Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments

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Expectations in Economic Theory

- economy is an expectation feedback system
  - expectations affect our decisions and realizations
  - expectations are affected by past experience
- expectations play the key role in most economic models

30s-60s	naive and adaptive expectations
70s-90s	rational expectations
90s-	models of learning and bounded rationality

  - adaptive learning (OLS-learning)
  - belief-based learning
  - reinforcement learning

Example 1: Model of Financial Market

- demand for the risky asset (available in zero supply)

$$D_h(p_t) = \frac{\mathrm{E}_{h,t}[p_{t+1} + y_{t+1}] - (1+r)p_t}{a \ \mathrm{V}_{h,t}[p_{t+1} + y_{t+1}]}$$

- solving market clearing eq. at time t find the equilibrium price

$$\sum\nolimits_h D_h(p_t) = 0 \quad \rightsquigarrow \quad p_t = \frac{1}{1+r} \sum\nolimits_h \mathrm{E}_{h,t}[p_{t+1} + y_{t+1}]$$

- rational expectations

$$p_t = \frac{1}{1+r} \mathrm{E}_t[p_{t+1} + y_{t+1}] \quad \rightsquigarrow \text{(for i.i.d. dividends)} \ \ p_t = \frac{\bar{y}}{r}$$

- heterogeneous expectations (Brian Arthur, 1991)

$$p_t = \frac{1}{1+r} \sum_h \mathrm{E}_{h,t} \left[\frac{1}{1+r} \sum_{h'} \mathrm{E}_{h',t+1}[p_{t+2} + y_{t+2}] + y_{t+1} \right]$$

This paper:

- Heuristic Switching Model with heterogeneous expectations
- model is inspired by and tested on the experimental data

Key features:

in forecasting agents use simple rules of thumb, heuristics
(Tversky and Kahneman, 1974)

in learning agents switch between different forecasting rules on the basis of their performances
(Brock and Hommes, 1997)

Experiments about expectations

- Earlier experiments: indirect focus / expectations on exogenous time series: Schmalensee (1976), Hey (1994), Marimon and Sunder (1994)
- Learning-to-forecast experiments: Hommes et al (2005, RFS; 2008, JEBO), Adam (2009, EJ), Heemeijer et al (2009, JEDC)

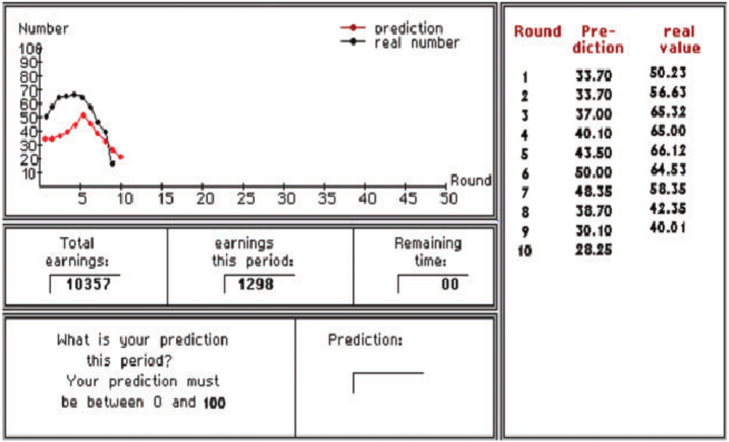
Model of asset-pricing (Campbell, Lo and MacKinlay, 1997)

- riskless asset with interest $r = 0.05$
- risky asset with price p_t and i.i.d. dividend y_t with mean $\bar{y} = 3$

$$p_t = \frac{1}{1+r}\left(\bar{p}^e_{t+1} + \bar{y} + \varepsilon_t\right) = \frac{1}{1+r}\left(\frac{p^e_{t+1,1}+\dots+p^e_{t+1,6}}{6} + \bar{y} + \varepsilon_t\right)$$

Experiment: 6 human subjects know only qualitative features

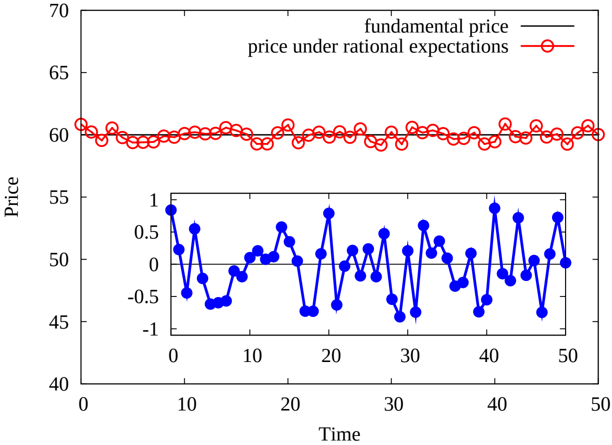
- submit forecasts $p^e_{t+1,h}$ and are paid according to the precision
- observe past prices (up to p_{t-1}), own forecasts and payoffs

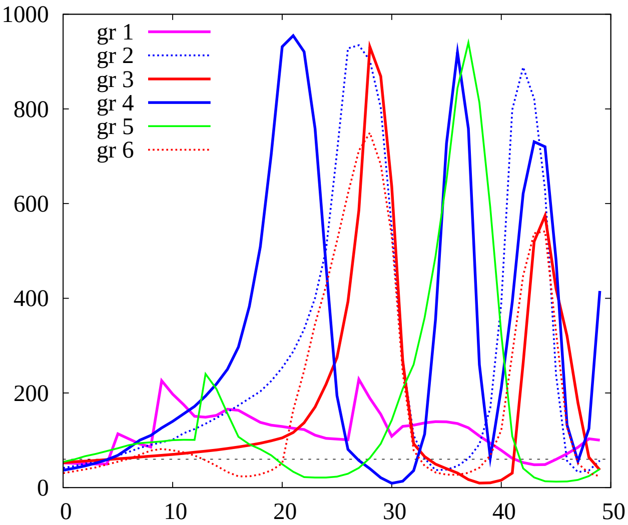


earnings per period: $e_{t,h} = \max\left(1 - \frac{1}{49}(p_t - p^e_{t,h})^2, 0\right) \times \frac{1}{2}$ euro

Rational Benchmark

If everybody predicts fundamental price $p^f = \frac{\bar{y}}{r} = 60$, then $p_t = p^f + \frac{\varepsilon_t}{1+r}$

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gr 4
gr 5
gr 6

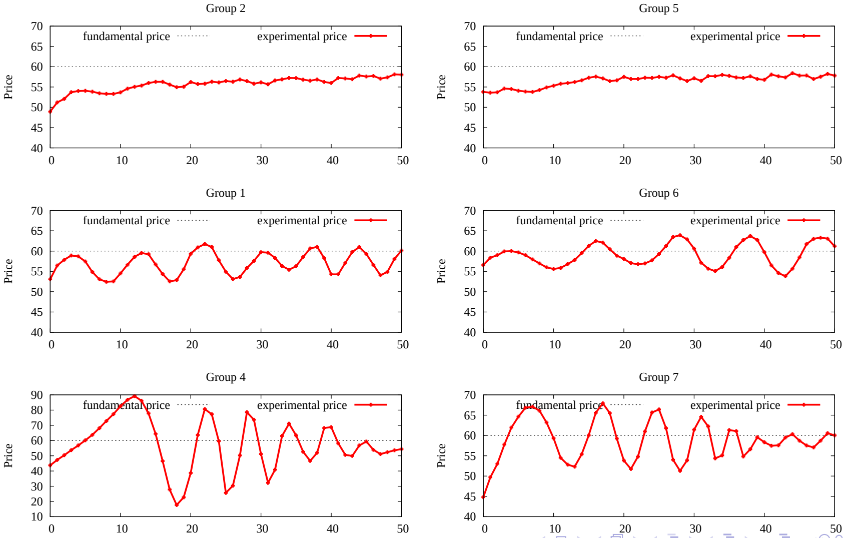
Experiment with stabilizing fundamentalists

- pricing equation

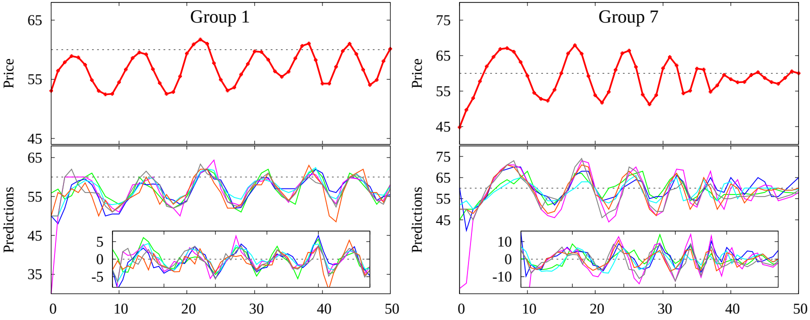
$$p_t = \frac{1}{1+r}\Big((1-n_t)\bar{p}^e_{t+1} + n_t p^f + \bar{y} + \varepsilon_t\Big)$$

- fraction of fundamental traders

$$n_t = 1 - \exp\Big(-\frac{1}{200}|p_{t-1} - p^f|\Big)$$


Group 2
Group 5
Group 1
Group 6
Group 4
Group 7
fundamental price
experimental price
Price

Results (individual predictions)



Estimation of individual prediction rules

OLS regression of predictions on the lagged prices and predictions

$$p^e_{i,t+1} = \alpha + \sum_{k=1}^{5} \beta_k p_{t-k} + \sum_{k=0}^{5} \gamma_k p^e_{i,t-k} + \epsilon_{i,t}$$

leaving insignificant coefficients out

- adaptive expectations

$$p^e_{t+1,h} = w\, p_{t-1} + (1-w)\, p^e_{t,h}$$

- trend-extrapolating rules

$$p^e_{t+1,h} = p_{t-1} + \gamma\, (p_{t-1} - p_{t-2})$$

- anchoring and adjustment rule

$$p^e_{t+1,h} = \frac{1}{2}\left(60 + p_{t-1}\right) + \left(p_{t-1} - p_{t-2}\right)$$

Learning-to-forecast experiments: Summary

"Stylized facts"

- large bubbles in the absence of fundamentalists
- qualitatively different patterns in the same environment
  - (almost) monotonic convergence
  - constant oscillations
  - damping oscillations
- coordination of individual predictions
- forecasting rules with behavioral interpretation are used

Model: four forecasting heuristics

- adaptive rule

  ADA $$p^e_{1,t+1} = 0.65\, p_{t-1} + 0.35\, p^e_{1,t}$$

- weak trend-following rule

  WTR $$p^e_{2,t+1} = p_{t-1} + 0.4\,(p_{t-1} - p_{t-2})$$

- strong trend-following rule

  STR $$p^e_{3,t+1} = p_{t-1} + 1.3\,(p_{t-1} - p_{t-2})$$

- anchoring and adjustment heuristics with learnable anchor

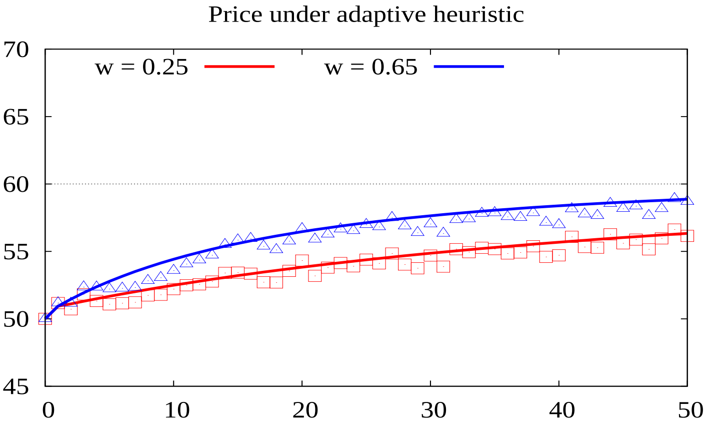
  LAA $$p^e_{4,t+1} = \frac{1}{2}\left(p^{av}_{t-1} + p_{t-1}\right) + \left(p_{t-1} - p_{t-2}\right)$$

price dynamics

$$p_t = \frac{1}{1+r}\left(\left(n_{1,t}p^e_{1,t+1} + n_{2,t}p^e_{2,t+1} + n_{3,t}p^e_{3,t+1} + n_{4,t}p^e_{4,t+1}\right) + \bar{y} + \varepsilon_t\right)$$

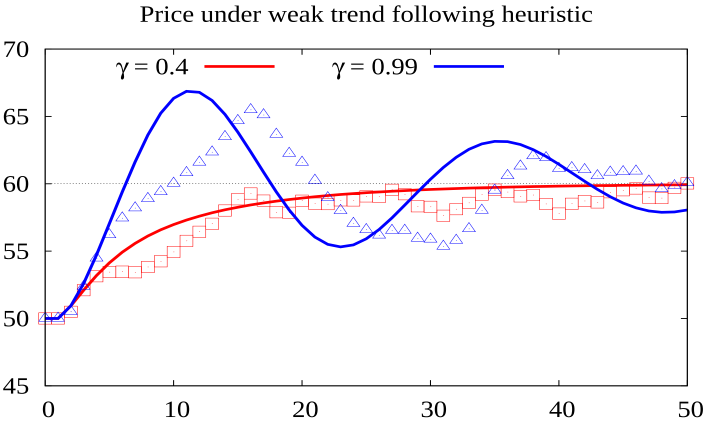
Adaptive Expectations: $p^e_{t+1} = w\, p_{t-1} + (1-w)\, p^e_t$

Dynamics globally converge to fundamental price.



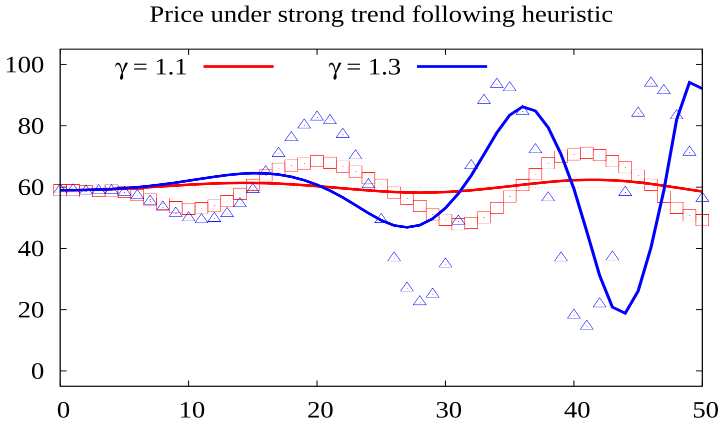
Weak-Trend Extrapolation: $p^e_{t+1} = p_{t-1} + \gamma\left(p_{t-1} - p_{t-2}\right)$

Dynamics converge to fundamental price.



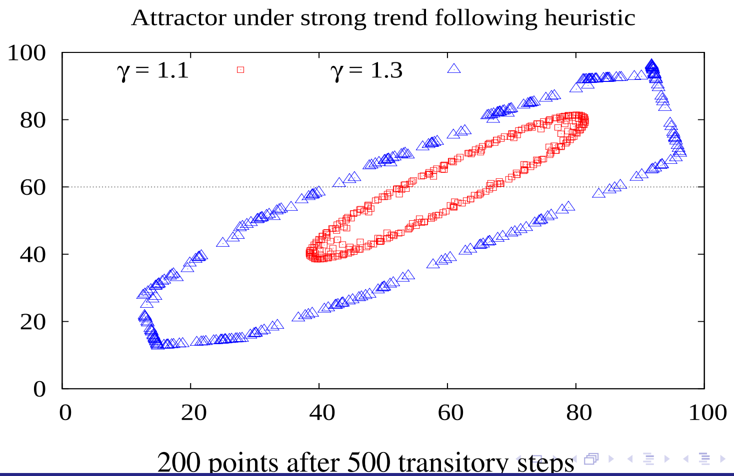
Strong-Trend Extrapolation: $p^e_{t+1} = p_{t-1} + \gamma\,(p_{t-1} - p_{t-2})$

Dynamics diverge from fundamental price...

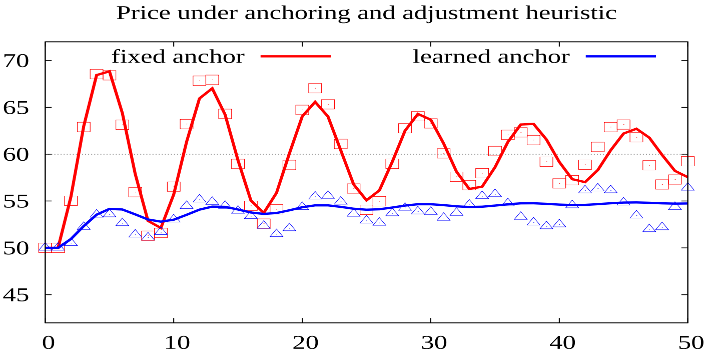


Strong-Trend Extrapolation: $p^e_{t+1} = p_{t-1} + \gamma\,(p_{t-1} - p_{t-2})$

...and settles on the quasi-periodic attractor.



Anchoring and Adjustment: $p^e_{t+1} = \frac{p^f + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$

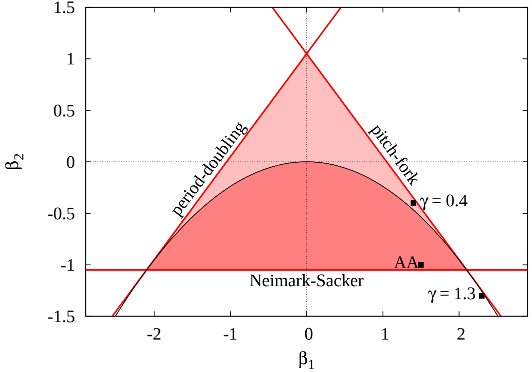


with learning of anchor: $p^e_{t+1} = \frac{p^{av}_{t-1} + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$

Model with Homogeneous Expectations

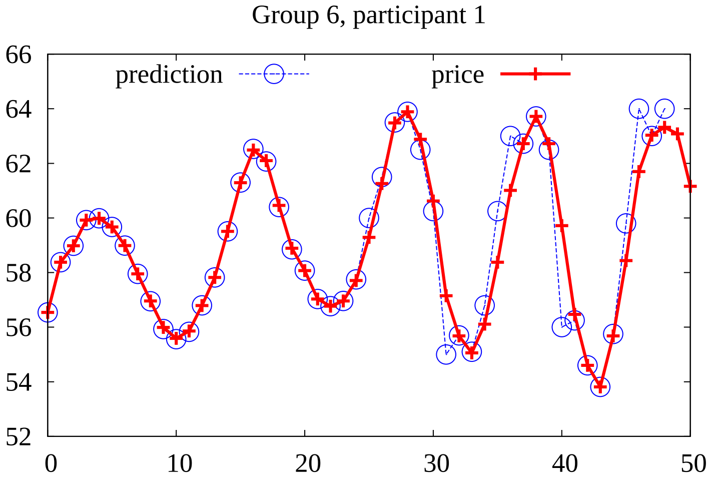
- pattern of monotonic convergence can be easily reproduced
  adaptive rule, weak trend extrapolation
- pattern of constant oscillations can be reproduced
  anchoring and adjustment rule without learning
- pattern of damping oscillations is reproduced (very imperfectly)
  strong-trend extrapolations

Stability conditions

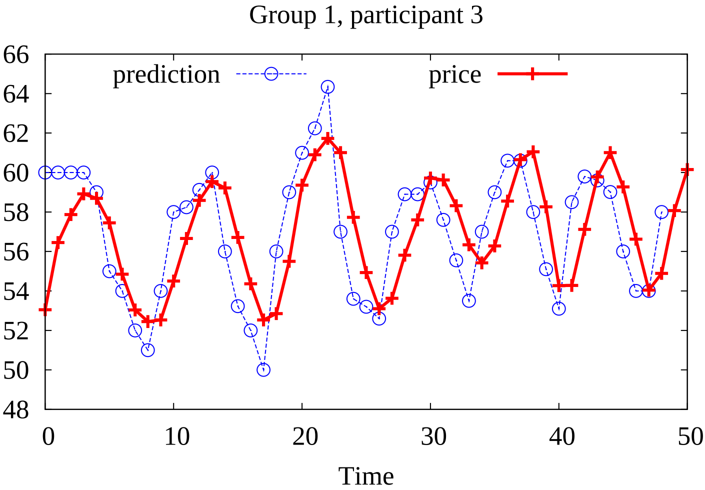


$$p^e_{t+1} = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2}$$

Evidence for switching I



Evidence for switching II



Modelling switching behavior

impacts of heuristics $n_{i,t}$ are evolving

- performance measure of heuristic i is

$$U_{i,t-1} = -\left(p_{t-1} - p^e_{i,t-1}\right)^2 + \eta\, U_{i,t-2}$$

parameter $\eta \in [0,1]$ – the strength of the agents' memory

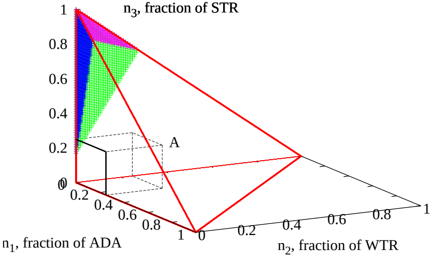
- discrete choice model with asynchronous updating

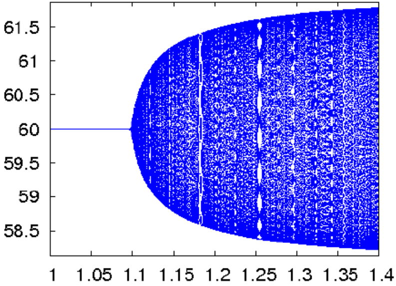
$$n_{i,t} = \delta\, n_{i,t-1} + (1-\delta)\, \frac{\exp(\beta\, U_{i,t-1})}{\sum_{i=1}^{4} \exp(\beta\, U_{i,t-1})}$$

parameter $\delta \in [0,1]$ – the inertia of the traders
parameter $\beta \geq 0$ – the intensity of choice

Stability and Instability of the model

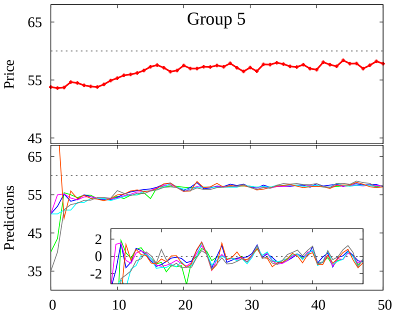
Stability region for model with fixed fractions

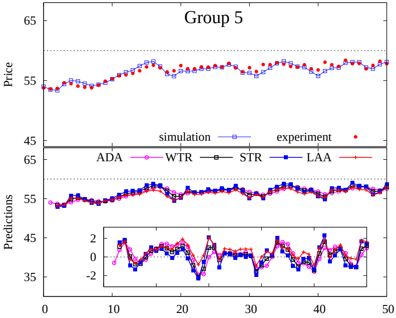




Deterministic path: Monotonic convergence

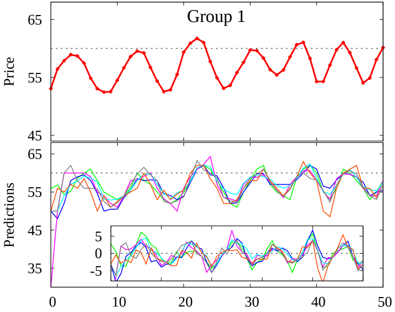
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$

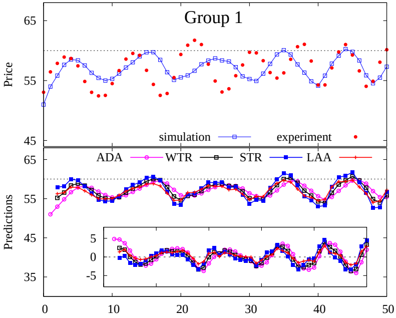




Deterministic path: Constant oscillations

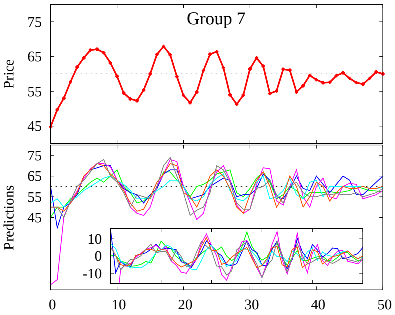
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$

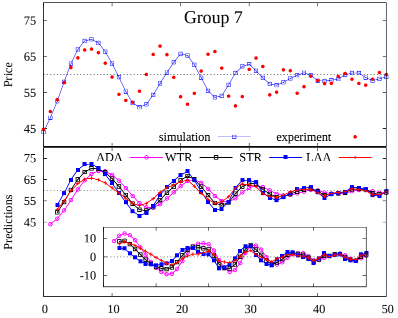




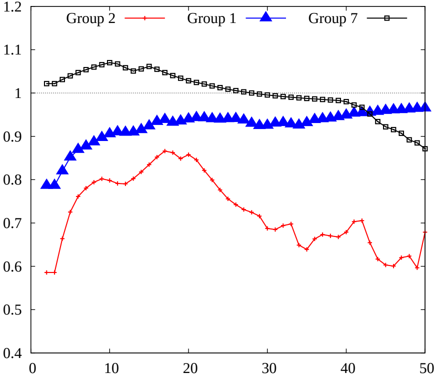
Deterministic path: Damping oscillations

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$





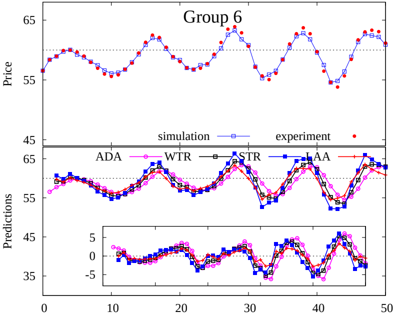
Evolution of instability

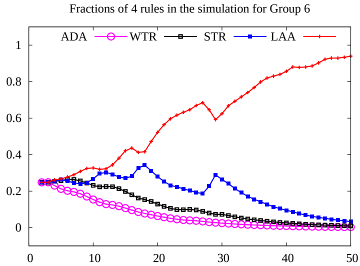


Largest modulus of eigenvalue of HSM

One-period ahead prediction: gr. 6 (constant oscillations)

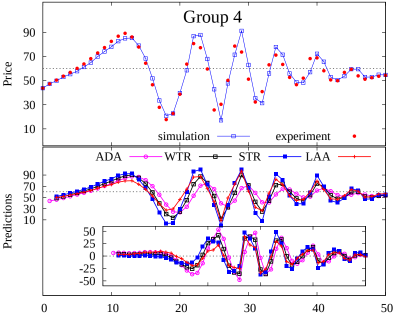
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$

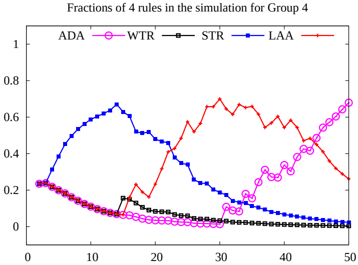




One-period ahead prediction: gr. 4 (damping oscillations)

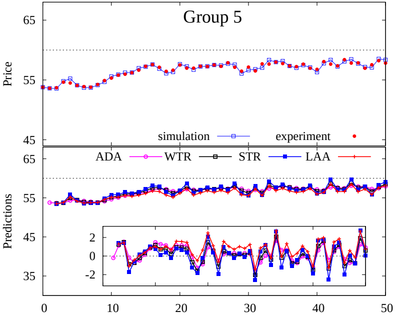
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$

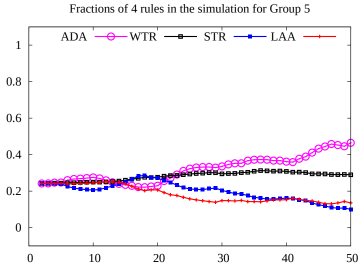




One-period ahead prediction: gr. 5 (convergence)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$





In-sample performance

MSE over 47 periods for 8 different models

Specification	Group 2	Group 5	Group 1	Group 6	Group 4	Group 7
Fundamental	16.6231	10.8238	15.7581	9.3245	300.9936	21.9123
ADA	**0.0712**	**0.0378**	5.6734	4.6095	210.3313	19.5158
WTR	0.0862	0.1419	2.0905	1.1339	92.2163	9.2932
STR	0.5001	0.6605	2.9071	0.8131	124.3494	14.7224
LAA	0.4588	0.4756	**0.456**	**0.6591**	**66.2637**	**5.8635**
constant weights	0.0814	0.1698	1.2417	0.6618	70.8516	7.0956
HSM	**0.0646**	**0.1108**	**0.4672**	**0.2917**	**47.2492**	**4.3154**
HSM (fitted)	0.0493	0.0353	0.4423	0.1655	34.4932	2.9358
$\beta \in [0, 10]$	10	10	0.1	10	3	0.2
$\eta \in [0, 1]$	0.4	0.9	1	0.1	0.8	0.5
$\delta \in [0, 1]$	0.9	0.6	0.5	0.7	0.6	0.4

Out-of-sample performance

The Heuristic Switching Model vs. AR(2) model

	Group 2	Group 5	Group 1	Group 6	Group 4	Group 7
HSM						
1 p ahead	0.0122	0.0321	0.479	0.1921	15.0395	0.7857
2 p ahead	**0.0122**	**0.0901**	**1.8599**	**1.0792**	57.5144	**1.5543**
AR(2) Model						
1 p ahead	0.3732	0.4431	0.9981	0.5568	13.4616	0.6682
2 p ahead	0.5052	0.4045	3.5823	1.4944	**44.6453**	2.0098

Conclusion

- the model with evolutionary switching between simple heuristics
- dynamics of the model is path-depended (different patterns of the experiments have been reproduced)
- good in-sample and out-of-sample performance
- Anufriev and Hommes (2012, AEJ-Micro and 2012, KER)

Model applications

- macroeconomics
- financial bubbles and crashes
- agent-based modelling

Conclusion

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Further Work

- theoretical relation of an outcome with Restricted Perception Equilibrium
- further analysis (other experiments, other methods)
- comparison with other learning methods (e.g., with GA)
- direct experiment on switching to estimate switching parameters
- classification of behavioral types on the basis of individual predictions

Learning in a Complex Environment

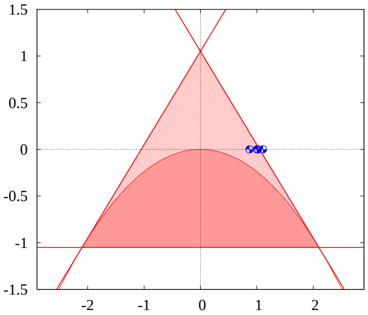
Mailath (JEL, 1998) *Do people play Nash equilibrium? Lessons from evolutionary game theory*, p. 1349-1350:

> The typical agent is not like Gary Kasparov, the world champion chess player who knows the rules of chess, but also knows that he doesn't know the winning strategy.
>
> In most situations, people do not know they are playing a game. Rather, people have some (perhaps imprecise) notion of the environment they are in, their possible opponents, the actions they and their opponents have available, and the possible payoff implications of different actions.
>
> These people use heuristics and rules of thumb (generated from experience) to guide behavior; sometimes these heuristics work well and sometimes they don't. These heuristics can generate behavior that is inconsistent with straightforward maximization.

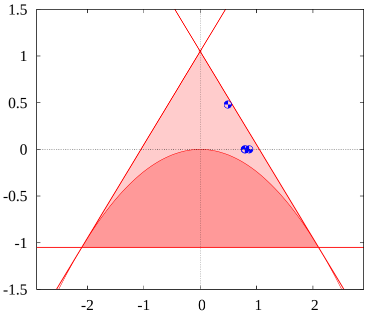
Dynamics for Individual Rules: Converging Groups

$$p^e_{t+1} = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2}$$

Stability region and group 2

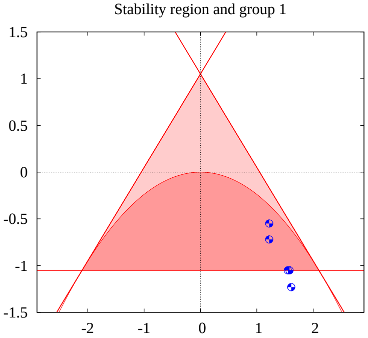


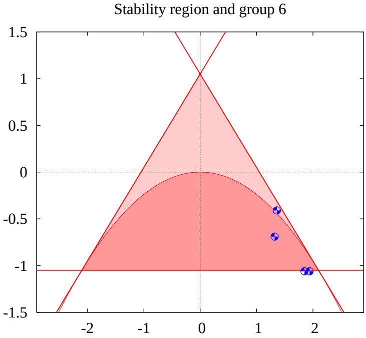
Stability region and group 5



Dynamics for Individual Rules: Oscillating Groups

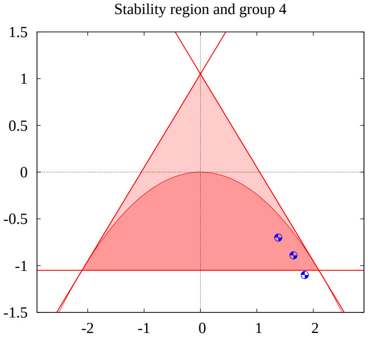
$$p^e_{t+1} = \alpha + \beta_1 \, p_{t-1} + \beta_2 \, p_{t-2}$$

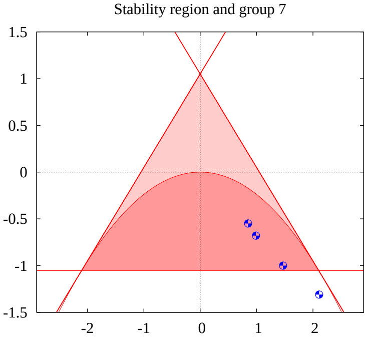




Dynamics for Individual Rules: Damping Groups

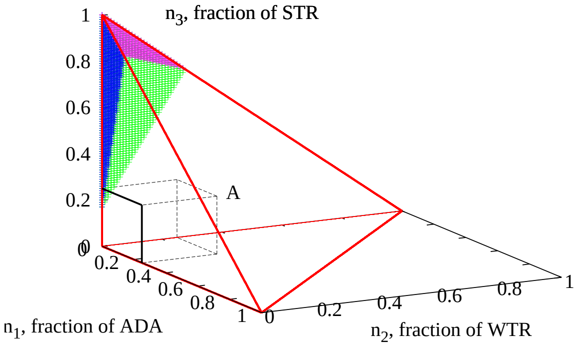
$$p^e_{t+1} = \alpha + \beta_1\, p_{t-1} + \beta_2\, p_{t-2}$$





Stability for the Model with Fixed Impacts

Stability region for model with fixed fractions



Comparison with Homogeneous Expectations: MSE

"Direct fit" with parameters $\beta = 0.4, \eta = 0.7, \delta = 0.9$

Specification	Group 2	Group 5	Group 1	Group 6	Group 4	Group 7
Fundamental Prediction	18.037	11.797	15.226	8.959	291.376	22.047
ADA – exp prices	0.841	0.200	7.676	8.401	330.101	51.526
WTR – exp prices	4.419	1.983	8.868	6.252	308.549	30.298
STR – exp prices	585.789	478.525	638.344	509.266	1231.064	698.361
AA – exp prices	39.308	21.760	17.933	17.345	289.134	87.878
LAA – exp prices	5.475	3.534	5.405	14.404	307.605	69.749
ADA – fitted prices	0.514	0.199	6.832	7.431	312.564	36.436
WTR – fitted prices	4.222	1.844	8.670	6.228	292.150	19.764
STR – fitted prices	413.435	42.488	182.284	29.200	580.543	579.141
AA– fitted prices	26.507	13.228	11.117	13.981	258.010	63.777
LAA – fitted prices	2.055	1.859	4.236	13.433	284.880	45.153
4 heuristics (plots)	0.449	0.302	8.627	14.755	526.417	29.520
4 heuristics (fitted)	0.313	0.245	7.227	7.679	235.900	18.662

Comparison with Homogeneous Expectations: AR2

"Indirect fit" with parameters $\beta = 0.4, \eta = 0.7, \delta = 0.9$

Specification	Group 2	Group 5	Group 1	Group 6	Group 4	Group 7
Fundamental Prediction	0.946	0.671	2.673	3.610	2.311	2.002
ADA – exp prices	0.239	0.006	2.182	2.898	1.691	1.494
WTR – exp prices	0.066	0.529	0.383	0.627	0.203	0.165
STR – exp prices	1.494	2.583	0.112	0.020	0.240	0.342
AA – exp prices	1.095	1.848	0.010	0.038	0.045	0.094
LAA – exp prices	0.747	1.544	0.003	0.050	0.003	0.013
ADA – fitted prices	0.100	0.000	1.584	2.159	1.385	1.157
WTR – fitted prices	0.068	0.343	0.262	0.435	0.174	0.139
STR – fitted prices	1.358	2.192	0.078	0.001	0.147	0.242
AA– fitted prices	1.036	1.755	0.005	0.029	0.038	0.083
LAA – fitted prices	0.640	1.277	0.000	0.033	0.000	0.004
4 heuristics (plots)	0.383	0.744	0.011	0.008	0.157	0.239
4 heuristics (fitted)	0.144	0.499	0.009	0.003	0.121	0.048

Measure of coordination

- average prediction error =

  average dispersion + common prediction error
- in the experiment

$$\frac{1}{6}\sum_{i=1}^{6}\left(p_{i,t}^{e}-p_{t}\right)^{2}=\frac{1}{6}\sum_{i=1}^{6}\left(p_{i,t}^{e}-\bar{p}_{t}^{e}\right)^{2}+\left(\bar{p}_{t}^{e}-p_{t}\right)^{2},$$

  where $\bar{p}_t^e = \frac{1}{6}\sum_{i=1}^{6} p_{i,t}^e$
- in simulations

$$\sum_{h=1}^{4} n_{h,t-1}\left(p_{h,t}^{e}-p_{t}\right)^{2}=\sum_{h=1}^{4} n_{h,t-1}\left(p_{h,t}^{e}-\bar{p}_{t}^{e}\right)^{2}+\left(\bar{p}_{t}^{e}-p_{t}\right)^{2},$$

  where $\bar{p}_t^e = \sum_{h=1}^{4} n_{h,t-1} p_{h,t}^e$

Coordination

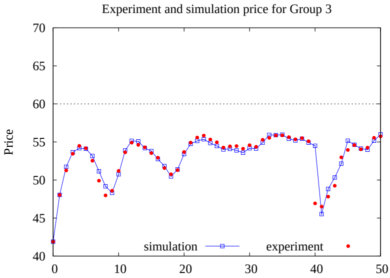
Table: Coordination in the experiment and over the simulations.

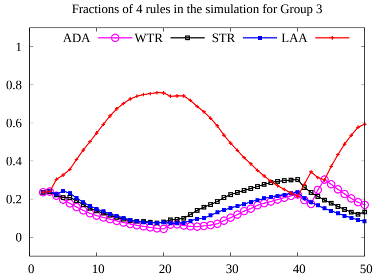
	Group 5		Group 1		Group 7	
period	exp	sim	exp	sim	exp	sim
3-10	44.91	94.23	79.16	71.04	61.30	79.43
11-20	81.54	84.42	78.03	91.28	72.72	90.35
21-30	71.75	80.50	78.62	89.48	66.14	94.27
31-40	81.83	84.14	81.98	97.98	67.01	94.74
41-50	84.85	86.51	94.90	93.84	21.95	74.38

	Group 2		Group 6		Group 4	
period	exp	sim	exp	sim	exp	sim
3-10	40.88	69.98	58.59	77.00	76.44	87.45
11-20	67.25	86.48	78.94	80.96	90.41	88.44
21-30	75.73	80.53	76.16	79.69	83.41	96.70
31-40	83.88	80.51	79.33	88.72	48.47	88.03
41-50	91.44	84.66	72.09	92.26	31.74	65.78

The same experiment: group 3

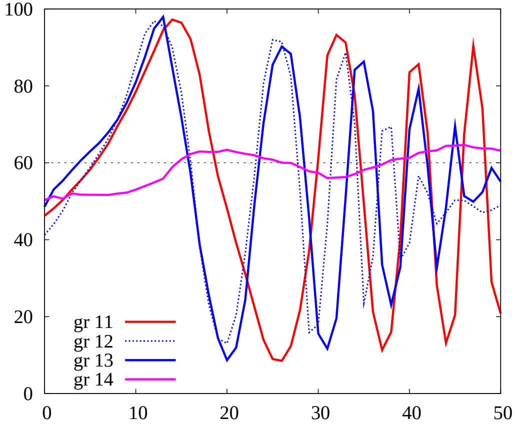
Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$





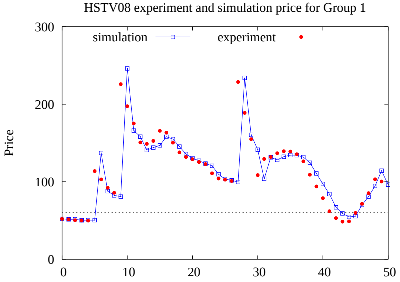
Other Experiments

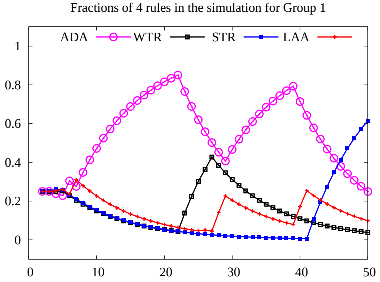
$$p_t = \frac{1}{1+r}\left(\bar{p}^e_{t+1} + \bar{y} + \varepsilon_t\right)$$



Bubbles in Experiment I

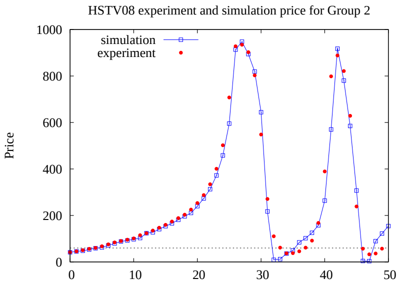
$$p_t = \frac{1}{1+r}\left(\bar{p}_{t+1} + \bar{y}\right)$$

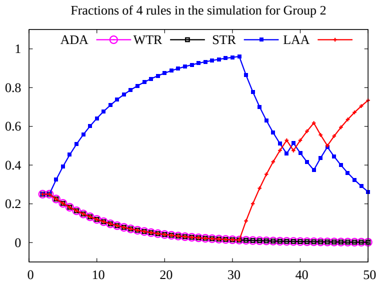




Bubbles in Experiment II

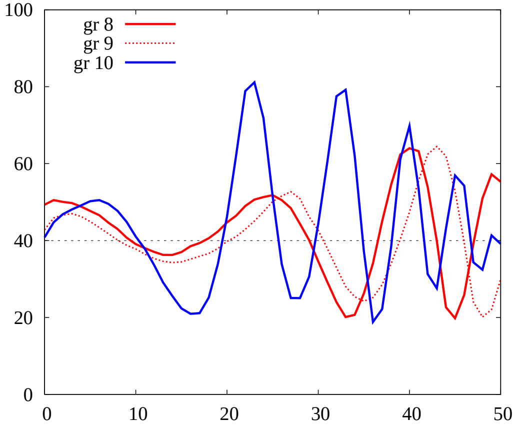
$$p_t = \frac{1}{1+r}\left(\bar{p}_{t+1} + \bar{y}\right)$$





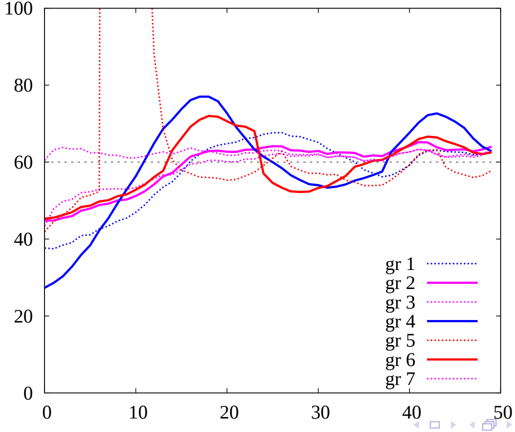
Other Experiments: Smaller Fundamental Price

$$\bar{y} = 3 \;\rightarrow\; \bar{y} = 2$$



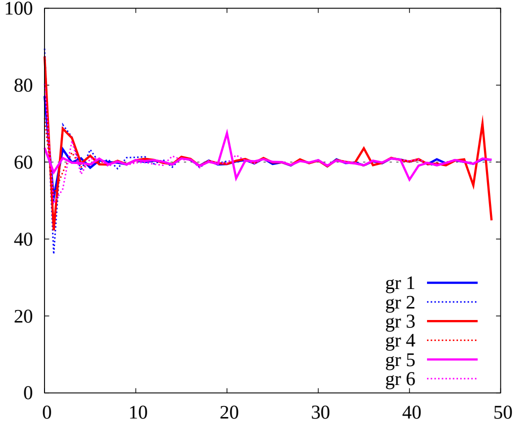
Other Experiments: One-period ahead forecast

$$p_t = \frac{1}{1+r}\left(p^{AE}_{t+1} + \bar{y}\right) \quad \rightarrow \quad p_t = \frac{1}{1+r}\left(p^{AE}_{t} + \bar{y} + \varepsilon_t\right)$$



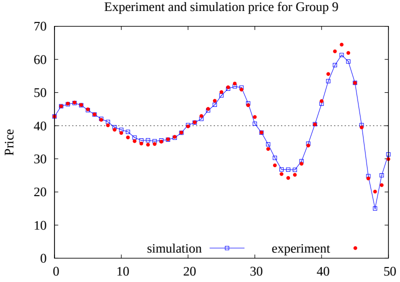
Other Experiments: Negative Feedback

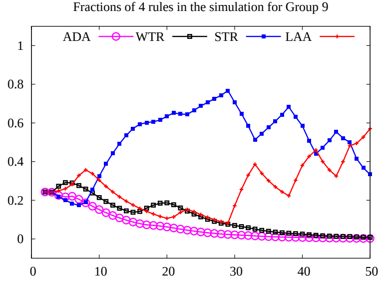
$$p_t = a + bp^{AE}_{t+1} + \varepsilon_t \quad \rightarrow \quad p_t = a' - bp^{AE}_{t+1} + \varepsilon_t$$



Other Experiments: Smaller Fundamental Price I

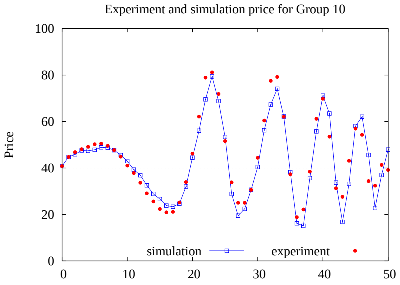
$$\bar{y} = 3 \rightarrow \bar{y} = 2$$

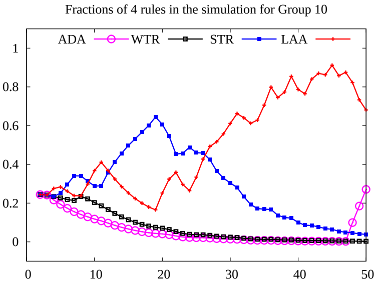




Other Experiments: Smaller Fundamental Price II

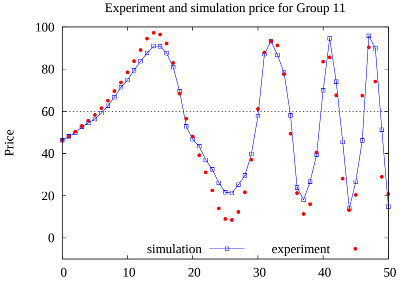
$$\bar{y} = 3 \ \rightarrow \ \bar{y} = 2$$

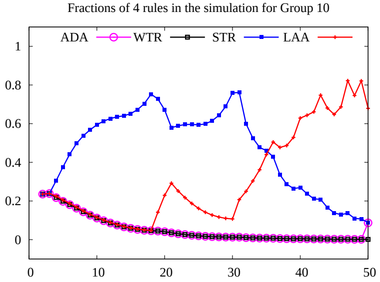




Other Experiments: No Robots

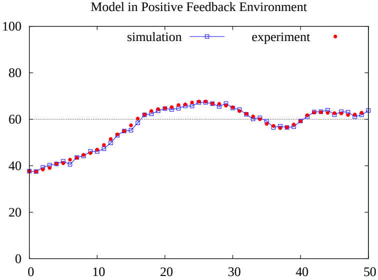
$$p_t = \frac{1}{1+r}\Big((1-n_t)p^{AE}_{t+1} + n_t p^f + \bar{y} + \varepsilon_t\Big) \;\rightarrow\; p_t = \frac{1}{1+r}\Big(p^{AE}_{t+1} + \bar{y}\Big)$$

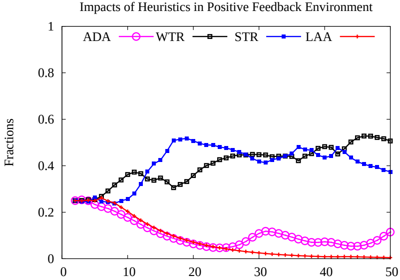




Other Experiments: One-period ahead forecast

$$p_t = \frac{1}{1+r}\left(p^{AE}_{t+1} + \bar{y}\right) \quad \rightarrow \quad p_t = \frac{1}{1+r}\left(p^{AE}_t + \bar{y} + \varepsilon_t\right)$$





Other Experiments: Negative Feedback

$$p_t = a + bp^{AE}_{t+1} + \varepsilon_t \quad \rightarrow \quad p_t = a' - bp^{AE}_{t+1} + \varepsilon_t$$