Oral communication

A PHENOMENOLOGICAL STUDY OF THE FORMATION PROCESS OF VALUES IN MATHEMATICS LEARNING

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In mathematics education research, values are seen as the internalisation, recognition and decontextualisation of beliefs and attitudes by individuals in a socio-cultural context (Seah & Wong, 2012). International comparative 'Third Wave Project' research on values has been multifaceted, but the process of value formation remains a research question (Seah & Wong, 2012). This study addresses the exploration of the value formation process that is emphasised within mathematics learning classrooms.

The method of interpretive phenomenology is used to articulate the process of value formation (Eddles-Hirsch, 2015). Specifically, the researcher is first immersed in the phenomenon in order to understand it. Then, the researcher interprets the phenomenon based on inferences of its meaning from the participants' writings, words and actions. Specifically, the author, who has 20 years of teaching experience at elementary, junior high and special-needs schools, enters one fourth-grade class at a primary school attached to a national university in Japan for one hour every week for six months to conduct observations. During the observation, the author writes in the Fieldnotes about his findings on values related to mathematics education. The author then reconstructs the observations into episode descriptions using phenomenology. These episode descriptions are compared chronologically and the formation process of values emphasised in the classroom is discussed.

As a result of the observations, 11 episode descriptions were collected in six months. For example, children contrasted and discussed their own ideas with the teacher's statements based on their absolute trust in the teacher; they felt an increased satisfaction in creating their own learning because their opinions were valued even at the end of the class; they proceeded with the class in a child-centred manner while speaking freely; they had heated discussions when they created their own volume unit; and they murmured, "It's interesting that everyone's opinions are different. The descriptions of these episodes show that in this classroom, the values of satisfaction with learning with the teacher, valuing different ideas in the classroom and valuing independent learning have emerged.

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VALUING IN MATHEMATICS CURRICULUM AND TEXTBOOKS FOR GRADES 1 AND 2

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Through learning mathematics, students not only understand mathematical knowledge, but also recognize the values of mathematics. Values and valuing are related but distinct concepts. "*Valuing* is clearly a behavior but with no specification of what is to be valued. *Values* on the other hand represent what is to be valued (Bishop, 2014)."

Given that mathematics textbooks are one of the resources to support mathematics learning, they can be analyzed not only in terms of how they help students understand mathematical concepts, but also in terms of how they intend students to value mathematics or mathematics learning. However, prior research analyzing mathematics textbooks has been focused on the former, with little research on the latter. Research on the valuing of elementary school mathematics textbooks is still very limited.

Recently, there have been several changes in mathematics education in Korea. The national mathematics curriculum was revised for the first time in seven years, and the revised curriculum explicitly emphasizes the values of mathematics by introducing 'values and attitudes' as a category of mathematics learning. In addition, textbooks based on the revised mathematics curriculum are being developed in sequence, and new textbooks for grades 1 and 2 will be applied in the field starting this year.

In this context, this study aimed to is to explore the valuing in mathematics curriculum and textbooks for grades 1 and 2 of South Korea. This study analyzes the valuing in curriculum and textbooks based on Seah and Bishop's (2000) mathematical values and mathematics educational values. Mathematical values include the pairs of rationalism-objectivism, control-progress and openness-mystery. Mathematics educational values include the pairs of formalistic-activist, instrumental-relational, relevance-theoretical, accessibility-specialism, evaluating-reasoning.

This study can contribute to discussion on the issue of what mathematical and mathematics educational values are emphasized in Korea, and the issue of how to align the valuing in curriculum with the valuing in textbooks.

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THE USE OF PERSON-ORIENTED RESEARCH METHODOLOGY IN IDENTIFYING STUDENT AND TEACHER VALUES IN MATHEMATICS EDUCATION

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This paper argues for the use of a phenomenological Person Oriented Research (POR) methodology when studying student and teacher values in mathematics education. In POR, participants are considered as a ‘system’, a product of the inter-relationships that exist between and around them (Bergman & Wangby, 2014). POR offers a means to overcome the paradigm of research seeking generalisations which may not consider the broader environment and sociocultural factors by exploring individuals within their encompassing systems. The insights from Bergman and Wangby (2014) emphasise the uniqueness of individuals, the prevalence of non-linear models in personality traits, and the importance of identifying patterns and emergent themes in students' ascribed values. Studies of student and teacher values in relation to learning mathematics employ thematic analysis for coding and analysis (Hill et al., 2021). However, the results of these similar studies show that there has been little consensus on the values that are identified. POR methodology may address this limitation.

In the proposed study, POR classification methods as outlined by Bergman & Wangby (2014) would be employed to qualitatively examine how five secondary mathematics teachers interact with up to 20 students, focussing on the values demonstrated by students and teachers. The data to be collected in this study includes surveys, observations and semi-structured interviews. In this context, a POR methodological approach could provide a nuanced understanding of values. Whilst thematic analysis provides a basis for observing quantitatively collected data in real world, the rigidity in qualifying and categorisation does not fit the needs of this mixed methods study. On the other hand, a POR approach to coding and analysis may provide a fresh lens to observing student and teacher values in secondary mathematics classrooms.

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MATH WELLBEING AND MATH VALUE AMONG TAIWANESE UPPER ELEMENTARY SCHOOL STUDENTS IN MATHEMATICS LEARNING

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Taiwanese students excel in mathematics tests; nevertheless, it is worth noting that there exists a prevalent negative affection towards mathematics (Kell et al., 2020). Recently, a framework consisting of seven dimensions for mathematical well-being and values has been proposed (Hill et al., 2021). Nonetheless, to date, no similar investigation has been conducted in Taiwan. This study aimed to explore the circumstances in which upper elementary school students experience well-being during the process of learning mathematics and to understand the values that students consider crucial in mathematics learning. The two research questions were proposed in this study: (1) Under what circumstances do students feel happy or confident during the process of learning mathematics? (2) What values do students consider most important in mathematics learning?

Fifty-two upper elementary school students (27 boys and 25 girls) were recruited from a school in the southern region of Taiwan, all in the sixth grade. Two open-ended questions were employed to explore students' well-being and values in the context of mathematics learning. Nvivo 12 was used to conduct the thematic analysis.

The results revealed four key themes in students' mathematical well-being: Competency (41%), Accomplishment (40%), Relationship (13%), and Engagement (6%). In terms of mathematical values, nine themes emerged, including Competency (29%), Performance (18%), Engagement (14%), Attitude (14%), Methods (13%), Relationship (6%), Positive Emotions (3%), Creativity (2%), and Utility (1%). These findings align with the values identified in Hill et al. (2021) and other previous studies while also highlighting cultural differences. The most significant distinction is that Taiwanese students emphasize Competency, differing from Accomplishment, which refers to achieving good marks, completing tasks, etc. Competency emphasizes the understanding of mathematical knowledge and skills. Further discussion will provide suggestions on how these insights can be transferred for use in the math classroom.

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THE DIFFERENCES IN INTEGRATED STEM TASK VALUES BETWEEN MATHEMATICS AND OTHER STEM TEACHERS

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Teachers' values of integrated STEM (iSTEM) education strongly influence their manner to engage and implement integrated STEM education (Margot & Kettler, 2019). Nonetheless, few studies focus on teachers' perceived values on iSTEM tasks implemented in two different teaching settings (major teaching course and elective/alternative course). On the other hand, mathematics teachers tended to have lower identity and self-efficacy than science teachers (Polizzi et al., 2021). Thus, it is worth further comparing mathematics and other STEM secondary teachers' iSTEM task values. Based on expectancy-value theory, the research questions include (1) What are the differences in iSTEM task values between mathematics and other STEM teachers? (2) What kinds of patterns can be identified from the classes of STEM teachers' perceived values on iSTEM tasks in two different teaching settings? (3) What is the association between teachers' teaching subject (mathematics or other STEM disciplines) and the class of teachers' iSTEM task values?

143 Taiwanese mathematics teachers and 175 other Taiwanese STEM teachers were surveyed regarding their perceived values of attainment, interest, utility and cost of implementing iSTEM tasks in their own major teaching course and in elective/alternative course. A mixed MANOVA was performed to examine the differences in the eight scales. Latent class analysis was conducted to identify the number of classes, and three-step BCH approach was used to estimate the comparative influence of teaching subjects on classes of iSTEM task values.

The main results showed that (a) mathematics teachers perceived significantly less iSTEM task values than other STEM teachers ($p = .033$) ; (b) four classes were characterized as high task value (HTV, 67%), relatively low cost value (rLCV, 12%), low task value (LTV, 11%) and setting-specific cost value (SCV, 10%); (c) mathematics teachers more likely belonged to the SCV class than other STEM teachers ($OR = 2.197$, *95% CI: 1.01 to 4.77*, $p = .047$). It indicates that mathematics teachers' cost values were lower and more influenced by the teaching setting.

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EFFECTIVE TEACHING IN MATHEMATICS CLASSROOM: FROM THE LOW-ACHIEVERS' PERSPECTIVES

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Most of research findings related to effective teaching in mathematics provide evidences which come from the teachers' perspectives, while from the students' perspectives, there are relatively few studies on effective teaching (e.g. Seah, 2010). Some research reported high, moderate and low academic performance pupils' view for effective teaching (e.g. Tan & Lim, 2010), but there are even still fewer studies focusing on the low-achieving students' perspectives.

This study aims to investigate the perspectives on effective teaching of the low-achievers' in fifth grade. 10 low-achieving fifth-grade students from 2 classes (5 from each class) and their mathematics teachers participated in this study. Low-achieving students are asked to capture the moment of effective teaching, taking pictures immediately by iPad when they thought a good mathematics teaching moment occurred. In addition, Video recording and interview with teachers and students were used to collect data. During the interviews, each photo will be shown to the student who took the photo, and the student will be asked to describe his or her thoughts on the events that occurred at that moment, and then discourse analysis method were used to analyse the transcribed interview data.

The findings of this study showed as follows. What low-achieving students value most is teaching methods and strategies, followed by teaching materials, class management and group discussion and presentation. In the aspect of teaching methods and strategies, they prefer to listening to teachers' lecture, followed by "listening to classmates' opinions" and "visual representations". In the aspect of teaching materials, math task worksheets and interesting or daily-life math problems are their favourites. With regard to classroom management, students pay attention to whether their teacher has established a good class management system to correct classmates' behaviors. In term of group discussion and presentation, they prefer to seek help from classmates or act as assistants rather than express themselves indiviually. However, teaching methods of the two mathematics teachers are slightly different, so students' views on these four aspects are also somewhat different, more details will be presented in the conference.

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EXPLORING PERCEIVED VALUE DIFFERENCE SITUATIONS IN AUSTRALIAN MATHEMATICS CLASSROOMS

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Recent research has highlighted how students might disengage from classroom activities when their values and their teachers' are different. The present study examined 625 secondary school students' perspectives and experiences to better understand what these value difference situations look like. Analysis of the 29 identified instances of value differences revealed the need to propose a new category - mathematical content value – to add to the existing classification of values. The findings reveal an issue where students often struggle to appreciate the value of learning specific mathematical content, even when teachers emphasise its practical usefulness. The results also highlight that teacher' excessive reliance on textbooks runs counter to student' mathematics educational values, hindering their learning.

INTRODUCATION

Previous research (Seah & Andersson, 2015) revealed how students and teachers bring their personal values (that is, regarding what is important) in mathematics education into the classroom. Since individuals' values in mathematics education stem from the nature of mathematics as well as their sociocultural and educational experiences (Seah, 2019), students and their teachers develop and interpret these values in diverse ways. This phenomenon becomes more complex in multicultural societies such as in Victoria, Australia, where teachers, students and families hail from a diverse ethnic and racial profile. It is thus reasonable to assume that interactions in the (Victorian) mathematics classroom is characterised by the coming together of a range of teacher and student values relating to mathematics, mathematics teaching and learning, and education generally.

Given that decisions and actions (in mathematics education) are driven by values (Seah, 2019), the alignment of students' values with their teacher's influences the quality of classroom interactions, and thus, of the lessons. For instance, a teacher who values *group work* would incorporate collaborative activities in their lessons, but any of their students valuing *independent work* might disengage with or avoid such activities. Indeed, students who hold different values from their teachers are likely to resist or disengage, negatively affecting their interest and performance in mathematics learning (Kalogeropoulos, 2016). Effective teaching thus requires an understanding of value differences, as teacher intentions alone are unlikely to ensure successful lessons. This calls for a thorough exploration of the attributes valued by students and their teachers in mathematics pedagogy.

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Previous studies assumed that students would express their values through feedback to their teachers (Kalogeropoulos et al., 2021). However, students may conceal their values for a variety of reasons. Even when students have choices, these choices are usually constrained within the classroom context (Clarkson et al., 2019). Thus, to gain a more comprehensive understanding of value differences in the mathematics classroom, it is important to consider whether students are able to contemplate, compare, and express their values during classroom interactions. In this context, the current study seeks to address this gap, establishing a comprehensive overview of the types of value differences in mathematics classrooms, based on Victorian students' perspectives.

The Research Question guiding our study here is: What sorts of values espoused by students and their teachers most often end up in value difference situations?

THEORETICAL FRAMEWORK

Instead of being viewed as an objective body of knowledge awaiting discovery, mathematics has been acknowledged as being socialised knowledge (Bishop, 1988). Research into values and valuing in mathematics education from the late 1980s (see Bishop, 1988, 1996) acknowledges that the discipline and its pedagogy is culture-dependent. Early research literature in this area reflects a conception of values as an affective variable (Bishop, 1996). Seah (2019), inspired by the tripartite model of the human mind, later redefined values as being conative in nature. In doing so, Seah could explain why people often passionately embrace their values and why values can be made visible through decision processes. Hence, this study adopted Seah's (2019) definition of values in mathematics education, in which valuing is concerned with "an individual's embracing of convictions in mathematics pedagogy...[shaping] the individual's willpower to embody the convictions in the choice of actions" (p. 107). Since a value directs an individual's course of action, values can be regarded as being motivational (E, 2023). However, motivation might not fully explain persistence—the character of will and determination embedded in values empower determination amid obstacles (Seah & Andersson, 2015), not just guiding actions but defending them. As such, value difference situations arising from interactions between teachers and students are characterized not just by motivation, but also involve the will inherent in values.

Bishop's (1996) categories of values in mathematics education, namely, general educational, mathematical, and mathematics educational values, provided a useful framework for categorising the types of value differences perceived. Mathematical values relate to the mathematics discipline itself, and they have been identified to be the three complementary pairs of *rationalism* and *objectism, control* and *progress,* and *mystery* and *openness* (Bishop, 1988). On the other hand, mathematics educational values refer to the objects, experiences, and pedagogies that students and teachers consider important to learning and teaching mathematics (Seah, 2019). Last but not least, general educational values cover the moral, ethical, citizenship, and sometimes, religious values that a given educational system aims to impart to its students. Unlike the first two categories of values, general educational values are part of the educational process but are not directly related to mathematics instruction.

METHODS

Data were collected online using the 'What I Find Important Too' questionnaire, accessible at https://www.surveymonkey.com/r/VASstu_v4. The questionnaire draws on hypothetical situations to encourage student respondents to delineate instances where disparities exist between their own values and those emphasised by their teachers during mathematics lessons (E, 2023). Here we focus on the open-ended question posed: What was the value differences situation like?

Participants were chosen via stratified probability sampling of schools across Victoria, ensuring diversity in terms of gender, ethnicity, and educational background. The 625 participants (50% female) were from urban and regional secondary schools located in Victoria. Students self-identified their ethnicities as Australian (349), European (116), Asian (73), North African and Middle Eastern (17), Americans (9), Indigenous Australian (6), Sub-Saharan African (4), and Other (43).

Students' responses were analysed using thematic analysis (Braun & Clarke, 2006). Initial nodes were generated and coded inductively. For example, "I asked her to teach in a way that we could all understand" was coded as *understanding* to reflect this valuing. Subsequently, nodes were organised deductively into value categories, guided by a checklist that included three pairs of mathematical values (Bishop, 1988) and seven pairs of complementary mathematics education values (Kinone et al., 2020). For example, the value nodes *understanding* and *textbook* were categorised as 'mathematics educational values'. The checklist provided flexibility, allowing for the identification of additional value categories.

RESULTS

29 instances of value differences were reported. While 10 (34.48%) of these were either maths educational or general educational values, a majority could not be located within the three value categories which Bishop (1996) identified. Rather, they appeared to relate to mathematical content. Bishop's (1996) three-category system does not seem to adequately describe the range of value differences perceived by secondary school mathematics learners. As a result, an additional category – mathematical content values – is being proposed here.

Mathematical content value differences refer to those incidents when specific mathematical content that is important to the teacher is not important to the students, and vice versa. Of the 29 value differences reported, 19 of these were related to student and teacher differences in valuing particular mathematical content. Specifically, within this category, 11 students referenced they did not find algebra to be important. For instance, one student noted, "It [algebra] is just numbers and letters mixed up together; it is not reality". Five students did not specify the name of content, but they also expressed disagreement with their teachers' view that some specific mathematics knowledge was important, noting for instance, "we were learning something in maths that we are never going to use in everyday life and my teacher said it was important". The rest of the mathematical content value differences pertained to numerals, coordinates, addition/subtraction, geometry, trigonometry, equations, data and stats.

Our data suggested that these mathematical content value differences often stemmed from students' questions about the usefulness of the specific content. The thought process of students revealed that while they would try to assess the value of these mathematical concepts based on their perceived usefulness, this approach often fails to convince them. For example, one student remarked, "we will never use it [algebra] in the future because why would people make people waste their time when there [sic] shopping and try and figure out what the hidden number or something." Another student questioned the importance of trigonometry, stating, "but teacher thinks it is important. How will we use [this] mathematics in day-to-day scenarios?" Interestingly, teachers' justifications for the mathematical content value, as mentioned in students' responses, also emphasised its usefulness in careers, everyday life, and future. One student emphasised, "we were learning something in maths that we are never going to use in everyday life and my teacher said it was important, but I didn't take it seriously because it was useless. She [my teacher] gave me a situation of how we would use this maths content in everyday life, and it was something that would never happen". In this instance, the concerned teacher attempted to address the disparity in values related to mathematical content by highlighting its practical relevance in everyday life, but she was not successful in engaging their students in her endorsement.

Mathematics educational value differences were next most commonly reported by the secondary school participants, where the source of difference rooted in the differing perceptions of teachers and students regarding approaches to mathematics teaching and learning respectively. There were nine of the 29 value differences documented which pertained to the nature of mathematics educational values. Specifically, there were 5 instances which revealed differences in valuing between teachers' teaching relying on textbook questions and students' embracing of other pedagogies. For example, a student pointed out, "My teacher doesn't listen to us and she thinks by doing textbook questions it'll help us, which isn't true". From what a student wrote, "I think that doing the same type of questions all the time is unimportant. My teachers make us repeat them all the time", it appears that students interpreted their teachers' value as repetition unnecessarily. Indeed, other students wrote their maths education values that "I want constant and different types of problems", "I asked her to teach in a way we could all understand and not just do workbook everyday", "I have difficulties taking in information because as a class we just do book work and other teachers use different methods to teach their classes … I told her [my teacher] to use different examples and/or teachings skills" and "It becomes boring to do that many questions, which means we don't concentrate well in class". Notably, these responses reflected Kalogeropoulos's (2016) assertion that students who hold different values from their teachers are likely to resist or disengage from pedagogical activities, negatively impacting their interest and performance in mathematics.

Only one student referred to value differences of the general educational value nature. It involved the differing valuing of *equity*: “We had a deadline and most of us met it, but some didn’t, so the deadline was extended. I thought that was unfair to the people who had worked hard to meet the deadline”. Perhaps one possible explanation is that generally these values are already accepted by the (educational) community, such that the chance of any such value being involved in value difference situations was less likely. In fact, Seah (2019) suggested that general educational values are already being inculcated in students as part of fulfilling the professional and ethical responsibilities of teaching.

DISCUSSION AND CONCLUSION

Although our data about mathematical content value difference situations suggested that some teachers emphasise the importance of mathematical content due to its usefulness, it cannot be definitively categorised as valuing *application*. *Application* as a mathematics educational value refers to “valuing application of mathematics in various problematic situations in mathematics learning” (Kinone et al., 2020, p. 44), which is more aligned with emphasising pedagogies that can enhance students’ abilities to recall factual knowledge and concepts readily and flexibly to find solutions for mathematical questions. However, teachers highlighted the importance of mathematical content in everyday life and future jobs/careers. This emphasis seems to extend beyond mathematical learning and appears unrelated to pedagogy, making it incompatible with the mathematics educational value as defined by Bishop (1996).

In addition to defying categorisation within existing value categories, there are also arguments for acknowledging the mathematical content value category. Current knowledge about values (Seah, 2005) suggested that cultures construct and develop mathematics in different ways, resulting in educational systems that reflect what cultures consider valuable, that is, what they value. In other words, mathematics is socialised knowledge; knowledge that has been cultivated and developed in response to particular needs within cultures, not objective knowledge (Clarkson et al., 2019). This implied that mathematical knowledge is selected and organised to become content knowledge for teaching and learning, a process inherently embedded with values. Illustrating this is the stated aims for revising the Australian mathematics curriculum (ACARA, 2021), which highlighted the need to “remove outdated and non-essential content, add new content that has been identified as important for students to learn, better sequence student learning and give teachers greater clarity and guidance about what they are expected to teach” (p. 1). Although this document does not explicitly state why certain contents are considered important for teaching and learning, what it does highlight is the value attributed to different mathematical content.

This study has emphasised the need to focus on mathematical content values. It is noteworthy to reflect on the fact that prior studies have often approached the body of mathematical knowledge as a whole, emphasising its importance as long as the existence of specific content that can be applied in everyday life or future careers can be justified. However, clear indications suggest that students may encounter difficulties in discerning the value of each piece of mathematical content. This aligns with Atweh et al.'s (2010) observation that students persist in believing that some content is largely meaningless, particularly when teachers cannot demonstrate how all mathematical concepts can be applied to real-life situations. Recently, Rosenzweig et al. (2020) emphasised that a more effective way to increase student utility is to help students think about the value of course material, which seems to suggest a new trend: a return from considering body of knowledge as a fixed whole to a focus on specific content knowledge. Therefore, the urgent next step is to concentrate on mathematical content values and find a way to respond to situations in which students do not see the value of learning particular mathematical content.

In most value difference situations involving mathematics educational values, teachers tend to value practicing problems in textbooks, but students generally do not value this way of learning mathematics. Notably, our findings demonstrate how problematic it can be with teachers' heavy dependence on the mathematics textbooks, as students appeared to associate textbook use with mechanical or habitual repetition. This result may not be surprising given that secondary-level mathematical textbooks are often filled with relatively low-level, repetitious exercises (Stephens, 2019). Rather than repeatedly solving the same types of problems found in the textbooks, students are more likely to seek diverse examples and instructional methods that are aligned with their mathematics educational values. What this implies is that educators and teachers who rely on textbooks need to consider how to meet their students' mathematics educational values by carefully selecting or redesigning textbook problems (e.g., by changing just one or two numbers to extend learning opportunities). Otherwise, these students with conflict values from their teachers would exhibit resistance or disinterest in pedagogical activities.

Inherently, repetitive and low-complexity problems are not necessarily bad, because students learn procedures through sufficient repetition. However, this pedagogy relying on overwhelming prevalence seems to limit students' opportunities to feel and contemplate the value of mathematical content. An example can be found in early research; Goldin (2004) expressed concern over the inclusion of activities related to solving discrete mathematics problems in the secondary school mathematics curriculum. In his words, if these problems were added to standardized achievement tests with the intention of evaluating nonroutine problem-solving skills, then routine methods for solving them would be explained and utilised in numerous parallel practice problems in school workbooks. This means that this particular knowledge is included in the mathematics curriculum as a set of information to be memorised and strategies to be used routinely, which is exactly the aspect that is not valued by students frequently mentioned in mathematics educational value differences situations. More importantly, in this way, the potential of discrete mathematics in terms of triggering attributes such as experimentation, logical reasoning, and problem-solving would certainly be diminished (Goldin, 2004). This seems to reveal the mathematical content value of learning discrete mathematics, but it is obscured by certain pedagogy. It is also worth reflecting on the fact that these attributes have been described in previous studies as characteristics of the mathematical discipline as a whole, rather than analysing the relationship of the individual essential mathematical content to these attributes. The question raised here is whether specific mathematical contents containing these attributes are the precise reason the discipline of mathematics exhibits these characteristics at the macro level. Therefore, it is necessary to unpack the values embedded in each mathematical content.

To conclude, the present study investigated Australian secondary school students' experiences of perceived value differences in mathematics lessons, and identified 29 perceived value differences in the data collected, which revealed the need to add a new category, namely, mathematical content value, to the existing classification. The mathematical content category was reported most frequently in this research study, further indicating an issue: students do not value particular mathematical content. Conversely, the mathematics educational values perceived by secondary students nearly all relate to their teachers' values related to teaching from textbooks. Students in Australia often face struggles when their teachers rely solely on mathematics textbooks for teaching. This is due to the textbooks being filled with repetitive and straightforward problems, which is in conflict with students' mathematics educational values. Thus, the findings of this study could offer teachers inspiration to enhance their pedagogy, aligning it with students' values in mathematics education.

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CHINESE STUDENTS' MATHEMATICAL WELLBEING THREE YEARS ON: A RE-ASSESSMENT IN GRADE 6

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The mathematical wellbeing (MWB) of 76 students in a suburban elementary school in Chengdu, China were assessed twice, once in 2020 when they were part of a bigger Grade 3 participant group, and again in 2023 when they were in Grade 6. The same questionnaire was used, with its presentation adjusted to match students' ages. Variable/facet parameters were determined using Many Facet Rasch Measurement, and the Rasch-Welch t-test was employed to compare differences between Grades 3 and 6. Analysis found that the fulfilment of the same values contributed to students' MWB at both grade levels. However, at Grade 6, MWB was associated with more experiencing of the valuing of accomplishment and perseverance, less experiencing of engagement and bliss, and similar levels of relationship and meaningfulness.

INTRODUCTION

Given the enabling effect of general wellbeing on human flourishing (Chaves, 2021), the fostering of mathematical wellbeing (MWB) amongst students can promote effective mathematics learning while reducing the likelihood of disengagement and mathematics anxiety. While MWB (and other affective traits) might be cultivated in early and elementary school years, we are concerned that it might be eroded as students progress up the grade levels. Especially since MWB is an expression of the extent to which relevant values are fulfilled, how might such values fulfilment be affected by mathematics topics and/or pedagogies in upper elementary or high school curricula, which would in turn impact on MWB?

This paper reports on a study conducted with a group of Grade 6 students in the Chinese city of Chengdu, whose MWB had been assessed in 2020 in a previous study when they were in Grade 3, and which was assessed again in their final year of elementary schooling (i.e., Grade 6) in 2023. Thus, this study design incorporates the advantage of surveying from the same students a few years apart, rather than making inferences from surveying students of different grade levels at any one time period.

MATHEMATICAL WELLBEING (MWB)

We regard MWB as being "the fulfilment of core values … within the mathematics learning experience, accompanied by positive feelings (e.g., enjoyment, pride) and functioning (e.g., accomplishment, engagement) in mathematics" (Hill & Seah, 2023, p.386). Developing and maintaining positive MWB amongst students is important not just because mathematics is one of a few subjects that is studied by all students

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globally, but also because so many students experience disengagement in – or negative attitude to – mathematics lessons, and/or mathematics anxiety. Intervention approaches can be costly and yet success is not guaranteed. On the other hand, if we proactively develop and maintain students' MWB, more students around the world can get to learn mathematics with positive affect, as well as effectively, to help them navigate the complexities and uncertainties of our current world.

Data collected and analysed in Australia, China and New Zealand had validated a set of seven ultimate values the fulfilment of which is needed to achieve MWB (Hill et al., 2022). These values are *accomplishment, cognitions, engagement, meaning, perseverance, positive emotions,* and *relationship*. While these ultimate values might be the same across cultures, the instrumental values serving them have been found to be different (Hill & Seah, 2023).

THE PREVIOUS STUDY

The 'previous study' mentioned above refers to a similar study (Pan et al., 2022) conducted in 2020 when the same student participants were in Grade 3 in the same school. In fact, they were part of a larger group of 258 Grade 3 students in six classes in the Chengdu suburban school, taught by three mathematics teachers. The teachers had nominated 21 classroom learning moments (e.g., 'when you are given an interesting mathematics learning task', 'when your mathematics teacher praises you) to which students indicated the extent to which they valued each and were able to live it. There was also an 'other' option for students to indicate classroom learning moments associated with positive MWB. Engaging in these learning moments enabled the students to fulfil and live some or all of 15 instrumental values (Figure 2). The instrumental values together would serve the realisation of a smaller set of 6 terminal or ultimate values (Figure 1). For example, the learning moment 'when you are given an interesting mathematics learning task' was considered to help students fulfil their instrumental valuing of *interestingness*, which was in turn considered to serve the ultimate valuing of *engagement*.

Three findings were of particular importance in this previous study. Firstly, the students' MWB corresponded to the fulfilment of a set of seven ultimate values which are similar to the set that Hill et al. (2021) observed in Australia, namely, *relationship, engagement, bliss, accomplishment, perseverance, meaningfulness,* and *learning.* Secondly, four of these - *engagement, relationship, bliss* and *accomplishment* – were especially emphasised by the students for positive MWB. Thirdly, teachers' facilitation of these values which fostered positive MWB was generally consistent across different teachers and different classes.

Given that student affect often becomes less positive as they progress through grade levels (e.g., Thomson et al., 2020), this current study is interested to find out what the MWB of some of these 258 students were like in their final year of elementary schooling. In particular, the Research Questions guiding the conduct of this study are:

RQ1: What are the ultimate values that need to be fulfilled in order for Grade 6 students in Chengdu to experience mathematical wellbeing? How do these compare with the ultimate values associated with these students when they were in Grade 3?

RQ2: For each of the ultimate values associated with Grade 6 students' mathematical wellbeing, how similar or different are the corresponding instrumental values compared to the time when the students were in Grade 3?

METHODOLOGY

Participants

The student participants in this study were 76 Grade 6 students in two classes in a Chengdu suburban elementary school. They (and their mathematics teacher) were also part of the 258 participants who provided data in 2020, when they were in Grade 3. They have had the same mathematics teacher throughout the six years of elementary education in the same school, but the different mathematics topics and the different pedagogies associated should affect individuals' mathematics learning experience. For example, as mathematics topics become more abstract in the upper elementary school year levels, and as different teaching approaches need to be introduced, how might these affect the extent to which students were able to engage in classroom learning moments that reflect the fulfilment of relevant values? How might these affect MWB?

The Questionnaire Method

Just like when they were in Grade 3 three years before the current study, the students' MWB was assessed through the questionnaire survey method. Compared to alternative methods such as interviews and journals, the questionnaire approach would have facilitated efficient collection of data from a large group of participants at the same time. In both times, the students completed the questionnaires during mathematics lesson time, with the same mathematics teacher administering the exercise.

The questionnaire (in Chinese) is accessible at https://www.wjx.cn/vm/POhZjXH.aspx. While the items are the same as the questionnaire which the student participants completed three years prior (see Hill & Seah, 2023), there were necessarily some changes in the way it was presented, considering that the students had become older and more matured. Firstly, students had indicated in the earlier questionnaire if 21 given learning moments were associated with positive MWB through a colouring exercise. The argument then was that the activity would help maintain the young students' attention span. In the current questionnaire, students only needed to click on bullet points adjacent to the 21 learning moments statements to indicate that they were still associated with their experiencing of positive MWB. Like the Grade 3 questionnaire, there was an additional 'other' option too. (For a list of the 21 learning moments, refer to the English translated version of the questionnaire, accessible at: https://melbourneuni.au1.qualtrics.com/jfe/form/SV_doQ5pV3ruEwyZTw) Secondly, while the questionnaire was administered in hardcopy version in the earlier exercise in 2020, it was presented to students as an online survey in the current exercise in 2023.

The questionnaire responses were exported in the form of a Microsoft Excel spreadsheet. The content (i.e., raw data) were cleaned and organized in preparation for Many Facet Rasch Measurement [MFRM] analysis. The MFRM is a measurement model in the item response theory that extends the Rasch model (Toffoli et al., 2016). Thus, the codes were written as guided by the FACET software to facilitate our investigation of the interaction between grade level and instrumental / ultimate values.

With the Research Questions listed above in mind, we focussed on Item 6 of the questionnaire. Each student was scored according to whether each of the 21 learning moments contributed to their MWB, based on their indications in the Grade 6 questionnaire. The same question in the Grade 3 questionnaire, however, gave the students three choices of which to colour-in one: contributes a lot, contributes, and does not contribute. In our analysis, responses to either one of the first two choices were counted together. In other words, all student responses were recorded as either 1 or 0, thus implying that the data were dichotomous.

In contrast to classical test theory, MFRM allows for the independent and objective estimation of facet parameters without being influenced by item, rater, test, and group characteristics (Toffoli et al., 2016). In this study, individuals (students), grade level, instrumental values, ultimate values, and learning moments were determined as facets. MFRM enables the separate estimation of each facet and allows for relevant comparison by calibrating and standardizing the facets within a logit scale (Prieto et al., 2014), where scores generally fall between -3 and +3.

Item response theory is relevant in our study as it helps us to examine the relationship between the latent MWB and the observable learning moments which made up the item responses. With the dichotomous data coded, the Rasch-Welch t-test was performed to compare the difference between ultimate values experienced in Grades 3 and 6, because this test is more effective in controlling Type 1 error rates when the assumption of equal variance is not fulfilled (such as in this study), while maintaining a strong level of reliability compared to Student's t-test if the assumptions are met (Delacre et al., 2017).

An informal conversation was also set up with the classroom teacher to share with her what the analysed data looked like, to stimulate her thoughts and opinions in response.

RESULTS

Point-measure (point-biserial) correlation values of the items for the 21 learning moments vary between .43 and .74. Infit MNSQ values range from .82 to 1.23, and outfit MNSQ values range from .59 to 1.28 except for one item that is .44. These values being between .5 and 1.5 are productive in terms of measurement. Values lower than 0.5 are not as productive for measurement, but they do not cause degradation. (Linacre, 2002). These results show that our data fit the Rasch model.

The interaction of grade level and ultimate values is shown in Figure 1, while the interaction of grade level and instrumental values is shown in Figure 2.

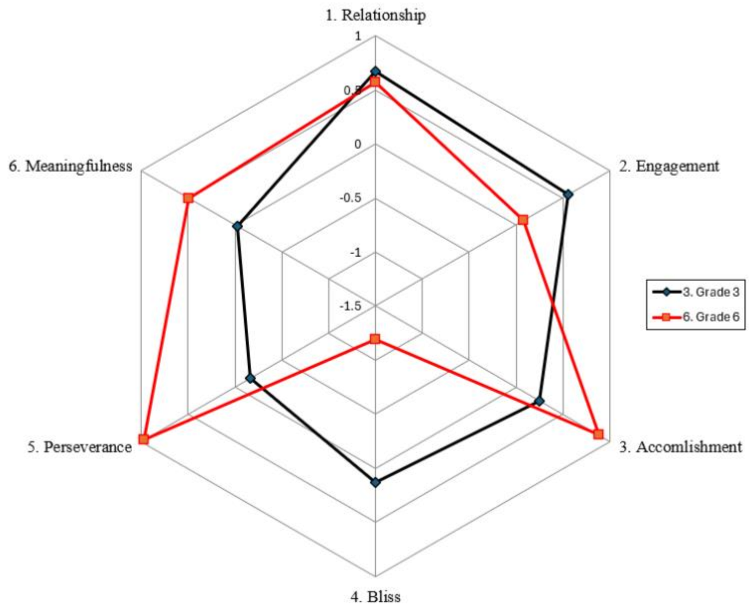

Figure 1: Interaction of Grade Level and Ultimate Values.

Rasch-Welch (logistic regression) t-test results show that there are statistically significant differences between the Grade3 and Grade 6 students in favour of the Grade 3 students for the ultimate values of engagement ($t_{(1115)}$=3.02, p<.05) and *bliss* ($t_{(273)}$=4.41, p<.05), and in favour of the Grade 6 students for the ultimate values of *accomplishment* ($t_{(882)}$=-3.62, p<.05) and *perseverance* ($t_{(347)}$=-4.20, p<.05).

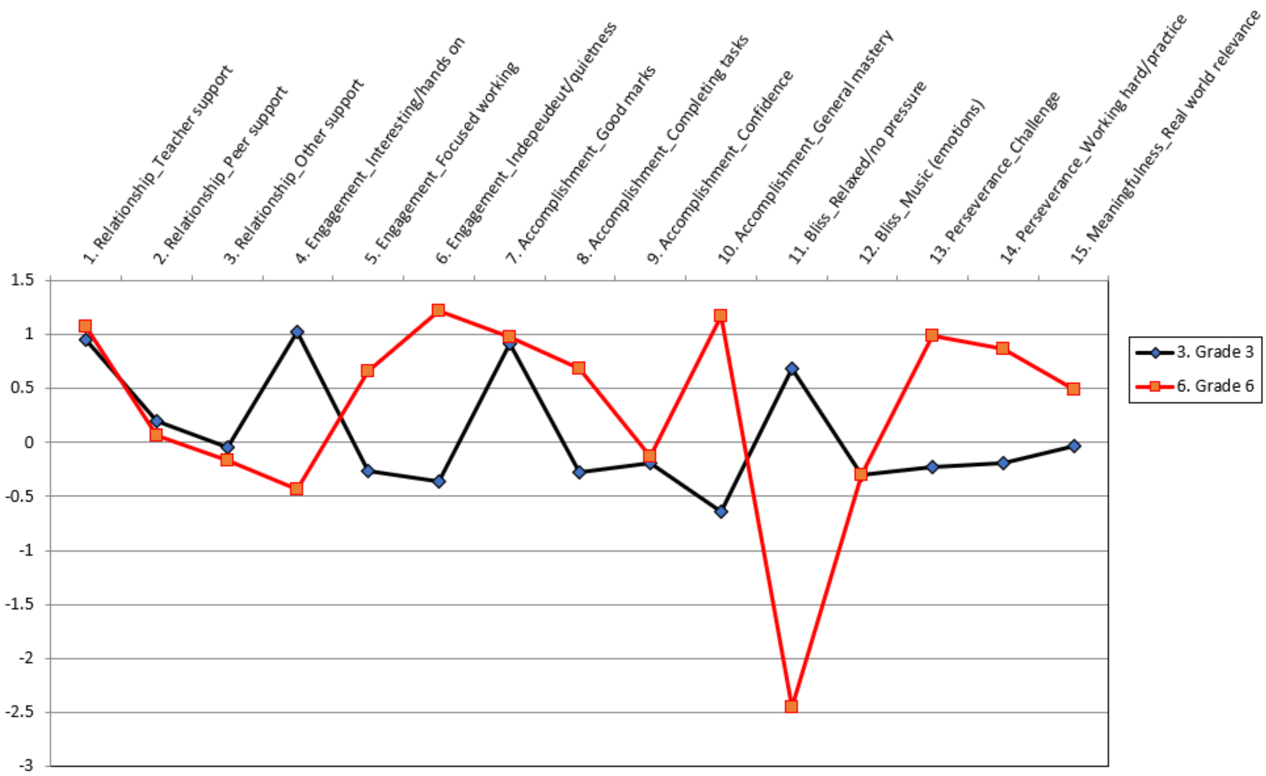

Figure 2: Interaction of Grade Level and Instrumental Values.

Rasch-Welch (logistic regression) t-test results show that there are statistically significant differences between the Grade 3 and Grade 6 students in favour of the Grade 3 students for the instrumental values associated with ‘interesting/hands-on’ ($t_{(758)}$=6.97, p<.05) and ‘relaxed/no pressure’ ($t_{(110)}$=5.66, p<.05); and in favour of the Grade 6 students for the instrumental values associated with ‘focused working’ ($t_{(172)}$=-2.52, p<.05), ‘independent/quietness’ ($t_{(166)}$=-4.09, p<.05), ‘completing tasks’ ($t_{(174)}$=-2.58, p<.05), ‘general mastery’($t_{(162)}$=-4.78, p<.05), ‘challenge’($t_{(171)}$=-3.16, p<.05), and ‘working hard/practice’ ($t_{(173)}$=-2.78, p<.05).

DISCUSSION

76 Grade 6 students in a Chengdu suburban elementary school were asked to identify, from a teacher-nominated set of learning moments, those which accompany their experiencing of MWB. A similar assessment was carried out with this group of students three years prior when they were in Grade 3. In responding to Research Question 1, it was found that at this upper elementary level, MWB was associated with the fulfilment of six of the seven ultimate values identified earlier, that is, without *learning*. This was to be expected, since none of the 21 teacher-nominated learning moments reflected the valuing of *learning*, and the reason why it was an ultimate value associated with MWB three years prior was that two students (of the 258) then had identified them in the open-ended ‘other’ item. This is not to suggest, however, that students’ MWB did not involve experiencing of *learning*: conversations with the mathematics teacher suggest that the students were not short of opportunities to experience the valuing of *learning*. In other words, even though learning moments reflecting *learning* might have been too obvious for the classroom teachers to have listed them in the questionnaire, this current study lends further support for the same set of seven ultimate values governing MWB as was identified in Hill et al. (2022).

Specifically, over the three years from Grade 3 to Grade 6, two each of the six ultimate values were associated more with MWB and experienced more by students (*accomplishment, perseverance*); less associated and experienced (*engagement, bliss*); and similarly associated and experienced (*relationship, meaningfulness*).

Research Question 2 aimed to understand which instrumental values experienced changes in fulfilment that led to changes in the fulfilment of the associated ultimate values. The statistically significant drop in students’ experiencing of *bliss* in Grade 6, for example, could be the result of a drop in the fulfilment of being *relaxed* (one of two instrumental values assessed), whereas the fulfilment of (listening to) *music* (the other instrumental value assessed) remained the same over the three years. Similarly, the increase in students’ fulfilment of *accomplishment* and *perseverance* was due to changes in two instrument values each: *completing tasks* and *general mastery* for the former, and *working hard* and *challenge* for the latter. Notably, the drop in expression of students’ *engagement* seemed to be caused by all three instrumental values assessed, namely, *interestingness*, *focussed work*, and *independence*.

Despite the changing nature of mathematics topics at upper elementary levels, despite the demands and needs of adolescence, the data suggest that the students’ experiencing of their valuing of *relationship* has not been affected. Perhaps this is because students in China have the same teachers and peers throughout their elementary school years. Students' experiencing of *meaningfulness* has also remained stable.

It is not surprising that even as *bliss* continued to be a value underlying MWB, Grade 6 students were experiencing less of it. Not only have mathematics topics become more difficult (and abstract), parental pressure on results, teachers having less opportunities for positive reinforcements, and more complex question types all contributed. The changing nature of classroom activities away from fun ones such as origami (in Grade 3) probably also explained the less fulfilment of *bliss* and *engagement* at Grade 6.

The classroom teacher was aware that her students' opportunities to experience the valuing of *engagement* were being threatened. In response, she introduced group-based mathematics projects to her students annually, recognising that these would stimulate students' interest in hands-on tasks, promote focussed working, and provide students with the independence they enjoyed in completing the respective projects. These three aspects are in fact the instrumental values (see Figure 2) promoting student engagement in their mathematics learning. Yet, the projects probably did not exert sufficient influences to the students' engagement. Another point to note is that at the time of collecting the Grade 6 data in 2023, the year's project had not been announced yet.

The classroom teacher had been surprised that her students were fulfilling *accomplishment* and *perseverance* more, when she was expecting these to slide in Grade 6. According to her, this concern had probably led her to over-compensate, by consciously building into her lessons more opportunities for students to exercise perseverance, and to feel accomplished. This suggests that intentional teacher actions in their professional practice can effectively affect values fulfilment, and thus, MWB.

The data suggest that as students progress through the elementary school years in China, the development / maintenance of their MWB does not require the fulfilment of different values. However, the changing nature of the curriculum and the changing preferences of growing children have meant that opportunities for relevant instrumental values – and thus, the learning moments in class – to be experienced by the students are different across grade levels. Teacher awareness of these are important, for as the mathematics teacher in this study showed, teachers can use this knowledge to orchestrate student experiencing of targeted learning tasks to facilitate the fulfilment of particular values. Furthermore, the learning moments are commonly found in mathematics classrooms, implying that teachers need not introduce intervention activities into their lessons, disrupting established lesson structures.

CONCLUSION

This paper reports on the second assessment of students' MWB for a longitudinal study in an elementary school in Chengdu, China. Three years on after the first assessment in Grade 3, the Grade 6 students' MWB were supported by six ultimate values which were also documented three years prior, namely, *accomplishment*, *perseverance*, *meaningfulness*, *relationship*, *bliss*, and *engagement*. A seventh value, *learning*, was neither surveyed nor identified by the Grade 6 students, although we were not surprised when the classroom teacher believed that students' experiencing of it would also contribute towards their MWB. Amongst the six identified values, the students reported experiencing more of the first two ultimate values three years on, equivalent experience with the middle two, and less experiencing of the last two. The instrumental values underlying these changes were identified, with students experiencing less of all the three instrumental values feeding into *engagement* in particular.

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FIVE WORDS FOR RETHINKING MATHEMATICS EDUCATION

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Drawing on my research and experience over the past 20 years, this plenary lecture offers five words for rethinking mathematics education together: alignment, resources, culture, understanding, and specificity. Each word depicts an area of research that I engaged in over the decades. By looking back at these I offer considerations of how my thinking continues to move forward as a mathematics educator. As such, this talk provides a platform for reflection on my research journey and a way forward on reshaping my earlier thoughts in how we can be responsive to our change in an ever-changing world.

INTRODUCTION

It is a great honour to give a plenary lecture at PME 47. In order to prepare this talk, I looked back on my studies and experiences as a mathematics educator. I have been teaching both pre-service and in-service teachers at Korea National University of Education since 2002. My research areas include teacher education and professional development, mathematics textbook development and research, and various topics in elementary mathematics, especially early algebraic thinking. Reflecting on my research and teaching experiences over the last 20 years or so, I have come up with five key words–alignment, resources, culture, understanding, and specificity–that provide some thought for rethinking mathematics education. For each word, I start with the research background and then present some selected studies to raise implications for mathematics education and our responsiveness in moving forward.

ALIGNMENT

I have been heavily involved in curriculum and textbook development and research, but the word alignment was not really on my radar until recently. In the following section, I describe how alignment emerged in the design of a particular unit in a new textbook (Pang & Sunwoo, 2022). Also, alignment emerged when I was exploring students' perspectives on what they valued in mathematics learning while, at the same time, investigating how students would perceive what their teachers valued in teaching mathematics (Pang et al., 2024).

Three successive alignments from curriculum to student learning

We continue to change our mathematics curriculum to make mathematics better to teach and learn while meeting the new needs of an ever-changing world. We then develop a new series of textbooks to make the ideal or abstract aspects of the curriculum concrete enough to be implemented in the classroom. Alignment between the curriculum and the textbooks is emphasised. However, we often overlook the critical link in the alignment chain that connects textbooks (or any other instructional

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resources) to actual classroom instruction and student learning outcomes. This is especially true in Korea, where research on curriculum or textbooks has been the most common topic in recent years, but the majority of studies have focused on analysing the curriculum or textbooks themselves rather than on how the intended curriculum is implemented in the classroom (Pang, 2022). With this in mind, the following is an example of how alignment was seriously considered in the design of a pattern and correspondence unit to promote functional thinking in a Korean textbook.

In Korea, the national mathematics curriculum requires students in Grades 5 and 6 to (a) identify and explain the pattern from a table showing the correspondence relationship in which one quantity changes and the other quantity depends on it and (b) represent the relationship by an equation using symbol variables. Four key instructional elements were extracted from a review of the literature on functional thinking: correspondence relationships in real-life contexts, various pattern tasks, exploration for a correspondence relationship, and symbol variables to represent a correspondence relationship. The previous textbook unit on pattern and correspondence was analysed through the four elements. The analyses led to the new textbook, including more geometric patterns using shape blocks and additive relationships, giving students many opportunities to explore the relationship between two covarying quantities and to think about the usefulness of using symbol notations to represent a correspondence relationship along with the meanings of such symbols.

Once textbook activities are aligned with the curriculum expectations, the next step is to align the intent of the textbook activities with an actual lesson. The new pattern and correspondence unit was implemented in an elementary classroom (the intervention group) to explore its appropriateness for students to develop functional thinking. As it is important for a teacher to understand the intentions of new textbook activities, a teacher's guide was also developed to include the background knowledge of the unit, such as three modes of student thinking when exploring the relationship between two quantities (i.e., recursive thinking, covariation thinking, and correspondence thinking) and content-specific pedagogical strategies (e.g., using a non-sequential function table). Five lessons in implementing the new unit in the intervention group were videotaped and qualitatively analysed using the four key instructional elements described above. Students were able to (a) notice and articulate various correspondence relationships between objects in the classroom, (b) find a correspondence relationship in both numerical and geometric pattern tasks, (c) use correspondence thinking beyond their initial recursive thinking, and (d) use symbols for variables when expressing a function rule.

A final alignment needs to verify that the implementation of the new unit has resulted in positive student learning outcomes. For this verification, the intervention group (with the new unit) and the non-intervention group (with the same unit from the previous textbook) were compared using two types of written assessment: Type A assessment was to confirm whether the students understood the main content of the unit, while Type B assessment was to compare and contrast the functional thinking of both groups. There were no statistically significant differences between the two groups in the pre-test for the two types of assessment. There were also no statistically significant differences between the two groups in the post-test for the Type A assessment. However, there were statistically significant differences for the Type B assessment, indicating that the intervention group could develop a better understanding of functional thinking than the non-intervention group.

This example is significant because it goes beyond the development research trend of focusing mainly on the alignment between a curriculum and textbook activities, which is often the case, and looks at whether textbook activities are translated into classroom instruction and lead to student learning outcomes. Whenever a new curriculum and its accompanying textbooks are changed, they should be examined to see whether the intentions of the curriculum are ultimately linked to student learning outcomes. In turn, student learning outcomes should be an important catalyst for curriculum revision, if the three types of alignment are done as intended.

Value alignment for effective mathematics instruction

Mathematics educational values are regarded as any attributes of mathematics teaching and learning that are considered personally important (Seah, 2019). As students' values in learning mathematics are key to understanding and facilitating their learning, the importance of considering what students value in learning mathematics cannot be overemphasised. Furthermore, if what the teacher considers important in teaching mathematics is aligned with what students consider equally important, the foundation for effective teaching can be laid.

Korea's participation in the Values Alignment Study (VAS) identified value alignment between students' personal values of learning mathematics and their perceived teacher values of teaching mathematics. A total of 832 Grade 9 students participated. Two specific items of the VAS questionnaire were the following: (a) Think about your own experience of learning mathematics. What do you think is important when you learn mathematics? (b) Think about your own mathematics teacher this year. What do you think is important to him or her in mathematics teaching? Students were asked to list and explain up to three attributes corresponding to what they considered important. In order to capture the richness of the data in the students' writing, we analysed it based on the connectivity between words and the centrality of specific words.

The frequency analyses between students' personal values of learning mathematics and their perceived teacher values of teaching mathematics show a remarkably similar trend: The top three values (i.e., *problem*, *understanding*, and *review*) were the same. The centrality analysis revealed three groups of students' personal values and four groups of their perceived teacher values. As shown in Table 1, the groups had the following values in common when comparing students' personal values and their perceived teacher values: Group 1 included *problem, concept, formula, solution,* and *basics*; Group 2 included *understanding* and *concentration*; and Group 3 included *review* and *preparation*. The value *problem* was centralised as being the most important for students' own learning of mathematics and for their teachers' teaching of mathematics. These findings show an alignment between what the students thought was important for their mathematics learning and what they thought was important for their mathematics teachers in teaching mathematics. However, there were subtle differences, such as more values adjacent to the value *problem* in describing students' own values of mathematics learning, as well as a new group with *oneself* and *lesson* in describing teacher values.

Grou p	Students' personal values of learning mathematics	Students' perceived teacher values of teaching mathematics
1	problem, formula, concept, thinking, solution, ability, computation, basics, type	problem, concept, solution, formula, basics, method, variety
2	understanding, concentration, persistence	understanding, concentration, thinking, explanation
3	review, preparation, content	review, preparation
4	·	oneself, lesson

Table 1: Students' values of learning mathematics and their perceived teacher values

The striking similarities between students' values of learning mathematics and their perceived teachers' values of teaching mathematics suggest that teachers need to be aware that students' perceived values are related to what teachers consider important. Therefore, teachers need to better understand their values of mathematics education and be able to balance different values when interacting with students to achieve intended pedagogical goals.

RESOURCES

As I said earlier, because of my heavy involvement in the development of instructional resources, primarily textbooks, the word resources clearly came to mind as I reflected on my research and experience. Here, I use the word resources to include not only textbooks, but also workbooks, teacher guides, digital materials, and so on that are developed to support teaching and learning. Effective mathematics textbooks are key to students' learning and teachers' teaching. This is particularly true in Korea, where 99% of fourth graders (the international average was 75%) used textbooks as the basis for instruction, and 97% of eighth graders (the international average was 77%) did so (Mullis et al., 2012). It is not surprising, therefore, that Korea is putting its best efforts into developing effective textbooks. Similarly, research on curriculum and textbooks has accounted for about 15% of all articles published in mathematics education journals in Korea over the past 50 years (Pang & Kwon, 2023).

Textbook development and related issues

In Korea, many textbook analysis studies have been conducted to develop effective textbooks. In fact, about 76% of textbook studies in Korea are textbook analysis studies, mainly focusing on the specific mathematical concepts or the overall organisation of a mathematical content strand, including how and when to teach it (Pang, 2022). Such studies tend to identify the changes in textbooks in relation to the revisions of the national mathematics curriculum in Korea. Also, some of these studies compare Korean textbooks with foreign counterparts in order to identify alternative methods or activities for addressing and presenting specific mathematical concepts.

However, most comparative studies focus on textbooks rather than teacher guides, which makes it difficult to fully understand the intentions of textbook construction and activities, and they are limited to foreign textbooks written in English or translated into Korean. To overcome these limitations, collaborative studies involving foreign researchers need to be activated. In addition, if international comparative studies are conducted not only in textbook analysis but also in textbook use, it will be easy to analyse socio-cultural factors related to the interaction between textbooks and teachers beyond the characteristics of the textbooks themselves.

Another effort to develop effective textbooks in Korea involves multiple experts (e.g., mathematics educators, mathematicians, teachers, and designers) researching, writing, reviewing, and discussing textbooks, which are then pre-tested in actual classrooms to check their appropriateness. This has been possible because Korea has only one set of elementary mathematics textbooks under the national curriculum. However, this government-issued textbook system has recently been changed to a government-approved textbook system for Grades 3 to 6. On the one hand, this change invites the development of diverse and creative textbooks that address and present the same mathematics topics with different approaches. On the other hand, it raises many concerns, such as the decentralisation of research and writing, the weakening of the field review process, the professionalism and transparency of the approval committee, excessive competition among publishers, and textbook adoption influenced by non-mathematical reasons, such as whether textbook publishers have an online platform.

Another recent issue related to textbook development is the development of digital textbooks using artificial intelligence (AI). To be clear, the educational community is already using various digital resources and platforms, but these tools have only served as a supplement to textbooks. However, government-approved AI digital mathematics textbooks will be used for the first time in Grades 3 and 4 next year, so they are currently under development. Much discussion has revolved around how AI digital textbooks will be built, what technologies will be used to maximise student learning of mathematics, and how best to support teachers in teaching mathematics while respecting their ability to plan and implement lessons.

Once AI digital textbooks are developed and deployed in the field, it will be necessary to study how teachers select and use digital textbooks compared to traditional book-based textbooks. Since AI digital textbooks will diagnose individual students' mathematical skills and dispositions, it is expected that a teacher can effectively use AI digital textbooks to provide differentiated and responsive instruction according to students' needs. Similarly, students' actual use or interaction with AI digital textbooks is a research area to be further investigated.

A teacher's guide and related issues

Compared to the development of textbooks, the development of teacher guides has been relatively neglected, mainly because most of the tasks in the textbook are used by a teacher and students in mathematics classes, whereas teacher guides are intended only for teachers and are used selectively by the teachers. Nevertheless, the development of teacher guides in Korea has recently gained importance in terms of teacher learning and professional development (Pang, 2022). On the one hand, pre-service teachers study teacher guides thoroughly in order to pass the highly competitive national teacher employment test, making them an important resource for novice teachers' learning. On the other hand, in-service teachers can strengthen their content-specific pedagogical knowledge by reviewing the intentions of the tasks, understanding the teaching strategies with their rationales, and looking for alternative activities in the teacher guides if necessary.

The Korean teacher guides have both strengths and weaknesses in supporting teacher learning (Pang et al., 2023c). For example, the teacher guides for Grades 3 and 4 are effective in that the mathematical content knowledge for teaching is well provided, and a variety of manipulative materials are presented along with the rationale or strategies for using them, especially in the areas of operations or geometry. However, the sample teacher-student dialogues for each lesson are only useful for getting a sense of the overall flow of the lesson. They do not include a wide range of student responses due to space limitations. Recent changes in the development of the teacher guides should address these weaknesses.

As instructional resources for elementary school mathematics move towards a government-approved system, teacher guides will also be approved separately from textbooks, resulting in richer materials and more research-based guidance for teaching than before. Another change associated with the government-approved system concerns the main textbook publishers, who will provide teachers with condensed versions of the core content of the teacher guides, called teacher textbooks (i.e., teacher editions of student textbooks), to help them use the textbooks more effectively.

The development and dissemination of digital resources can free authors or teachers from the space constraints of typical teacher guides. Korea has a popular online community of teachers, and they have been sharing their own resources voluntarily and collaboratively for over 20 years. In addition, major textbook publishers have recently set up an online platform to provide teachers with almost all instructional resources, including video clips to explain the main concept in each lesson, which they can easily select according to their daily needs.

The increasing availability and variety of instructional resources require teachers to have expertise in their use. No matter how effective the resources are, their effectiveness depends on how teachers use them. Therefore, teachers' interactions with different instructional resources need to be further explored, especially in relation to how recent new features of such resources affect teachers' lesson planning, implementation, and reflection.

CULTURE

It seems obvious that mathematics education should be understood within the socio-cultural context in which it takes place. Nevertheless, I was surprised to find the power or influence of culture in unexpected research contexts, particularly in studies of effective mathematics teaching (Pang, 2012; Pang & Kwon, 2015; Pang et al., 2023b).

Teachers' overall perspectives on effective mathematics pedagogy

One of my main roles as a teacher educator is to help teachers implement effective mathematics lessons. The question of what constitutes effective mathematics pedagogy can be answered from a variety of perspectives, but essentially, it is important to know what the teachers who deliver the lessons think. The initial target participants for my related study were elementary school teachers, but this population was extended to secondary school mathematics teachers.

A questionnaire was developed to explore teachers' perspectives on effective mathematics pedagogy. In the first part of the questionnaire, teachers were asked to describe any aspects that they considered important for effective mathematics pedagogy. In the second part, they were asked to indicate the extent to which they agreed with 48 items related to effective mathematics pedagogy using 5-point Likert scales: A score of 5 meant strongly agree, and 1 meant strongly disagree. The participants were 135 elementary school teachers, 132 middle school mathematics teachers, and 124 high school mathematics teachers. Figure 1 shows the mean scores of the 48 items according to the three groups of teachers (Pang & Kwon, 2015, p. 149).

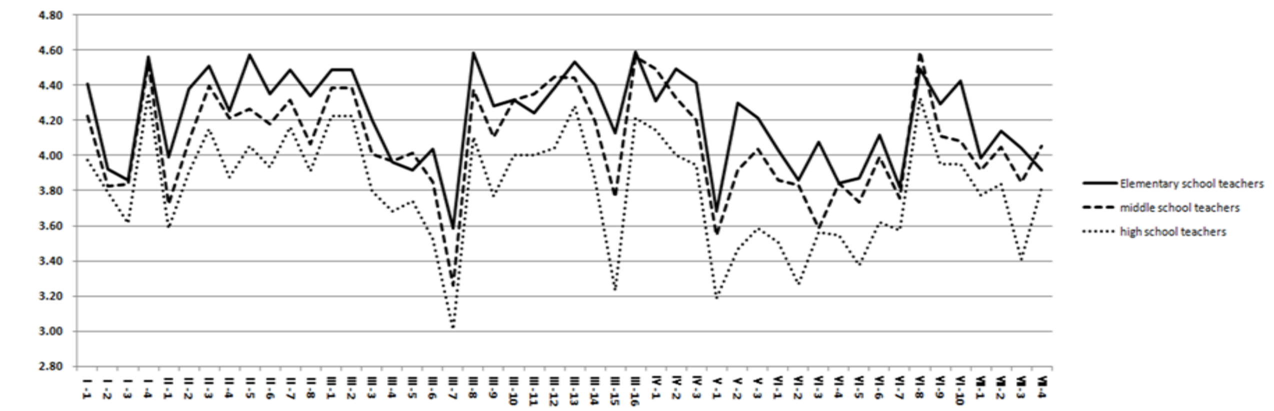

Figure 1: Mean scores of the 48 items by the groups of teachers

The most striking aspect of Figure 1 is the similar patterns across the groups of teachers. To be sure, the extent to which each group of teachers agreed with each item varied. Elementary school teachers tended to agree more than their middle or high school counterparts. In fact, there were statistically significant differences between elementary and middle school teachers on 11 items and between middle and high school teachers on 29 items. However, the overall agreement about what constituted effective mathematics pedagogy was quite similar.

On the one hand, the items the three groups of teachers agreed upon the most included (a) teaching by constructing the curriculum according to students' different levels, (b) teaching based on mathematical communication between the teacher and students, (c) teaching to improve students' self-directed learning ability, (d) providing students with appropriate feedback, and (e) teaching the essential concepts in mathematics. These perspectives are influenced by the recent revisions of the mathematics curriculum in Korea. On the other hand, the least agreed-upon items included (a) teaching students to calculate proficiently, (b) teaching by using technology, (c) having a good physical environment, (d) teaching while managing problematic students, and (e) emphasising human relationships. The similar patterns in Figure 1 suggest that teachers' perspectives on effective mathematics pedagogy were deeply rooted in their socio-cultural contexts. Therefore, it is necessary to explore what effective mathematics pedagogy means when we discuss what we are aspiring to in mathematics education, and we must explore how it is the same as or different from teachers' perceptions of effective mathematics pedagogy, especially among those who actually teach it. This exploration needs to be interpreted within the socio-cultural context of each country.

Changing teaching practices towards effective mathematics teaching

For me, the research context in which the word culture clearly emerged was when I was studying the process by which teachers changed their teaching practices to implement better mathematics teaching. For instance, I analysed how an elementary school teacher (Ms Y) changed her teacher-centred teaching to a student-centred approach by participating in a year-long research project (Pang, 2012). An analytical framework of five dimensions with 24 sub-dimensions was developed, and five lessons of Ms Y's teaching practice were selected to trace her instructional changes. I focused on what had changed and what had not changed in the process of incorporating student-centred instructional approaches (e.g., using instructional strategies tailored to students' differences) into an ordinary teaching practice.

In terms of what had changed, three types of changes were identified: (a) *dramatic changes* were sudden and noticeable changes that occurred in the early stages of teacher change, (b) *substantial changes* were less dramatic but considerable changes that occurred in the middle stages of teacher change, and (c) *gradual changes* were changes that occurred over a longer period. For instance, dramatic changes were noted in the use of manipulatives, the opportunities for students to present their own ideas to the whole class, and the use of small-group or individual activity formats. Substantial changes were noted in the focus on promoting students' mathematical reasoning skills and on soliciting and using students' ideas. Gradual changes were identified for focusing on fostering students' positive dispositions towards mathematics and reducing teacher question/answer and demonstration.

In terms of what had not changed in Ms Y's teaching practice, two aspects were noticeable. On the one hand, there were some positive changes that were not fully achieved as recommended. These less fully achieved changes occurred in the use of teaching strategies tailored to students' differences and in emphasising the importance of mathematical communication. On the other hand, there were unchanged practices throughout the year. Ms Y used many new recommended approaches, but the overall characteristics of her lessons were still consistent, progressive (i.e., from easy/concrete to difficult/abstract forms), and systematic (i.e., a lesson flow included learning motivation, learning objectives, main activities, practice, and evaluation/summary). A notable constant was Ms Y's emphasis on important mathematical content. For instance, she encouraged students to use manipulative materials, but she made sure that such activities were connected to the conceptual structure behind them. She also solicited students' multiple ideas but did not forget to emphasise a mathematically significant idea by orchestrating the path of classroom discourse to explore it or otherwise directly introducing it when students did not come up with such an idea. In other words, despite the change in the form of instructional approaches, the pedagogical priority of Ms Y's teaching was her students' conceptual understanding of mathematical content. The strong emphasis on mathematical content reflects one of the most prominent cultural activities of Korean teaching.

Another similar research context in which the word culture emerged was when I explored the challenges Korean teachers faced in implementing Smith and Stein's (2018) five practices for orchestrating productive mathematics discussions in their classrooms: *anticipating, monitoring, selecting, sequencing*, and *connecting.* The researchers associated these five practices with *setting goals* and *selecting tasks* as a foundation. The research participants were 15 elementary school teachers who were keen to implement the five practices through an iterative lesson study cycle of lesson planning, implementation, and debriefing. They had overall success in implementing the five practices, particularly those related to lesson design (i.e., from setting goals to anticipating student responses), but they also experienced various challenges. Some challenges were similar to those reported in the U.S. context, such as identifying the core mathematical ideas of the learning objectives and planning specific questions and feedback tailored to student responses (Smith et al., 2020).

Other challenges not explicit in the literature were also identified in the study I worked on (Pang et al., 2023b). For instance, one challenge identified was selecting and presenting tasks appropriate to the student levels and the classroom environment, probably due to the specific characteristics of the lesson study. The challenge of taking on multiple teacher roles at the same time may be a problem inherent in monitoring student work. Still other challenges, such as clearly stating the learning goals in sentences, writing a lesson plan for effective use, or visually sharing student presentations, seem to be related to the Korean classroom culture. It was interesting that these challenges manifested themselves when the challenges faced by Korean teachers were compared and contrasted with those of their U.S. counterparts. The process of implementing new approaches, such as the five practices, can reveal subtle but significant differences and variability that exist between cultures. Changes in teaching practices tend to occur within socio-cultural expectations and educational values long maintained in a particular education system. Therefore, further research is needed to identify the different challenges teachers face in implementing the five practices in different education systems and to explore whether these are inherent in the practices themselves or in the socio-cultural context in which they are implemented.

UNDERSTANDING

The importance of understanding in mathematics education cannot be overemphasised. The curriculum that defines what should be taught in school mathematics has been revised periodically in response to societal changes, but what has remained constant is students' understanding of mathematical concepts, principles, or laws. It is important to examine how students understand a mathematical concept. In my recent research on early algebraic thinking, I have been particularly interested in students' understanding of the equal sign and variables (e.g., Lee & Pang, 2021; Pang & Kim, 2018; Pang et al., 2023a).

Students' opposing conceptions of the equal sign

As a basis for developing algebraic ideas, students need to have a relational understanding of the equal sign. In Korea, the equal sign is first introduced in Grade 1 when teaching addition and is then used in many contexts. Using the 27 assessment items from Matthews et al. (2012), I investigated students' understanding of the equal sign, expressions, and equations (Pang & Kim, 2018). Specifically, 695 students from Grade 2 to Grade 6 (ages 7 to 12 years) were included in the study. The results show that students were quite successful in almost all items of three different types (i.e., equation-structure items, equal sign items, and equation-solving items). More importantly, a statistically significant difference was found among the grades except between Grades 5 and 6, indicating that students' understanding improves as their grade levels increase up to Grade 5. Nevertheless, there were some items that needed further consideration.

The most difficult of the equation-structure items was deciding whether the number that goes into the box is the same number in two given number sentences: $2 \times \square = 58$ and $8 \times 2 \times \square = 8 \times 58$. Even among Grade 6 students, about 57% of them got the correct answer, and only about 15% of them could use relational thinking for the answer. Another item that needs attention concerns the meaning of the equal sign. When asked to determine whether the given definition of the equal sign is true or false, more than 80% of the students from Grade 3 onwards chose "true" for the following sentence: The equal sign means "the same as". In contrast, about 22% of the students chose "false" for the following sentence: The equal sign means "the answer to the problem”. Students considered the equal sign to mean both “the same as” and “the answer to the problem”.

In a separate study I conducted, the coexistence of two conceptions of the equal sign in a single student’s mind was found in a whole-class discussion of equations with two equal signs (Lee & Pang, 2021). Thirty students in Grade 4 were studying multiplication and division based on textbook activities. Specifically, students were first exposed to equations with two equal signs while checking $162 \div 20 = 8R2$ and solving $20 \times \square + \bigcirc = \square + \bigcirc = \square$. Students came up with different equations, and the teacher guided students to discuss why they agreed or disagreed with each equation. Even after a lengthy discussion of the meanings of the equal sign, as many as 22 students said that they agreed not only with $20 \times 8 + 2 = 160 + 2 = 162$ but also with $20 \times 8 = 160 + 2 = 162$, insisting that the equal sign could be interpreted as both “the same as” in the former equation and “is” in the latter. Note that it was helpful for these students to parenthesise the expression on each side of the equal sign to treat it as an entity or as a whole. It was also helpful for them to explore the structure of the equation or to use the transitive properties of equivalence.

The two studies above indicate that many students have simultaneously opposing (but apparently compatible) conceptions of the equal sign, which are operational and relational conceptions. In this respect, when investigating elementary school students’ understanding of the equal sign, it seems a better option to present them with a specific equation and ask them what the equal sign means in that equation rather than to ask them to write a free-form description of the meaning of the equal sign or to give them the meaning of the equal sign and ask them to judge it as true or false. In addition, for concepts encountered across all grades in elementary school, such as the equal sign, it is worth investigating whether an understanding of the concept grows as students progress through the grades, specifically in terms of what misconceptions older students continue to have and what features of equation-structure and equation-solving items they find particularly challenging.

Similarly, to develop students’ understanding of the equal sign, it is important to cover a variety of contexts in which the operational or non-relational interpretations of the equal sign do not apply and to encourage students to think about how best to define the equal sign in different contexts. Even if students see the meaning of the equal sign as sameness, it is important to be clear about which components of the equation are equal. Despite numerous studies of students’ understanding of the equal sign, the complexity of this understanding remains elusive.

Students’ understanding of variables

Variables are an important concept common across the two content areas of early algebra: the generalised arithmetic perspective and the functional perspective (Kieran et al., 2016). Building on the previous finding that Korean students had difficulty representing unknown quantities with variables (Pang & Kim, 2018), I investigated students’ overall understanding of variables across the two content areas. Specifically, from the generalised arithmetic perspective, my study's questionnaire included the property of "1", the community property of addition, the associative property of multiplication, and a problem context with indeterminate quantities. From the functional perspective, the questionnaire covered additive, multiplicative, squaring, and linear relations.

A total of 246 Grade 5 students were included in the study (Pang et al., 2023a). The results show that most students could find specific values for variables and understood that equations involving variables could be rewritten using different symbols. However, students struggled to use variables to represent generalised properties or contexts of arithmetic problems. For example, almost all students in the study found a specific value related to the property of "1" (i.e., 10,293 × 1 = 10,293), but only about 66% of them could represent it with a variable (i.e., $\square \times 1 = \square$). Also, about 30% of the students knew that the variable could be not only natural numbers but also fractions or decimals. This suggests that students need to explore whether the properties of numbers and operations remain the same when the range of numbers covered in elementary school extends from natural numbers to fractions or decimals.

The most difficult item in the study was to represent problem contexts with indeterminate quantities in equations with variables. The problem was as follows: Minsoo's current hair length is 7 cm, and his hair grows at a rate of 2 cm per month. Think about how Minsoo's hair will change in the future: (a) How many months will it take until Minsoo's hair is 15 cm long? (b) Write an expression for the length of Minsoo's hair after some ($\square$) months have passed. The study found that 72% of the students were able to answer the first question correctly, but only about 29% of them were able to answer the second question correctly. Students' incorrect responses included writing the rule using a specific number without using a variable (e.g., $7 + 4 \times 2 = 15$), not expressing it in the form of a completed equation (e.g., $7 + \square \times 2 =$), or writing a specific value to the right side of the equal sign (e.g., $7 + 2 \times \square = 15$). While this tendency (i.e., success in finding specific values of variables but difficulty in writing an expression using variables) was consistent for correspondence relations from the functional perspective, it was found to be more challenging for the items associated with the generalised arithmetic perspective. These findings highlight the need to investigate students' understanding of a particular mathematical construct comprehensively by presenting different problem contexts from the two perspectives and within the same perspective.

SPECIFICITY

The last word I would like to mention is specificity. This word emerged as important when analysing the level of implementation of each of Smith and Stein's (2018) five practices as Korean teachers applied them to their mathematics teaching (Pang et al., 2022). Another research context in which the word emerged was in developing a teacher's guide that emphasised "process-focused assessment" (or formative assessment) during instruction (MOE, 2021).

Specificity for higher implementation levels of the five practices

There is a growing consensus that mathematical discussions should be a key part of mathematics teaching. Despite many suggestions to support teachers in transforming a discourse pattern in the mathematics classroom, general pedagogical ideas still make it challenging for teachers to implement productive content-specific mathematical discussions. The five practices make mathematics discussions more manageable for teachers by managing the content discussed and by reducing the burden of improvisations while honouring student contributions (Smith & Stein, 2018).

The five practices were expected to be suitable for Korean teachers to apply in their mathematics classes, as the pedagogical priority in Korean mathematics classes is conceptual understanding of important mathematical content, as mentioned earlier. Indeed, I have conducted several classroom studies with teachers who have attempted to apply the five practices in their mathematics classes (e.g., Pang, 2016; Pang et al., 2022). Overall, the teachers were quite successful as they participated in an iterative cycle of a lesson study. However, a closer analysis of the implementation of each practice led to four levels of implementation based on the extent to which the key components of each practice were implemented: Level 1 was assigned to a study participant's performance in the lesson study when only one component of each practice was considered or undesirable aspects were included; Level 2 when two or three components were implemented or undesirable aspects were included; Level 3 when all the components were implemented but insufficient aspects were included; and Level 4 when all components were implemented faithfully.

Because the teachers who participated in the lesson studies were committed to implementing the five practices, they rarely fell into Level 1 but, instead, implemented each practice at Levels 2 to 4. For example, regarding *task selection*, Grade 6 teachers changed a simple percentage problem to one of mixing two chocolate milks of different thicknesses and finding the thickness of the mixture. The task was aligned with the learning objectives and was cognitively challenging. However, in the first lesson, the students struggled with the task because they did not fully understand the meaning of thickness expressed as a percentage (Level 2). In response, the teachers added a sub-question in the second lesson to explore the meaning of thickness, which helped some students approach the task (Level 3). In the subsequent third and fourth lessons, the teachers adjusted the numbers of the task to make it more accessible to the students by drawing or calculating based on their understanding of percentages (Level 4).

Similarly, regarding *anticipating student responses*, Level 4 was given when a teacher (a) anticipated not only correct approaches to the given task but also errors or misconceptions that students might make in different and specific ways, (b) anticipated responses to student approaches in different and specific ways, or (c) identified the responses that might address the mathematical objectives. In summary, the higher implementation of each of the five practices ultimately depended on how *specifically* they prepared and implemented the key elements of each practice. Given this, specificity is key to maximising the potential of the five practices as content-specific pedagogical practices for orchestrating whole-group discussion.

Specificity of process-oriented assessment in a teacher's guide

Recent Korean mathematics curricula recommend process-oriented assessment as well as assessment of learning outcomes. When a curriculum only mentioned the term process-oriented assessment, the question arose as to how it could be implemented in the mathematics classroom. It was not enough to explain the intentions of the curriculum to teachers in a way that emphasised the importance of students' learning processes or to give brief examples of how process-oriented assessment could be used in a particular unit or lesson in the textbook. Therefore, the teacher's guide has been developed to provide teachers with specific ideas on how to use process-oriented assessment in each lesson.

For instance, in the pattern and correspondence unit for Grade 5 described above, the fourth lesson dealt with representing the correspondence relationship in an equation with symbol variables (Pang & Sunwoo, 2022). In the first task, students were asked to (a) fill in a table with the number of drones and the number of blades when making a drone with four blades and (b) represent the correspondence relationship between the number of drones and the number of blades in an equation by selecting the given cards with words and symbols, such as "the number of blades" or "=". For this task, the teacher's guide presents three possible student responses and gives specific teaching tips. Table 2 represents an example of teaching tips for one of these responses (MOE, 2021, p. 190).

Learning information	Example of teaching tips
Students incorrectly represent the correspondence between the number of drones and the number of blades using symbols and words.	Ask students to check the meaning of their equations. For example, the equation (number of drones) = (number of blades) × 4 means that the number of drones is equal to four times the number of blades. Have students check this using the table. Alternatively, students can plug in the numbers from the table to see if the equation is correct.
e.g., (number of drones) = (number of blades) × 4	This is a typical error response for many students, so it is suggested that you teach this explicitly to the whole class.

Table 2: Part of the process-oriented assessment described in the teacher's guide

Note that for the above task, the teacher's guide includes teaching tips for correct answers as well as incorrect ones. For example, even if students have correctly represented the relationship as (number of drones) × 4 = (number of blades), the guide suggests that a teacher might encourage students to represent it using a different equation and, if so, to compare the apparently different equations. The guide also suggests that students have the opportunity to explore whether the number of drones and the number of blades could be any number.

Process-oriented assessment was abstract in the curriculum, but it manifested in the teacher's guide by presenting teachers with a range of possible student responses to key tasks, along with teaching tips tailored to each response. One of the characteristics of an effective teacher's guide is that it is specific enough to allow teachers to anticipate multiple student responses to each task in the textbook and to provide appropriate feedback accordingly. It was this specificity that enabled teachers to implement the intentions of the curriculum in their classrooms.

CLOSING REMARKS

This talk has addressed five words based on my own research and experience. As such, these words are limited by my research and experience. In other words, the words that mathematics educators consider important may vary depending on their research and experience. What would your five words be based on your own research and experience? How can we rethink mathematics education from the words? According to socio-cultural changes, new content, processes, or ideas should be emphasised in mathematics education to prepare for the future society. So, our mathematics education tries to include more and more. However, we could move in another direction when rethinking mathematics education. Instead of the five words I have chosen, I think it would be possible to condense it down to fewer words or even just one word. Instead of adding new ideas, like Picasso's series of paintings, we could find the most important thing about mathematics education and subtract and subtract until we are left with the essence. I hope that this talk will stimulate your thinking about your own mathematics research journey, the research contributions you have made, and the directions you will take in your continued contributions to the mathematics education journey.

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