Asset Price Dynamics with Heterogeneous Types and Local Network Interactions

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The Scene

- Many *non-specialist* investors allocate money into funds
  - Mutual funds
  - Pension funds
- Funds can be classified into a number of types, e.g., *fundamentalists* and *chartists*
  - Types are not explicitly observed by investors
  - Funds do not typically switch strategies
- Investors are able to switch funds
  - Every period investors receive performance reports for the past period
  - Inventors talk to their *network* of friends and may switch to a different fund if it performed better

Adaptive behavior with interactive agents

- Start: Brock and Hommes (1998) - tractable model
  - heterogeneous expectations
  - evolutionary updating of the agents' behavior
- Introduce: local information exchange with various topologies

Stylized model of financial market

- two assets
  - riskless: risk-free interest rate r_f
  - risky: price p_t and dividend y_t
- mean-variance demand for the risky asset for type h

$$z_{h,t}(p_t) = E_{h,t}\left[p_{t+1} + y_{t+1} - (1 + r_f)\, p_t\right] \Big/ a\,\sigma^2$$

- market clearing: Walrasian auction

$$\sum_h n^h_{t-1} z_{h,t}(p_t) = z^s$$

$$\sum_h n^h_{t-1} E_{h,t}\left[p_{t+1} + y_{t+1}\right] - z^s\, a\,\sigma^2 = p_t(1 + r_f)$$

- REE fundamental solution $p^* = \frac{\bar{y}}{r_f}$ (assume $z^s = 0$).

Model with heterogeneous types

- heterogeneous types of funds
- agents may switch between fund types

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  - fundamentalists: $E_{f,t}[p_{t+1}] = p^*$
  - trend-followers: $E_{c,t}[p_{t+1}] = p^* + g(p_{t-1} - p^*)$
- agents may switch between fund types

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- agents may switch between fund types
  - past performance of type h,

$$U_t^h = (p_t + y_t - Rp_{t-1})z_{t-1} - c^h$$

    - performance of type h has observed component and idiosyncratic unobserved component

$$\widetilde{U}_t^h = U_t^h + \frac{1}{\beta}\varepsilon_t^h$$

    - β is the intensity of choice

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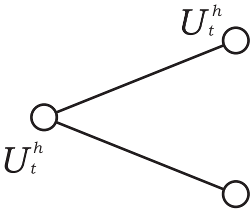
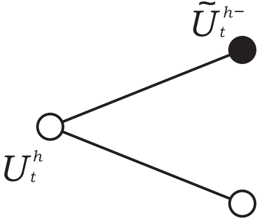
    - β is the intensity of choice
  - discrete choice model

$$n_t^h = P_t^h = P\big(\widetilde{U}_t^h > \widetilde{U}_t^k \text{ for all } k \neq h\big) = \frac{\exp(\beta U_t^h)}{\sum_{\xi=1}^{H} \exp(\beta U_t^{\xi})}$$

Local evolutionary updating of types

- no neighbor of different type - no switching

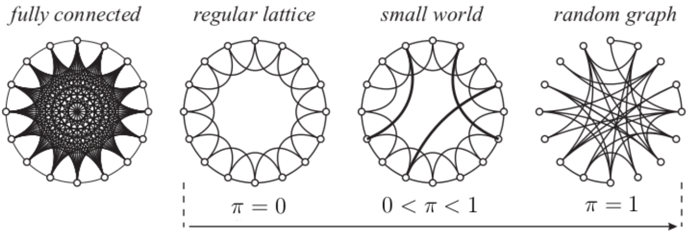




- potential switching

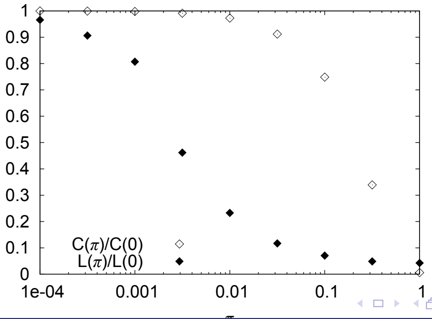
Topologies of networks

- agents are living in the nodes of different networks
- edges correspond to information exchange channels



Key network measures

- (average) degree, k - size of the neighborhood
- clustering coefficient, C - connectedness of the neighborhood
- characteristic path length, L - min distance between any two nodes



Evolution of types within network

$$P_{i,t} = P_{i,t-1} \prod_{j \in \mathbb{G}_i} P_{j,t-1} +$$
$$+ [P_{i,t-1}(1 - \prod_{j \in \mathbb{G}_i} P_{j,t-1}) + (1 - P_{i,t-1})(1 - \prod_{j \in \mathbb{G}_i} (1 - P_{j,t-1}))]\Delta_t,$$

where $\mathbb{G}_i$ is a neighborhood of agent i, the set of agents directly connected to i, excluding i, and

$$\Delta_t = \frac{1}{1 + \exp[\beta(U_t^{\bar{A}} - U_t^{A})]}.$$

Evolution of types within network: random graph

$$n_t = n_{t-1} n_{t-1}^k + [n_{t-1}(1 - n_{t-1}^k) + (1 - n_{t-1})(1 - (1 - n_{t-1})^k)]\Delta_t =$$
$$= n_{t-1}^{k+1} + [1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1}]\Delta_t,$$

where k is (average) degree of the network and

$$\Delta_t = \frac{1}{1 + \exp[\beta(U_t^{\bar{A}} - U_t^A)]}$$

.

System of 3D difference equations

$$x_t = \frac{g}{R} n_{t-1} x_{t-1}$$

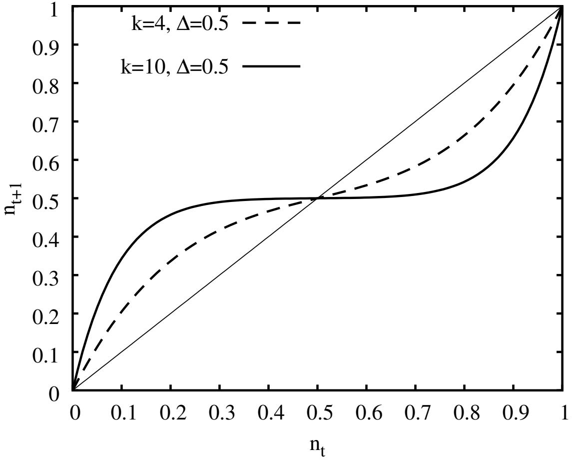
$$n_t = n_{t-1}^{k+1} + [1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1}] \times \\ \times [1 + \exp(\beta(-Dgx_{t-2}(x_t - Rx_{t-1}) - c))]^{-1},$$

where $x = p - p^*$, $D = 1/a\sigma^2$.

Ingredients for steady states:

- $x = 0$ and $n^\diamond(\beta)$
- $n^* = R/g$ and x^*

Evolution of fractions map



$$n_t = n_{t-1}^{k+1} + (1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1})\Delta_t$$

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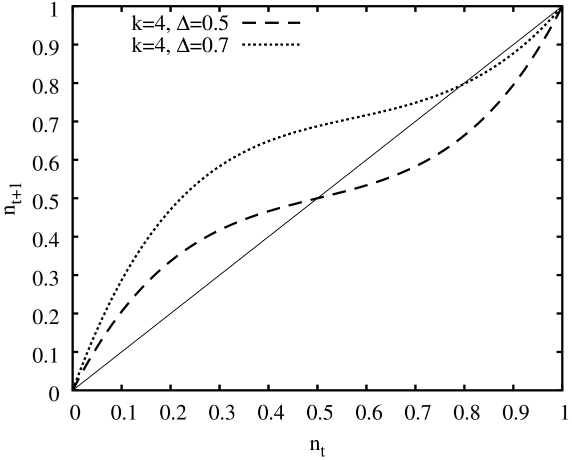

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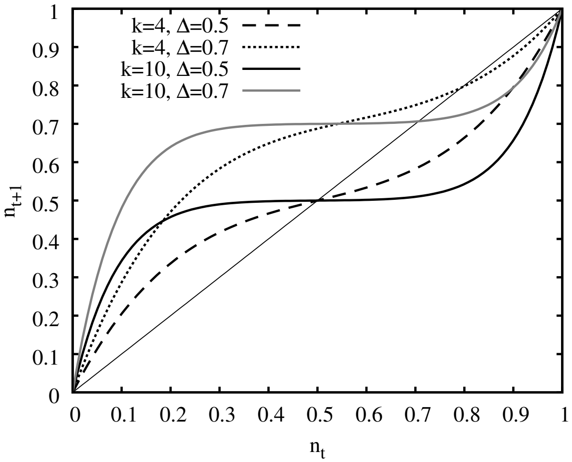

$$n_t = n_{t-1}^{k+1} + (1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1})\Delta_t$$

Characterization of steady states

- Fundamental steady states
  $E_\diamond = (0, n^\diamond), E_0 = (0, 0), E_1 = (0, 1)$.
- Non-fundamental steady states
  $E_+ = (x^*, n^*), E_- = (-x^*, n^*)$.

	$\beta < \ln(k)/c$	$\beta > \ln(k)/c$
$0 < g < R$	$E_0, E_\diamond, E_1$ exist E_0, E_1 are unstable	E_0, E_1 exist E_0 is unstable
$g > 2R$	$E_0, E_\diamond, E_1, E_+, E_-$ exist $E_0, E_\diamond, E_1$ are unstable	E_0, E_1, E_+, E_- exist E_0, E_1 are unstable
$R < g < 2R$ & $n^* > n^\diamond$	$E_0, E_\diamond, E_1$ exist E_0, E_1 are unstable	E_0, E_1 exist E_0 is unstable
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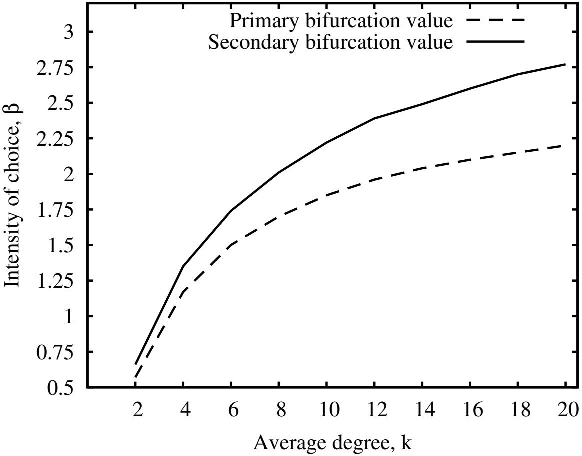
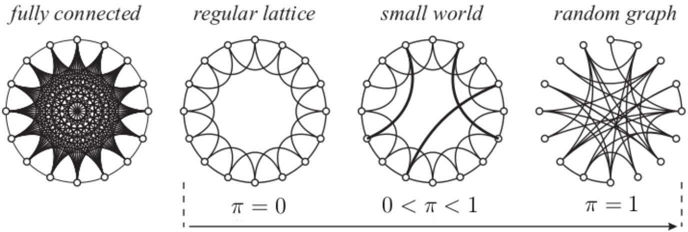
Dependence of bifurcations on k



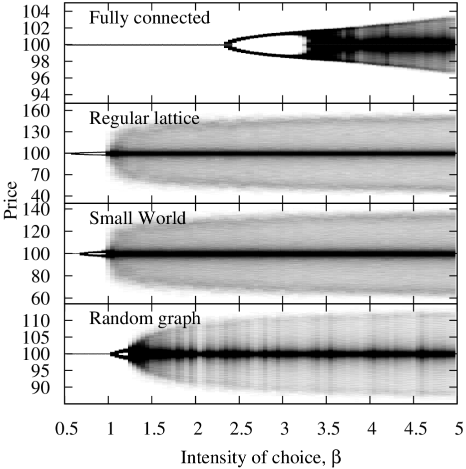
Other networks

- connected neighbors in the neighborhood reduce *informational content* or “effective” *k*.

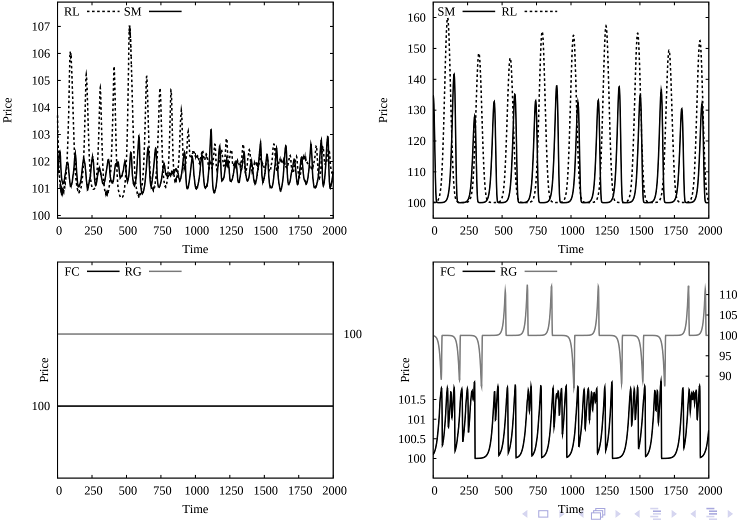


Simulations: bifurcation diagrams

$c = 1, g = 1.2, r_f = 0.1, \bar{y} = 10$

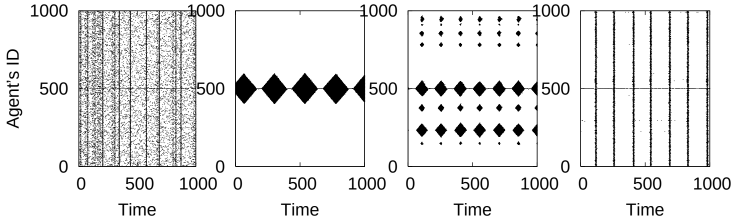


Time series $\beta = 1$ (left), $\beta = 3.5$ (right)



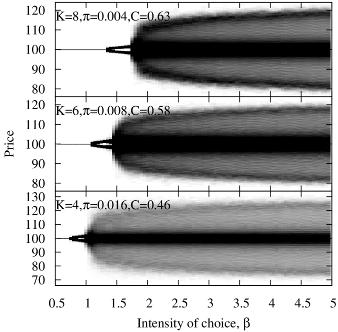
Spatio-temporal patterns of agent types evolution

$$\beta = 3.5, C = 1, g = 1.2, r_f = 0.1, \bar{y} = 10, N = 1000,$$

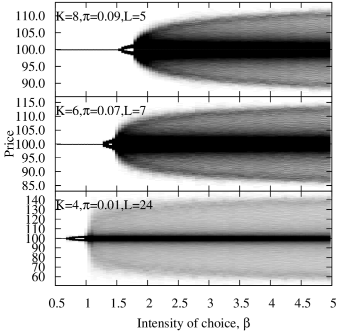


fully connected regular lattice small world random graph

Bifurcation diagrams types for varying k, π, L and C

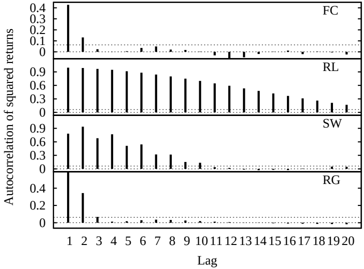


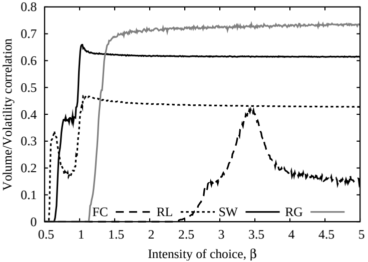
Fixed L=17



Fixed C=0.48

Statistical properties: correlations of squared returns, volume/volatility correlations





Conclusions

- Analytically tractable model for random network
  - importance of neighborhood size
  - importance of clustering
- Major features generated by the BH model are preserved under various communication structures
- Importance of the network structures
  - faster bifurcations - less stability
  - short period between primary and secondary bifurcations

My Related Contributions:

- Anufriev and Panchenko (2009), JEDC
- Goldbaum and Panchenko (2010), JEBO

My Related Contributions:

- Anufriev and Panchenko (2009), JEDC
  - BH-model under with different market architecture
  - Walrasian, call auction, continuous double auction
  - interaction between behavioral assumptions and market architecture
- Goldbaum and Panchenko (2010), JEBO

My Related Contributions:

- Anufriev and Panchenko (2009), JEDC
- Goldbaum and Panchenko (2010), JEBO
  - Grossman-Stiglitz paradox
  - BH-type switching between informed and uninformed (least squares learners)
  - endogenous fluctuations

Future directions

- More tractable models for small world network
- Dependence of the signal strength on the number of neighbors having same type
- Introduction of longer memory
- Endogenous network formation