Karlsruhe Institute of Technology (KIT)

Unanimity Overruled: Majority Voting and the Burden of History

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Centre of Mathematical Social Sciences
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- E.g., if votes are cast in the order (d, a), (a, b), (b, c) one obtains $d \succ a \succ b \succ c$, hence $d \succ c$ by transitivity, although there is unanimous consent that c is better than d.

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- Can the problem be avoided by an appropriate choice of a decision sequence?

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- SMV yields either $(1,1,0)$, $(0,1,1)$, or $(1,0,1)$.

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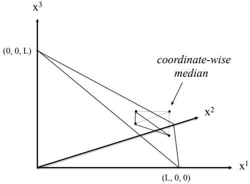

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Given a profile $\mu \in X^N$ of feasible views, the **Condorcet set** *$Cond(\mu) \subseteq X$ is the set of all $x \in X$ such that no feasible view coincides with the issue-wise majority view on a strictly larger set of issues than x.*

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$$x \in Cond(\mu) \ \Leftrightarrow \ x = SMV_\gamma(\mu) \text{ for some path } \gamma.$$

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As above, consider a, b, c, d and suppose that $\frac{1}{3}$ of the population endorses the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$ and $c \succ_3 d \succ_3 a \succ_3 b$, respectively.

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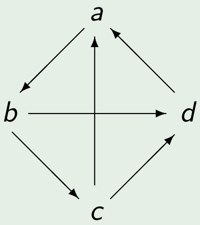

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  Hence, $X_{I,J;K}^{\mathrm{com}}$ is not strongly sequentially unanimity consistent whenever $K \geq 5$. On the other hand, e.g., $X_{2,2;4}^{\mathrm{com}}$ is strongly sequentially unanimity consistent.

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- Show that $SMV_\zeta(\mu)$ extends $\succ_\mu^*$.

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- We have characterized all spaces which are strongly sequentially unanimity consistent, i.e. in which sequential majority voting never overrides unanimous consent, no matter in which sequence the voting takes place.
- The characterizing condition is a simple generalization of the condition that is necessary and sufficient for the consistency of issue-wise majority voting (given any profile of individual views).
- Very few aggregation problems verify this condition.

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- Remarkably, some important aggregation problems that are not strongly sequentially unanimity consistent satisfy the weaker requirement that there *exists* some decision path along which unanimous consent is always respected.
- These include the aggregation of linear preference orders, weak orders, strict partial orders, weak partial orders, and equivalence relations.

Concluding Remarks

- Remarkably, some important aggregation problems that are not strongly sequentially unanimity consistent satisfy the weaker requirement that there *exists* some decision path along which unanimous consent is always respected.
- These include the aggregation of linear preference orders, weak orders, strict partial orders, weak partial orders, and equivalence relations.
- An open problem is a general characterization of all weakly sequentially unanimity consistent aggregation problems.

Thank you!