf{q} : m^*_p \geq m_p(\mathbf{q})$,* (30) *is true for all links.*

By Theorem 3, the *necessary condition* for the wrench-closure validity from (6) and (8) can be expressed as

$$WCV \Rightarrow m^*_a \geq n_a + 1\ \forall a. \tag{31}$$

The interpretation of (31) is that if there exists some pose in which the WCC is satisfied for an MCDM, then it is necessary that $m^*_a \geq n_a + 1\ \forall a$.

The necessary condition from (31) has two important characteristics. Firstly, it only provides a necessary condition for the wrench-closure validity of MCDMs expressed with respect to only the CRM and hence cable routing. Secondly, it is independent of manipulator pose and attachment locations of the cables.

5 Analysis of Example Manipulators

In this section, simple and intuitive examples have been included to assist the explanation of the physical meaning of the mathematical formulations. It should be noted that the proposed conditions be used on systems with arbitrary number of links and joint types.

5.1 Two Link Manipulators

For two link MCDMs, the number of degrees of freedom for the system can be denoted by $n = n_1 + n_2$, where n_1

Table 1. Analysis of cable routing for two link MCDMs

Type	**Description**	α^*_{i1}	β^*_{i1}	α^*_{i2}
C_1	base $\rightarrow$ link 1	1	0	0
C_5	base $\rightarrow$ link 2 $\rightarrow$ link 1	0	1	1

represents the number of degrees of freedom between link 1 and the base, and subsequently n_2 represents the number of degrees of freedom between link 2 and link 1. The number of cables m^*_1 and m^*_2 that have the potential to actuate link 1 and link 2, respectively, can be determined by (19), (20), (21) and expressed as

$$\begin{aligned} m^*_2 &= \sum_i \alpha^*_{i2} \\ m^*_1 &= \sum_i \alpha^*_{i1} + \min\left\{\sum_i \beta^*_{i1}, m^*_2 - n_2\right\} , \end{aligned} \quad (32)$$

where

$$\begin{aligned} \alpha^*_{i1} &= \begin{cases} 1 \ , \ \exists j : c_{ij2} \neq 0, c_{ij3} = 0 \ \forall j \\ 0 \ , \ \text{otherwise} \end{cases} \\ \beta^*_{i1} &= \begin{cases} 1 \ , \ \exists j : c_{ij2} + c_{ij3} \neq 0, \exists j : c_{ij3} \neq 0 \\ 0 \ , \ \text{otherwise} \end{cases} \\ \alpha^*_{i2} &= \begin{cases} 1 \ , \ \exists j : c_{ij3} \neq 0 \\ 0 \ , \ \text{otherwise} \end{cases} . \end{aligned} \quad (33)$$

For example, consider cables 1 and 5 in Figure 1(a), the cable routing C_1 and C_6, respectively, can be expressed by

$$C_1 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ C_5 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} .$$

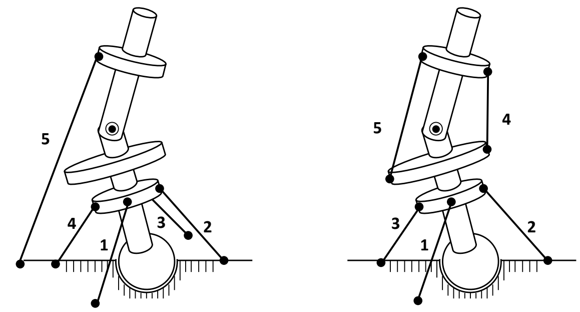
The element in the j-th row and k-th column of C_i corresponds to the element c_{ijk} of the CRM for the system. Using the CRM, the terms α^*_{ia} and β^*_{ia} from (33) could be determined and are shown in Table 1.

For the Spherical-Revolute (SR) manipulator, link 1 is constrained to the base by a spherical joint $n_1 = 3$ and link 2 is constrained to link 1 by a revolute joint $n_2 = 1$. Using (32) and (31), the wrench-closure validity conditions are

$$m^*_1 \geq 4, \ m^*_2 \geq 2 \ . \quad (34)$$

If either of the conditions from (34) is not satisfied, then the arrangement of cables are unable to satisfy the WCC for all of the manipulator poses. Table 2 shows the values of m^*_1 and m^*_2 for the various example manipulator cable arrangements.

Example 1 : The condition $m^*_2 \geq 2$ is satisfied if there are at least two cables where $\alpha^*_{i2} = 1$. For example, the SR manipulator shown in Figure 1(a) has only one such cable (cable number 5 in the figure). As a result, $m^*_2 = 1$ and hence it can be observed that wrench-closure could not be produced on link 2 regardless of the manipulator pose.



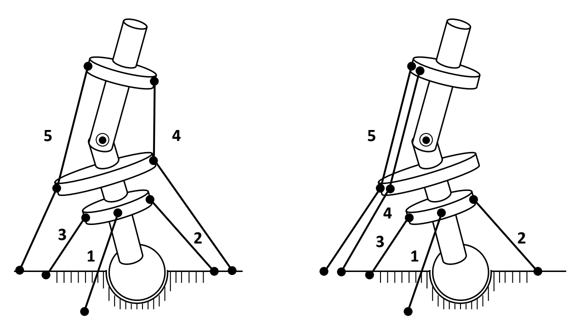
(a) Example 1 : $m^*_2 = 1 \ngeq 2$ (b) Example 2 : $m^*_1 = 3 \ngeq 4$

Fig. 1. Examples of cable arrangement for the two link SR MCDM that do not satisfy wrench-closure validity. (a) and (b) are examples where link 2 and link 1 do not satisfy the WCC, respectively.

Table 2. Analysis Results for SR Manipulator Examples

Example	m^*_1	m^*_2	**WCV**
1 (shown in Figure 1(a))	4	1	Invalid
2 (shown in Figure 1(b))	3	2	Invalid
3 (shown in Figure 2(a))	4	2	Valid
4 (shown in Figure 2(b))	4	2	Valid

Example 2 : Consider the cable arrangement shown in Figure 1(b). For cables 4 and 5, $m^*_2 = \sum_i \alpha^*_{i2} = 2$ and hence the manipulator satisfies the necessary condition for wrench-closure validity for link 2. However, as cables 4 and 5 from Figure 1(b) produce no resultant wrench on the spherical joint $\beta^*_{i1} = 0, \ i = 4,5$, only cables 1, 2 and 3 are capable of producing a resultant wrench on the spherical joint. As a result, the wrench-closure validity cannot be achieved for link 1 since $m^*_1 = \sum_i \alpha^*_{i1} = 3$.



(a) Example 3 : Valid (b) Example 4 : Invalid due to cable attachments

Fig. 2. Cable arrangement for the two link SR MCDM that satisfy the necessary conditions of wrench-closure validity.

Example 3 : Figure 2(a) is an example of cable routing that satisfies the necessary conditions for wrench-closure validity. The wrench-closure validity for link 2 is satisfied since $m_2^* = \sum_i \alpha_{i2}^* = 2$. The number of cables that are attached to link 2 and have the potential to actuate link 1 can be expressed as $\min\{\sum_i \beta_{i1}^*, m_2^* - 1\} = \min\{2,1\} = 1$. Hence, $m_1^* = 4$ due to the three cables attached to link 1 and the actuation redundancy inherited from link 2.

Example 4 : It should be noted that since the conditions are only necessary and not sufficient for wrench-closure validity. Figure 2(b) shows an example that satisfies (34) but clearly results in an empty wrench-closure workspace.

5.2 Three Link Manipulators

The number of cables m_1^*, m_2^* and m_3^* that have the potential to actuate link 1, link 2 and link 3, respectively, can be determined by (19), (20), (21) and expressed as

$$
\begin{aligned}
m_3^* &= \sum_i \alpha_{i3}^* \\
m_2^* &= \sum_i \alpha_{i2}^* + \min\left\{\sum_i \beta_{i2}^*, m_3^* - n_3\right\} \\
m_1^* &= \sum_i \alpha_{i1}^* + \min\left\{\sum_i \beta_{i1}^*, m_2^* - n_2\right\} ,
\end{aligned} \tag{35}
$$

where

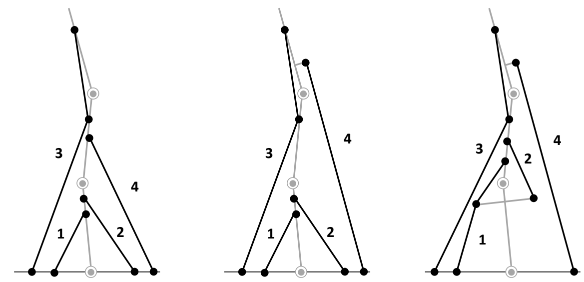
$$
\begin{aligned}
\alpha_{i1}^* &= \begin{cases} 1 \ , \exists j : c_{ij2} \neq 0, c_{ij3} = c_{ij4} = 0 \ \forall j \\ 0 \ , \text{otherwise} \end{cases} \\
\beta_{i1}^* &= \begin{cases} 1 \ , \exists j : c_{ij2} + c_{ij3} + c_{ij4} \neq 0, \\ \quad \exists j : c_{ij3} + c_{ij4} \neq 0 \\ 0 \ , \text{otherwise} \end{cases} \\
\alpha_{i2}^* &= \begin{cases} 1 \ , \exists j : c_{ij3} \neq 0, c_{ij4} = 0 \ \forall j \\ 0 \ , \text{otherwise} \end{cases} \\
\beta_{i2}^* &= \begin{cases} 1 \ , \exists j : c_{ij3} + c_{ij4} \neq 0, \exists j : c_{ij4} \neq 0 \\ 0 \ , \text{otherwise} \end{cases} \\
\alpha_{i3}^* &= \begin{cases} 1 \ , \exists j : c_{ij4} \neq 0 \\ 0 \ , \text{otherwise} \end{cases} .
\end{aligned} \tag{36}
$$

Figure 3 show example cable routing arrangements for the 3 link revolute joint (3R) manipulator that will be analysed. The manipulator possesses 3 DoF, where the number of DoF for each link is $n_a = 1$. Hence, the necessary conditions (31) for wrench-closure can be expressed as

$$
m_1^* \geq 2, \ m_2^* \geq 2, \ m_3^* \geq 2 \ . \tag{37}
$$

Table 3 shows the computed values of (35) for the examples in Figure 3.

Example 1 : Figure 3(a) shows an example of cable arrangement where cables number 1, 2 and 4 produce no wrench on link 3. Since only cable 3 is attached to link 3, $m_3^* = 1$ and hence the condition (34) is not satisfied.



(a) Example 1 : $m_3^* \ngeq 2$ (b) Example 2 : $m_2^* \ngeq 2$ (c) Example 3 : Valid

Fig. 3. Examples of cable arrangement for the three link 3R MCDM. (a) and (b) are examples where link 3 and link 2 do not satisfy the WCC, respectively. (c) an example of a valid arrangement.

Table 3. Analysis Results for 3R Manipulator Examples

Example	m_1^*	m_2^*	m_3^*	**WCV**
1 (shown in Figure 3(a))	2	1	1	Invalid
2 (shown in Figure 3(b))	2	1	2	Invalid
3 (shown in Figure 3(c))	2	3	2	Valid

Example 2 : In Figure 3(b), cable number 4 is attached from the base to link 3. As a result, $m_3^* = 2$ and hence the necessary conditions for wrench-closure of link 3 is satisfied. Since cables 1 and 2 are connected from the base to link 1, $m_2^* = 1$ and hence the WCC of link 2 is not satisfied.

Example 3 : Figure 3(c) shows an example of cable routing that satisfies (37).

5.3 Discussion

Through the examples shown in Section 5, it can be observed that the proposed necessary conditions can be used to reject cable arrangements that result in a manipulator with an empty WCW. In this section, the advantages in having such necessary conditions will be discussed.

Firstly, the proposed analysis approach uses the generalised model introduced in [4] and hence (31) can be applied for all types of MCDMs with any kinematic structure and arbitrary cable routing. As such, the necessary conditions do not have to be derived for different manipulator designs and (31) can form as a desired property on the CRM.

Secondly, the necessary conditions are expressed only with respect to cable routing and are independent to the exact attachment locations of the cables. This is particularly useful in the early stages of the design and synthesis of MCDMs, where appropriate cable routing could be first selected.

Finally, since only the cable routing is considered, the verification of valid cable routing from condition (31) is a very computationally efficient process. This means that arrangements in cable routing that will result in an empty WCW can be quickly determined without the need to performing WCW analysis.

6 Conclusion

Necessary conditions on the cable routing to achieve wrench-closure for multilink cable-driven manipulators (*MCDMs*) were derived. The conditions were mathematically derived with respect to the generalised Cable-Routing Matrix (*CRM*). It was shown through the examples on different manipulators that the proposed conditions were effective in identifying MCDM cable arrangements that cannot satisfy wrench-closure regardless of manipulator pose. Furthermore, it was shown that a significant number invalid cable arrangement combinations was identified using the necessary conditions. Future work will focus on extending the necessary conditions to consider the types of joints and consider both necessary and sufficient conditions.

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