J Econ Growth (2011) 16:71–98
DOI 10.1007/s10887-011-9062-5

Parental nurturing and adverse effects of redistribution

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Published online: 9 March 2011

Abstract This paper suggests that if parental nurturing is a dominating force in human capital formation then income redistribution may not promote economic growth. In particular, if, consistently with empirical evidence, parental human capital complements investment in a child's education and yields increasing returns in the intergenerational production of human capital, income redistribution may have an adverse impact on the growth rate of average human capital. Redistribution shifts resources towards the less educationally-productive families and thus in the presence of credit markets imperfections and increasing returns, it reduces the aggregate level of investment in human capital. Moreover, if the degree of increasing returns is sufficiently large to produce sustained growth, this adverse effect on human capital formation may outweigh the conventional beneficial effects of redistribution that arises from the interaction between a production technology exhibiting diminishing returns and credit market imperfections.

Keywords Heterogeneous ability · Parental input in education · Endogenous growth · Dynamic efficiency · Progressive income tax · Progressive education subsidy

JEL classification D61 · E24 · E62 · O11

1 Introduction

Evidence shows that early parental intervention in children's education contributes to children's human capital accumulation, significantly increasing returns on investment in schooling (e.g., Carneiro and Heckman 2003). Moreover, it appears that parental nurturing leads

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to increasing returns in the intergenerational production of human capital. This paper suggests that if parental nurturing is a dominating force in human capital formation then income redistribution may not promote economic growth.

In essence, our argument calls for a consideration of an unexplored macroeconomic cost of redistribution that mitigates various growth-promoting channels of income redistribution explored in the long strand of literature from Galor and Zeira (1993) to Mookherjee and Ray (2003). The general proposition in this literature emphasizes that redistributive policies promote growth by alleviating problems with allocative efficiency when: credit markets are absent, capital goods are not tradable and production technology exhibits diminishing returns (e.g., Galor and Zeira 1993; Aghion et al. 1999; Benabou 2002; Galor and Moav 2006). We argue that a consideration of increasing returns in the intergenerational production of human capital in the above modelling environment yields a unique qualification for the above proposition.[1] In particular, we find that if those increasing returns bring sustained growth into the economy, then the macroeconomic cost of redistribution may be large enough to make a redistribution scheme, progressive or regressive, dynamically inefficient.

The appropriateness of the assumption of increasing returns in the human capital accumulation process at the family level, especially in the context of examining the growth-promoting potential of redistribution, deserves a careful consideration on both theoretical and empirical grounds. Some theoretical studies of inequality and redistribution (e.g., Perotti 1993 and Benabou 1996b) rely on knowledge spillover to fuel long-run growth. In particular, Galor and Tsiddon (1997) suggest that the "global technological externality" that arises from knowledge spillover substitutes for redistribution because it mitigates the barriers of credit constraints by producing an "income equalizing effect". In contrast, however, they argue that a "local home environment externality", which arises from the complementarity between parental human capital and expenditure on a child's education, generates an "income widening effect". Thus, to enhance potential gains from redistribution, we allow inequality-widening role of parental human capital in a child's education and abstract from knowledge spillover.

Empirically, it has been argued since the influential study by Rosenzweig and Wolpin (1994), that the return to investment in education significantly increases with maternal schooling and results in increasing returns to the intergenerational production of human capital.[2] Heckman (2008) reports that parental socio-economic differences explain sizable test-score gaps already evident at school entry level in the U.S. In Latin American countries, Pastore et al. (1983) find that family origin plays a much stronger role in determining educational and labor market outcomes than it does in the U.S. In Asia, Glewwe et al. (2001) report that investments in early childhood programs could potentially return large and significant gains in academic achievement in the subsequent years. Sathar and Lloyd (1994) examine

[1] An alternative qualification of that proposition can be found in Ghatak et al. (2001) who note that preserving high rents for the rich would be desirable because with credit constraints, high rents motivate poor young agents to work hard and become rich in their old age. However, redistribution discourages working effort and thus reduces growth and welfare. Benabou (1993, 1996a) also characterizes a special economic condition, based on his location choice models, that would make "unequal societies" optimal. In those models a child's education process benefits from the interaction among the educated people in a rich neighbourhood. If increasing returns to scale arising from those interactions are sufficiently strong then segregation would foster growth. Conceivably, a redistributive policy would also deteriorate welfare under such special economic conditions. However, in both cases, redistribution would hurt growth because inequality (or segregation) would be beneficial to growth. We rationalize a case where inequality is indeed bad for growth. Nevertheless, redistribution to mitigate inequality makes it worse.

[2] Acemoglu and Pischke (2001) report that in the U.S., family income, which increases with parental human capital, significantly raises the probability of a child attending a four-year college, after controlling for factors unrelated to parental human capital, and explains almost 90% of variations in the enrolment rates.

survey data from Pakistan and report that household spending on children's education is up to 75% higher if the mother had ever attended school, relative to households wherein mothers had not, irrespective of various other economic factors controlled for in their empirical exercise. Brown (2006) concludes that irrespective of wealth, teacher quality and the child's cognitive development, more-educated parents in rural China allocate higher levels of goods and time in nurturing a child's education, suggesting higher returns from education for the children of more-educated parents.[3] Finally, Trostel (2004) reports strong complementarity between parental human capital and a child's education and the associated increasing returns, especially in low income countries.

With the above theoretical and empirical justifications, we maintain throughout our analysis a key assumption that there are increasing returns to human capital accumulation at the family level. Otherwise, our basic model closely resembles a stripped-down version of benchmark study of Benabou (2002) that estimates the optimal rate of progressivity that maximizes the long-run income per capita in a dynamic general equilibrium model. The equilibrium outcome of our model ensures, in the absence of global externality across families, that the state variables for each family dynasty evolve independently via education, and result in a unique path of income inequality and economic growth. An increase in income inequality, in the presence of credit constraints, increases human capital inequality in subsequent periods. The resulting increase in the variance of marginal product of human capital corresponds to a decrease in total factor productivity (TFP) and output, because the production technology follows the law of diminishing returns. Consequently, a reduction in income inequality with a redistributive policy has the potential for promoting growth in per capita income. Such a policy also provides a cushion against the risk associated with variation in income due to random allocation of inborn ability, or shocks to intergenerational production of human capital. Thus, redistribution mitigates efficiency loss due to the lack of an insurance market, and improves welfare by lowering the ex-ante variance of consumption.[4] In this specialized set-up, designed to allow the maximum possible beneficial effect of redistribution, no redistribution turns out to be dynamically efficient.

We argue that if the parental nurturing mechanism is the dominating force in human capital formation and economic growth then redistribution may be harmful for growth. In the presence of credit market imperfections, a progressive redistribution of income shifts resources for children's education from a parent with more human capital, and hence a more productive education provider, to a parent with less human capital, and hence a less productive education provider. If there are increasing returns to human capital accumulation then such reallocation of resources induces an adverse impact on the growth rate of average human capital and hence on the per capita income.

The macroeconomic cost of redistribution may outweigh its benefit, when the economy is in a sustained growth regime. On a balanced growth path, a greater progressivity lowers the growth rate of average human capital permanently, while it raises the TFP growth only temporarily. Endogenous growth typically arises from, or requires, long-run growth of human capital. Hence, a negative effect of progressivity on the growth rate of human capital is sustainable. However, progressivity ensures that the intergenerational correlations of

[3] Brown (2006) reports, in particular, that more educated parents generally have better access to jobs that have higher educational requirements and offer higher incomes than traditional farming. The study also finds positive correlations between income and investment in human capital that strengthened the empirical relevance for the parental nurturing mechanism, as an alternative to the "American Dream" mechanism, explored in Ghatak et al. (2001), for making a counter-argument to the idea of growth-promoting redistribution.

[4] In a similar vein, Owen and Weil (1998) note that the moral hazard and adverse selection impediments to full ability insurance may be prohibitively high, while progressive taxation may be a substitute.

income remain less than unity. Consequently, subsequent to a greater progressivity, income inequality and hence the variance of human capital converge, causing the growth in TFP to cease eventually. Thus, the potency of progressive redistribution in enhancing TFP turns out to be limited.

Interestingly, making redistribution regressive also does not generate any net output gain for the economy. Regressive redistribution generates benefit by increasing the growth rate of human capital. However, it also implies a cost, as it induces growth in income inequality that causes a decline in TFP over time. In fact, a regressive tax increases intergenerational correlation of income to greater than unity and that results in a non-stationary growth in income inequality. Moreover, the pace of increase in inequality and the consequent decline in TFP turns out to be faster than the pace at which the average human capital grows, because a greater inequality, by decelerating per capita income growth, also adversely affect aggregate investment in human capital. Consequently, any regressivity implies a faster rate of increase in cost than benefit.

Thus, the net cost of redistribution turns out to be a U-shaped function of progressivity reaching its minimum at zero, implying that both progressivity and regressivity are bad for growth. We find the above conclusion robust, even when we switch off distortionary effects of income taxes on economic growth, by restoring the investment in education to the undistorted level with a subsidy financed by consumption taxes, and even if parental investment is mostly time but the expenditure on education remains essential to accumulation of a child's human capital.

It is noteworthy, however, that if increasing returns from parental nurturing of education are not strong enough to produce endogenous growth, a dynamically efficient path could be consistent with a positive degree of redistribution, even in the presence of increasing returns in the human capital accumulation process, supporting Benabou (2002) and Galor and Moav (2006) conclusions. However, the new and yet unexplored cost of redistribution that we discuss in this paper may lower the output maximizing degree of redistribution in each of those models. Future work may shed light on the extent of decline in the quantitative estimates of the growth maximizing progressivity in similar models without endogenous growth.

In Section 2, we present our basic model and describe the individual optimization problem and the model's equilibrium outcome. In Section 3, we explore how greater income inequality lowers transitional economic growth, and, in Section 4, we focus on the effect of redistribution on growth to prove analytically the key proposition of our paper, followed by some considerations of various extensions to check the robustness of our findings. Section 5 comments on how our model's prediction offers a clue to interpreting the reported empirical findings regarding how a change in progressivity affects the rate of growth of income per capita. Section 6 adds concluding remarks and summarizes our contribution. We provide the proofs of our lemmas and propositions in the Appendix, followed by a complete list of references.

2 The model

We build our basic model on a stripped-down version of Benabou (2002), with a progressive redistributive scheme based on income taxes and lump-sum transfers. The basic model illustrates the main mechanism and establishes our essential finding for this paper. Subsequently, we consider relevant extensions by introducing into the model parenting time as an essential input to human capital accumulation, education subsidies and an alternative scheme of redistribution with zero income tax. As an important and significant distinction from Benabou

(2002), our basic model and all its extensions examine the crucial role that the assumption of increasing returns in the intergenerational production of human capital plays in the positive and normative theories of growth and redistribution.

2.1 Endowments, preference, technology and redistributive policy

The model considers a continuum of infinitely lived dynasties $i \in [0, 1]$. Following Loury (1981), each dynasty is made up of a sequence of families consisting of individuals who live for two periods, first as a child and then as a parent. At each date t, the dynasty is represented by a family consisting of a parent and a child. The parent at date t representing the dynasty i (or, the dynastical agent i) makes all decisions for that period. To make those parental decisions time consistent, we assume that the preference of the parent representative of the dynastical agent i at date t is given by

$$\ln U_t^i = E_t \left[\sum_{n=0}^{\infty} \rho^n \ln c_{t+n}^i \right], \tag{1}$$

where $c_t^i > 0$ denotes the consumption of the family of the dynasty i at date t, and $\rho \in (0, 1)$ denotes the intergenerational discount factor.

At each date t the output of the self-employed parent of the dynasty i as a function of her human capital h_t^i and labor supply l_t^i is given by

$$y_t^i = \left(h_t^i\right)^{\mu} \left(l_t^i\right)^{\varepsilon}, \quad \text{where } \varepsilon = 1 - \mu, 0 < \mu < 1. \tag{2}$$

Note, the above production technology implies diminishing returns to parental human capital and hence, by design, there is a negative association between inequality in the economy, measured by the variance of human capital among productive agents, and the per capita income (PCI). To increase PCI, the government tries to lower inequality with a redistributive policy, based on progressive income taxation and lump-sum transfers, such that the disposable income $\hat{y}_t^i$ of a typical agent i at a given date t satisfies

$$\ln \hat{y}_t^i = (1 - \tau) \ln y_t^i + \tau \ln \tilde{y}_t, \tag{3}$$

where τ measures the progressivity rate, or the degree of progressive redistribution, and $\tilde{y}_t$ represents the break-even level of income, such that people with income less than $\tilde{y}_t$ receive a net subsidy and may be viewed as poor, while those with income above $\tilde{y}_t$ pay net taxes and may be viewed as rich.[5] The government redistributes all the taxes it collects such that

$$\int_0^1 \hat{y}_t^i di = \int_0^1 y_t^i di \equiv y_t, \tag{4}$$

where, y_t denotes the per capita output or income at a given date t.

The parent cannot borrow against her child's future income but the above income redistribution scheme helps to relax the credit constraint faced by the poor parents. Consequently, the post-redistribution disposable income $\hat{y}_t^i$ of the parent i at each date t must equal the total expenditure on the family's consumption c_t^i and on the expenditure e_t^i for the child's education. In other words,

[5] The parameter τ captures the notion of *progressivity* that we consider in this paper. Earlier work of Benabou (2002), Jakobsson (1976) and Kakwani (1977) posits the appropriateness of such parameterization. Note the parameter τ is equal to the income weighted average marginal tax (and transfer) rate: $\tau = \int_0^1 T'\left(y_t^i\right) \cdot \left(y_t^i / y_t\right) di$, where $T\left(y_t^i\right) = y_t^i - \hat{y}_t^i$ is the net tax paid at income level y_t^i.

$$\hat{y}_t^i = c_t^i + e_t^i. \tag{5}$$

Parental nurturing of a child's education, as well as a natural random assignment of talents, play a crucial role in the intergenerational production of human capital. In particular, for each dynasty i, at each date t, the parental human capital h_t^i and her expenditure e_t^i for the child's education measure, respectively, the quality and quantity of parental nurturing of the child's education. The human capital h_{t+1}^i of the grown-up child who plays the parental role for the dynasty i at the date $t+1$ increases with the quality and the quantity of parental nurturing received as a child and is also subject to an idiosyncratic but uninsurable talent shock ξ_{t+1}^i, which is $i.i.d.$ and is drawn from a time invariant distribution given by $\ln \xi_t^i \sim N(-\sigma^2/2, \sigma^2)$. We assume the above process of intergenerational production of human capital within each dynasty i satisfies, at each date t,

$$h_{t+1}^i = \kappa \xi_{t+1}^i \left(h_t^i\right)^\alpha \left(e_t^i\right)^\beta, \kappa > 0, \alpha, \beta \in (0,1), \alpha + \beta > 1. \tag{6}$$

Note that the quality of parental nurturing is proxied by α which measures the elasticity of the child's human capital with respect to the parent's human capital. The quality of education is proxied by the parameter β which measures the elasticity of a child's human capital with respect to educational expenditure. Rosenzweig and Wolpin (1994) report that the sum of these two elasticities could be greater than one, presumably because of a cumulative positive effect of parental nurturing on the accumulation of a child's human capital for which, subsequently, Carneiro and Heckman (2003) and Heckman (2008) offer supporting evidence.[6] Note, the only source of uncertainty in our model comes from the talent shock, as described above, in the production of human capital, and not from an income shock. To promote this assumption, following Becker and Tomes (1979), we argue that "market luck", which performs like a shock to income, would not be as important as "endowment luck", corresponding to the random assignment of natural ability in determining agents' income. This conclusion follows from the fact that there are competitive markets readily available to insure the income shock, while the difficulty of verifying talent or the inborn ability of a child precipitates moral hazard and adverse selection problems, ruling out a similar insurance market for the talent shocks.[7] A policy of income redistribution does provide a cushion against the implied disutility for the risk-averse agents. Consequently, our modelling of the uninsurable risk that naturally arises in the human capital production process, provides a welfare improving role for the government's income redistribution policy.

We assume that the initial endowment of human capital h_0^i is lognormally distributed, and in each period the parent receives one unit of labor endowment.

2.2 Individual optimization

At each date t, let m_{ht} denote the mean and Δ_{ht}^2 denote the variance of $\ln h_t^i$, respectively. Suppose $M_t \equiv \left(m_{ht}, \Delta_{ht}^2\right)$. Then, for the agent's dynamic optimization problem, the state variables are $(h_t^i, M_t; \tau)$, the control variables are $\left(c_t^i, l_t^i, e_t^i\right)$ and the Bellman Equation is as follows:

$$\ln U\left(h_t^i, M_t; P\right) = \max_{c_t^i, l_t^i, e_t^i} \left\{ \begin{array}{l} (1-\rho)\ln c_t^i \\ +\rho E_t\left[\ln U(h_{t+1}^i, M_{t+1}; \tau)\right] \end{array} \right\}, \tag{7}$$

[6] Glomm and Ravikumar (1998) provide motivating economic reasons for increasing returns to scale in the production of human capital when an economy is in the early stages of its development.

[7] Note that unemployment insurance and income protection insurance are readily available to provide protection against income shock, whereas no such insurance exists to protect someone against ability shock.

subject to (2), (3), (5) and (6).

The first order conditions associated with the Bellman Equation described by (7) yield complete solutions to the agent's problem. In particular, it implies trivially that $l_t^i = 1$, and the following lemma holds regarding human capital accumulation:

Lemma 1 *The saving rate $s_t^i \equiv e_t^i/\hat{y}_t^i$ for investment in education is a time invariant constant and decreases with the progressivity rate τ:*

$$s_t^i = \frac{\rho\beta\mu(1-\tau)}{1-\rho\alpha} \equiv s(\tau), s'(\tau) < 0. \tag{8}$$

Proof See Appendix.

Lemma 1 explicitly spells out the typical disincentives or negative effects, measured by $s'(\tau)$, of changing the rate τ of progressivity of redistribution on the optimal saving rate $s(\tau)$ for the child's education. Also, as expected, the rate of parental saving for the child's education increases with the thriftiness parameter ρ and with the parameters α and β, which indicates, respectively, the quality of parental nurturing and educational institutions.

2.3 The equilibrium dynamics

By Lemma 1, (3) and (5), the parental decision rules for expenditure on the family's consumption and the child's education, respectively, satisfy:

$$\ln c_t^i = \ln(1 - s(\tau)) + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t, \tag{9}$$

and

$$\ln e_t^i = \ln s(\tau) + (1-\tau)\ln y_t^i + \tau \ln \tilde{y}_t. \tag{10}$$

Together with the government's budget constraint (4) the above decision rules imply a unique sequence of aggregate state variables $\{M_t\}$ that coincides with what agent i takes as given in solving (7), such that at each date $t = 0, 1, 2, ..$, the following aggregate consistency condition holds:

$$\int_0^1 y_t^i di = \int_0^1 c_t^i di + \int_0^1 e_t^i di. \tag{11}$$

In other words, at each date t the economy's output, measured by the sum of the output produced by the self-employed dynastical agents, equals the aggregate demand for consumption and investment in education.

By (2), (6), and (10) the dynamics of human capital and income for the dynasty i, respectively, satisfy:

$$\ln h_{t+1}^i = \ln\kappa + \beta \ln s(\tau) + \ln \xi_{t+1}^i + (\alpha + \beta\mu(1-\tau))\ln h_t^i + \beta\tau \ln \tilde{y}_t, \tag{12}$$

and

$$\begin{aligned} \ln y_{t+1}^i = {} & \mu \ln\kappa + \mu\beta \ln s(\tau) + \mu \ln \xi_{t+1}^i \\ & + (\alpha + \beta\mu(1-\tau))\ln y_t^i + \beta\mu\tau \ln \tilde{y}_t. \end{aligned} \tag{13}$$

Note that the intergenerational persistence of human capital or income as a function of τ is given by $\alpha + \beta\mu(1-\tau)$, which decreases with the rate τ of progressivity, and hence a policy of redistribution enhances intergenerational social mobility.

By (12) and (13), it follows, therefore, that at each date t, M_t satisfies

$$m_{ht+1} = \ln \kappa - \sigma^2/2 + \beta \ln s(\tau) + (\alpha + \beta\mu)m_{ht} + \beta\tau(2-\tau)\mu^2\Delta_{ht}^2/2, \tag{14}$$

and

$$\Delta_{ht+1}^2 = \sigma^2 + (\alpha + \beta\mu(1-\tau))^2\Delta_{ht}^2. \tag{15}$$

The block recursive nature of the equilibrium sequence of the vector M_t described by (14) and (15) implies that the sequence of the variance of the logarithm of human capital, Δ_{ht}^2, evolves independently of the sequence of the mean, m_{ht}, while the sequence of m_{ht} depends on the sequence of Δ_{ht}^2. This special feature of the equilibrium dynamics helps us to discern, in the following section, how "causality" flows from a measure of inequality, which is a 1-1 function of Δ_{ht}^2, to the growth rate of income per capita and not vice versa.

Given M_0, (14) and (15) yield a unique sequence $\{M_t\}_{t=1,2,..\infty}$ that characterizes the key equilibrium dynamics of our model.

3 Endogenous income inequality and economic growth

In this section we explore how income inequality evolves endogenously and makes a significant impact on the transitional dynamics of the growth rate of income per capita, and how increasing returns from parental nurturing counterbalance diminishing returns in production to produce perpetual economic growth.

3.1 Evolution of inequality

In line with Benabou (2002), we define, for each date t, an index of income inequality Λ_t as *the logarithm of the ratio of the mean to median income*.

Lemma 2 *The evolution of earnings of adults is governed by a lognormal distribution such that* $\ln y_t^i \sim N(\mu m_{ht}, 2\Lambda_t)$. *At each date* $t = 0, 1, 2, \ldots$, *the inequality index* Λ_t *equals half of the variance of the logarithmic earnings of agents, such that* $\Lambda_t = \mu^2\Delta_{ht}^2/2$.

Proof See Appendix.

Lemma 3 *The break-even level of income* $\tilde{y}_t$ *at which an agent's net tax obligation is zero satisfies:*

$$\ln \tilde{y}_t = \ln y_t + (1-\tau)\Lambda_t. \tag{16}$$

Proof See Appendix.

Lemma 4 *If and only if* $\alpha + \beta\mu(1-\tau) < 1$, *then, irrespective of the initial conditions, income inequality converges monotonically to its unique ergodic limit,* $\Lambda(\tau) > 0$. *Moreover,* $\Lambda'(\tau) < 0$.

Proof At each date t, by (15), the degree Λ_t of income inequality evolves as follows:

$$\Lambda_{t+1} = \frac{\mu^2\sigma^2}{2} + (\alpha + \beta\mu(1-\tau))^2\Lambda_t. \tag{17}$$

It follows, therefore, if $\alpha + \beta\mu(1-\tau) < 1$ then Λ_t converges to $\Lambda(\tau)$, where,

$$\Lambda(\tau) = \frac{\mu^2}{1-(\alpha + \beta\mu(1-\tau))^2}\frac{\sigma^2}{2}. \tag{18}$$

Clearly, by (12) and (13), a greater progressivity τ reduces the intergenerational persistence $(\alpha + \beta\mu(1-\tau))$ of human capital and income and hence, by (18), lowers the long-run index of income inequality. Consequently, $\Lambda'(\tau) < 0$. □

3.2 How inequality lowers growth

At each date t, output and human capital, respectively, satisfy $y_t = \int_0^1 y_t^i di$, and $h_t = \int_0^1 h_t^i di$. We define the date t total factor productivity (*TFP*) of the economy, following Solow (1957), as the ratio of the output to the weighted average of inputs such that by (2),

$$TFP_t \equiv \frac{\int_0^1 y_t^i di}{\left(\int_0^1 h_t^i di\right)^{\mu}}, \text{ or, equivalently, } y_t = TFP_t h_t^{\mu}. \tag{19}$$

Our assumption of a continuum of dynasties with measure one implies that output, human capital and their per capita values coincide. It follows, therefore, that at each date t, the per capita income growth rate $\gamma_t \equiv \ln y_{t+1} - \ln y_t$, the growth rate of *TFP*, $g_{TFPt} \equiv \ln TFP_{t+1} - \ln TFP_t$, and the growth rate of the average stock of human capital, $g_{ht} \equiv \ln h_{t+1} - \ln h_t$ satisfy

$$\gamma_t = g_{TFPt} + \mu g_{ht}. \tag{20}$$

Clearly, by (2) and (19), the negative effect of income inequality on the level of TFP arises from the combination of the diminishing returns in the production technology (i.e., $0 < \mu < 1$), and the absence of a credit market that helps to perpetuate allocative inefficiency due to the persistence of unexploited arbitrage opportunities from interpersonal differences in the marginal products of human capital. In particular, it follows from the production technology (2) and the characterization of inequality by Lemma 2, that the variance of the marginal product of human capital at any date t is given by,

$$\begin{aligned} var\left(\ln MPH_t^i\right) &= (1-\mu)^2 \Delta_{ht}^2 \\ &= \frac{2(1-\mu)^2}{\mu^2}\Lambda_t. \end{aligned} \tag{21}$$

Consequently, a higher inequality implies a greater variance of the marginal contribution of human capital across different production units of the economy and, hence, lowers TFP, corresponding to a lower level of allocative efficiency. The following lemma summarizes this negative effect of income inequality on TFP.

Lemma 5 *The date t TFP for this economy satisfies*

$$LnTFP_t = -\frac{1-\mu}{\mu}\Lambda_t, \tag{22}$$

and as Λ_t converges to $\Lambda(\tau)$, given by (18), so does the TFP and a higher income inequality corresponds to a lower TFP, both along the transition and across different steady states.

Proof See Appendix.

3.2.1 TFP growth

It is straightforward to note from Lemma 5 that an increase in income inequality would cause a decline in the growth rate of TFP. In particular, by (22), the growth rate of TFP satisfies for all date t,

$$g_{TFPt} = -\frac{(1-\mu)}{\mu}(\Lambda_{t+1} - \Lambda_t). \tag{23}$$

Note, however, that for all $\tau > 0$, the TFP growth rate diminishes to zero in the long-run as, by Lemma 4, income inequality converges to its long-run steady state. Thus, rising income inequality reduces the growth rate of TFP only temporarily along its transition path.[8]

3.2.2 Human capital growth

However, the effect of rising inequality makes a lasting positive impact on the growth rate of income per capita via a separate channel involving the growth rate of human capital. In particular, it follows from (14) and (15) that the growth rate of average human capital increases, as income inequality increases such that,

$$g_{ht} = g_h(\tau) - \sigma^2/2 + (\Lambda_{t+1} - \Lambda_t)/\mu^2 + \beta\tau(2-\tau)\Lambda_t, \tag{24}$$

where, $g_h(\tau) \equiv \ln\kappa + \beta \ln s(\tau)$. In other words, increasing returns to scale from parental nurturing imply that the average stock of human capital increases, if the income of a more productive education provider increases faster than others.

3.2.3 Mechanics of endogenous growth with parental nurturing

Intuitively, perpetual growth requires that the economy utilizes a mechanism to offset the diminishing marginal product of an input on the growth of output by supplying an additional amount of other factors of production. In the conventional models of endogenous growth, sometimes knowledge spillover, as in Lucas (1988), sometimes constant returns to capital as in Romer (1986), or sometimes research and development, as in Romer (1990), does that job. In our model, parental nurturing provides the necessary fuel for keeping the growth engine going by generating sufficient amounts of additional human capital to forestall slowing down of the output growth due to the diminishing returns to each additional unit of human capital. By Lemma 4, for all $\tau > 0$, on a balanced growth path inequality remains time invariant. It follows, therefore, from (20) and (23) that on a balanced growth path $\gamma_t = \mu g_{ht}$. In other words, output grows less than proportionately in response to the growth of human capital because $\mu < 1$, indicating diminishing returns to human capital in the technology of production. Note, in particular, that because of diminishing returns to human capital, every unit of growth in output requires $\frac{1}{\mu} > 1$ units of growth in human capital inputs, to offset exactly the loss in its marginal productivity. However, by (6) and Lemma 2, $g_{ht} = \left(\frac{\beta}{1-\alpha}\right)\gamma_t$. In other words, growth of output by one unit helps to create only $\frac{\beta}{1-\alpha}$ units of additional human capital input. It follows, therefore, that to sustain endogenous growth such that growth of output by one unit replicates itself for ever $\frac{\beta}{1-\alpha} = \frac{1}{\mu} > 1$, which implies $\alpha + \beta > 1$. Our assumption of increasing returns from parental nurturing of children in the production of human capital, thus, turns out to be a necessary condition for long-run growth. Clearly, it follows from the above discussion that the sufficient condition for endogenous growth is: $\frac{\beta}{1-\alpha} = \frac{1}{\mu}$. In other words, increasing returns in accumulating human capital must be large enough to offset diminishing returns from its use in production of consumer goods and, in particular, if $a \equiv (\alpha + \beta - 1)$ defines the strength or the extent of the increasing returns

[8] If $\tau < 0$, by (17) and (23), TFP decelerates forever without reaching a limit. Consequently, there would be a larger and longer lasting negative effect of rising inequality on the TFP growth with regressive taxes than with progressive taxes.

then to ensure endogenous growth $a = \left(\frac{1}{\mu} - 1\right)(1 - \alpha) > 0$. Thus, the necessary extent of increasing returns in the intergenerational production of human capital for sustaining endogenous growth varies inversely with the output elasticity of human capital and the elasticity of the quality of parental nurturing in the production of a child's human capital. The following Proposition formally establishes that result.

Proposition 1 *The model economy exhibits endogenous growth if, and only if, the pace of growth of the average human capital in the economy supported by the increasing returns to human capital accumulation at the family level sufficiently outweighs the negative effect of diminishing returns on the marginal product of human capital in the production of consumption goods to sustain a steady growth rate of output, such that the following condition holds:* $\frac{\beta}{1-\alpha} = \frac{1}{\mu}$*, or equivalently,* $a = (\frac{1}{\mu} - 1)(1 - \alpha)$*, where* $a \equiv (\alpha + \beta - 1)$ *measures the extent of increasing returns in the human capital accumulation technology.*

Proof See Appendix.

Proposition 1 implies that a more effective use of human capital in the production process (a higher value of μ) as well as a better scope of parent-child interaction (a higher value of α, given a) can trigger endogenous growth even if an insufficiently low extent of increasing returns in the intergenerational production of human capital stands in the way at a particular stage of economic development. A greater extent of increasing returns, however, by (12) and (13), corresponds to a greater intergenerational persistence and hence, by (18), a higher long-run income inequality, and that, as the following discussions reveal, can retard economic growth.

3.2.4 Endogenous growth–inequality relationship

By (20), (23) and (24) the overall effect of rising income inequality on the endogenous growth rate of output and income per capita is shown in the following lemma.

Lemma 6 *The growth rate of per capita income as a function of income inequality is given by:*

$$\gamma_t = \mu g_h(\tau) - \mu\sigma^2/2 + (1 - \Psi(\tau)L)\,\Lambda_{t+1}, \tag{25}$$

where L denotes the lag operator, and $\Psi(\tau) \equiv 1 - (1 - \alpha)\tau(2 - \tau) > 0$.

Proof See Appendix.

The above growth–inequality relationship implies that an economy forgoes its per capita income growth in proportion to the size of its existing income inequality, partially reflecting the forgone TFP due to interpersonal differences in the marginal product of human capital implied by the existing inequality. At the same time, future inequality impacts positively on the per capita income growth reflecting its positive influence in the growth rate of average human capital due to the presence of increasing returns in the education technology.

On the other hand, a higher progressivity lowers income inequality and a reduction in income inequality raises TFP growth but lowers the growth rate of average human capital. Consequently, the effect of redistribution on growth appears to be ambiguous. One question follows naturally: what rate of progressivity would maximize the long-run growth rate and ensure dynamic efficiency? The following section answers that question.

4 Progressive redistribution and economic growth

We begin this section with a discussion on how a change in progressivity affects various growth rates and establish a key result to show why zero progressivity would maximize the long-run growth rate of the income per capita. Afterwards, we examine the issues concerning dynamic efficiency and welfare but we do it only after switching off the standard micro-economic disincentive of redistribution on the investment in human capital with appropriate education subsidy. We switch off a specific negative effect of redistribution on growth at that juncture and not earlier in order to keep our analysis throughout the paper as general as possible until we derive a result which enables us to tighten our logical framework in a way to simplify the exposition of other results without any loss of generality.

4.1 Redistribution and the growth rates

In order to isolate the cost and benefit of increasing progressivity, we simplify the expressions for the growth rates derived in the previous section to incorporate the effect of changes in inequality due to changes in progressivity. In particular, by substituting the value of Λ_{t+1}, using (17), into (23) and (24) we find that the growth rates of TFP and average human capital satisfy

$$g_{TFPt} = -(1-\mu)\frac{\mu\sigma^2}{2} + \Omega_1(\tau)\Lambda_t, \tag{26}$$

where $\Omega_1(\tau) \equiv \beta(1-\mu)\tau(2-\beta\mu\tau) > 0$, and

$$g_{ht} = g_h(\tau) - \Omega_2(\tau)\Lambda_t, \tag{27}$$

where $\Omega_2(\tau) \equiv \beta\tau\,(2a/\,(1-\alpha) + (1-\beta)\tau) > 0$.

Note that $\Omega_1'(\tau) > 0$ and $\Omega_2'(\tau) > 0$. In other words, a greater progressivity improves TFP growth while it may reduce human capital growth, by lowering future income inequality Λ_{t+1} for any given level of existing income inequality Λ_t.[9] We, therefore, interpret $\Omega_1(\tau)$ and $\Omega_2(\tau)$, respectively, as the partial benefit and the cost of redistribution with progressivity rate τ corresponding to the size of the existing inequality.[10,11]

Substituting (26) and (27) into (20) gives

$$\gamma_t = \mu\left(g_h(\tau) - (1-\mu)\sigma^2/2\right) - \Omega(\tau)\Lambda_t, \tag{28}$$

where, $\Omega(\tau) \equiv \alpha\beta\mu\tau^2 > 0$. The above equation explicitly spells out the overall effect of redistribution and the existing income inequality on the growth rate of per capita income.

9 Potential ambiguity in the effect of changing progressivity on the human capital growth rate arises from the term $g_h(\tau)$ which captures the conventional negative effect of redistribution on the individual saving or investment in human capital, following Lemma 1.

10 Note if $\tau < 0$, $\Omega_1(\tau) < 0$ but the absolute value of $\Omega_1(\tau)$ increases with regressivity, given by the magnitude of τ. In other words, a higher regressivity decreases the growth rate of TFP, contrary to the case if $\tau > 0$. Note, however, the growth rate of human capital decreases with the progressivity if $\tau > 0$ and increases with regressivity if $\tau < 0$.

11 The partial cost and benefit of redistribution on economic growth, for a given level of inequality, show explicitly a separation of the effects of the education and the production technology. The parameters of education technology with parental nurturing that are subject to increasing returns capture the cost of redistribution $\Omega_2(\tau)$ to show explicitly how a higher cost results from a greater extent $a \equiv (\alpha + \beta) - 1$ of increasing returns. The benefit of redistribution $\Omega_1(\tau)$, on the other hand, becomes larger with a smaller value of μ, or, equivalently, with a greater strength of the diminishing returns in the production technology.

Note that the condition of endogenous growth as described in Proposition 1 implies that: $\Omega(\tau) = \mu\Omega_2(\tau) - \Omega_1(\tau)$. Therefore, it follows from our earlier discussion that $\Omega(\tau)$ combines the cost and benefits of redistribution that operates through two separate channels and, by (28), we discern that, for a given level of existing inequality, the cost of redistribution outweighs its benefit. Moreover, note that $\Omega(\tau)$ is a U-shaped function of the progressivity rate τ such that $\Omega(\tau)$ reaches its minimum at $\tau = 0$, where $-1 < \tau < 1$. In other words, neither progressivity nor regressivity is good for growth. This result is not obvious and hence calls for some explanations.

In our model the benefit of redistribution arises typically from the decrease in the variance of human capital in production which, in turn, increases the TFP growth, while the cost of redistribution arises from shifting of resources from more productive education providers to less productive ones, which, in turn, decreases the growth of average human capital. If $\tau > 0$ then, by (24), (23) and Lemma 4, the cost of redistribution turns out to be long-lasting, since its effect on the growth rate of human capital would be permanent, while its benefit turns out to be transitory, since it improves TFP growth rate only temporarily. If $\tau < 0$ then a greater regressivity delivers additional growth of human capital inputs. However, if $\tau < 0$, by (17), income inequality follows a non-stationary path of growth and that, by (23), causes TFP to decline forever, making the cost of a regressive redistribution to exceed its benefits. Consequently, the net cost $\Omega(\tau)$ of redistribution turns out to be an increasing function of progressivity as well as regressivity. Next, we formally establish that zero progressivity does indeed maximize the long-run balanced growth rate of income per capita.

We proceed by noting that, by Lemma 4 and (28), if $\tau > 0$ then the growth rate converges to a constant balanced growth rate $\gamma = \gamma(\tau) > 0$ such that

$$\gamma = \mu\left(g_h(\tau) - (1-\mu)\sigma^2/2\right) - \alpha\beta\mu\tau^2\Lambda(\tau), \tag{29}$$

where, $\Lambda(\tau)$ is given by (18).

Lemma 7 *The growth maximizing rate τ of progressivity must be equal to zero.*

Proof Note that, by (18), $\tau^2\Lambda = \frac{\tau\mu\sigma^2/2}{\beta(2-\tau\beta\mu)}$, which is a smooth function of τ for all $0 \le \tau < 1$, and equals 0 when $\tau = 0$. Thus, as τ decreases to zero, γ approaches a finite limit.

Differentiating (29) with respect to τ yields

$$\frac{\partial\gamma}{\partial\tau} = \beta\mu\frac{\partial \ln s(\tau)}{\partial\tau} - \frac{(1-\beta\mu)\mu^2\sigma^2}{(2-\beta\mu\tau)^2} < 0. \tag{30}$$

It follows, therefore, the growth maximizing $\tau = 0$, for all $0 \le \tau < 1$.

If $-1 < \tau < 0$, then, by (17), income inequality increases over time and, by (28), there exists a threshold level of inequality Λ^* such that once it exceeds that threshold then for all subsequent date t, $\gamma_t(\tau < 0) < \gamma$, where γ corresponds to the balanced growth rate with $\tau = 0$.[12] □

This ambiguity arises because of the possibility of the positive effect of a negative income tax on the parental investment in education to exceed the negative effect of inequality for limited periods and, it arises only if the inequality prior to the introduction of regressive taxes is

[12] If $\sigma^2 > \frac{4\beta}{\alpha\mu}$ then income inequality with negative progressivity or regressivity always exceeds the above threshold and hence under that condition zero progressivity unambiguously maximizes the long-run growth rate of PCI.

lower than the threshold. Given the long time horizon of our analysis, for the sake of simplicity of our exposition, we introduce education subsidy to eliminate the above microeconomic distortion caused by any income based redistribution.[13]

4.2 Redistribution with education subsidy

One may also reasonably wonder if the microeconomic disincentive of redistribution on the investment of human capital drives the above result. In fact, by Lemma 1, we do allow such negative effect of redistribution. However, contrary to what we find in Maoz and Moav (1999) and others, the negative effect of redistribution on the parental investment in the child's human capital does not drive the result described in Lemma 7. To make this specific point clear as well as to avoid the inessential details discussed above, we now expand the policy package for redistribution to include an education subsidy which offsets completely the above negative effect of redistribution described in Lemma 1 and thereby switch off its negative effect on growth via the microeconomic channel of individual optimization.

Suppose to offset the negative effect of redistribution on the optimal rate of parental investment in education, the government distributes d units of subsidy to each parent i per unit of the date t parental investment e_t^i in education and finances all of it with a non-distortionary tax, at a rate $\theta \in [0, 1]$, on consumption c_t^i such that the post-subsidy expenditure on education $\hat{e}_t^i = (1+d)e_t^i$ and the government's choice of tax and subsidy satisfies

$$\theta \int_0^1 c_t^i di = d \int_0^1 e_t^i di. \tag{31}$$

The new date t budget constraint for a parent i becomes,

$$\hat{y}_t^i = (1+\theta)c_t^i + e_t^i. \tag{32}$$

The modified human capital production function becomes,

$$h_{t+1}^i = \kappa \xi_{t+1}^i \left(h_t^i\right)^\alpha \left(\hat{e}_t^i\right)^\beta, \kappa > 0, \alpha, \beta \in (0,1), \alpha+\beta > 1. \tag{33}$$

Next, for any given progressivity rate τ, to switch off completely the negative impact of redistribution on the rate $s(\tau)$ of parental investment in education, we require that the subsidy rate d must satisfy

$$(1+d)s(\tau) = s(0), \tag{34}$$

where $s(\tau)$ continues to be given by (8) and $s(0)$ denotes its value at the laissez-faire state corresponding to zero progressivity.

Consequently, the new consumption path is given by

$$\ln c_t = \ln(1 - s(0)) + \ln y_t. \tag{35}$$

Thus the government ensures that with appropriate education subsidy, a redistributive policy does not distort the parental decision rule for allocating the budget between family consumption and the child's education.

[13] Also, the possibility for expropriation by or redistribution to the elite is limited even in countries without any democratic set-up. Consequently, consideration of this ambiguity due to the well-known microeconomic disincentive on economic growth but in an implausible way appears rather minor and inessential to the main point of this paper.

4.3 Redistribution and dynamic efficiency

In this section we identify the optimal progressivity corresponding to a dynamically efficient growth path.

We define an equilibrium to be *dynamically inefficient* if a social planner can redistribute resources to increase the equilibrium growth rate of per capita consumption at some date t without decreasing it at any other date. An equilibrium is *dynamically efficient* if, and only if, it is not *dynamically inefficient*.

By (35), it follows that the growth rate of consumption per capita must be equal to the growth rate of output per capita. The following lemma characterizes how lowering the extent of redistribution may raise the growth rate of per capita consumption.

Lemma 8 *A once and for all reduction in either progressivity or regressivity increases the common growth rate of output and consumption per capita.*

Proof Differentiating (28) w.r.t τ gives

$$\begin{aligned} \frac{\partial \gamma_t}{\partial \tau} &= -2\alpha\beta\mu\tau\Lambda_t \leq 0, \ \text{for } \tau \geq 0, \\ &> 0, \ \text{for } \tau < 0, \end{aligned} \tag{36}$$

since income inequality Λ_t is pre-existing and does not change when the government lowers τ. This means the common growth rate γ_t jumps up following a reduction in the magnitude of τ. □

Following a reduction in progressivity τ (or regressivity, i.e., decreasing the magnitude of τ when $\tau < 0$), the common growth rate of output and consumption per capita jump up immediately. This increase comes as a "free-lunch". By (28), as τ decreases, the adverse effect of inequality decreases and the growth rate increases. Also, such reduction in τ pushes the whole transition path upward, such that the growth rates remain higher in every period under the new regime. The following lemma establishes that claim explicitly, and elaborates on the channel through which this growth boost occurs.

Lemma 9 *Suppose that the economy is on a balanced growth path with progressivity rate $\tau = \tau_1 > 0$ such that for all t, the common time invariant growth rate $\gamma_t > 0$. If the government reduces progressivity by setting $\tau = \tau_2 < \tau_1$ at date $t = T$ then for all $n = 0, 1, 2, \ldots, \gamma_{T+n} > \gamma(\tau_1)$.*[14]

Proof By Lemma 8, the growth rate of consumption per capita jumps up immediately to a higher level in response to a reduction in τ. By Lemma 7, the growth rate must converge to a limit $\gamma(\tau_2) > \gamma(\tau_1)$. Next, following a reduction in τ, when income inequality increases, by (28), the growth rate must decline monotonically. Consequently, it must stay above the pre-tax reduction balanced growth path in every period. □

Proposition 2 *The economy's endogenous growth rate follows a dynamically efficient path if, and only if, the progressivity rate τ is set to zero.*

[14] If $\tau < 0$, a reduction in regressivity, by Lemma 8, lifts up the growth rate of consumption immediately. Also, by (17), the growth rate of inequality slows down from the subsequent periods. Consequently, by (28), the new sequence of growth rates of consumption per capita stays above its pre-tax reduction growth path.

Proof For any arbitrarily small magnitude of τ such that $-1 < \tau < 1$, a reduction in the absolute value of τ would, according to Lemma 9, lift the entire sequence of the growth rate of consumption above its previous state. Consequently, by Lemma 7, for the economy to reach its maximal possible growth path, or, equivalently, its dynamically efficient equilibrium, τ must be equal to zero. □

4.4 Redistribution and welfare

Following Benabou (2000), Benabou (2002) and Jones and Klenow (2010), we now broaden our focus from per capita consumption to economic welfare that takes into account the idiosyncratic risk in human capital accumulation and allows a positive role for a redistributive scheme to play in providing insurance against that risk. In particular, we define the date t social welfare W_t such that it increases with the logarithm of the equally-weighted average of utility of all dynasties and decreases with the variance of logarithm of utility as follows:

$$W_t \equiv \ln \int_0^1 U\left(h_t^i\right) di - \frac{1}{2} var\left[\ln U\left(h_t^i\right)\right]. \tag{37}$$

A lower degree of redistribution, by Proposition 2, increases the "size of the pie" (consumption per capita) and exerts a positive influence in the first component of the welfare function while, by Lemma 4, as Benabou (2002) characterizes, it also increases "the riskiness of individual slices". The following lemma reveals, however, that if the effects of parental nurturing are strong enough to produce endogenous growth, then the long-run growth rate of welfare turns out to be exactly equal to the long-run growth rate of per capita output under any progressive redistributive policy. The following lemma sheds some light on how a change in progressivity would affect economic welfare.

Lemma 10 *If $0 < \tau < 1$ the date t growth rate of welfare, $\gamma_{Wt} \equiv W_{t+1} - W_t$ equals the growth rate γ_t of income per capita.*[15]

Proof See Appendix.

The proof of Lemma 10 shows that an increase in income inequality has a direct negative effect on welfare growth only when the growth rate is non-stationary. If $\tau > 0$, by Lemma 4, income inequality converges to a stationary state and so does the growth rate of welfare. In that state inequality has no direct effect but only an indirect influence on welfare growth via its adverse effect on the growth rate of per capita income, described in (28). On the other hand, if $\tau < 0$ then income inequality increases at a faster rate than the converging sequence of inequality implied by a progressivity rate $\tau' = -\tau > 0$. Consequently, Lemmas 7 and 10 together imply that, for a given class of redistributive schemes with non-zero progressivity, a lower progressivity or a lower regressivity corresponds to a higher growth rate of welfare.

Two clear insights emerge from the above result regarding welfare evaluation of a scheme of redistribution with non-zero progressivity. First, the consideration for risk and insurance, when inequality follows a stationary process, turns out to be less important than the consideration of growing per capita consumption at a higher rate. Second, a regressive tax scheme turns out to be harmful, since it not only decelerates per capita consumption growth, and

[15] In the Appendix we show that if $-1 < \tau < 0$, there exists $\tau' = -\tau$ such that, at each date t, $\gamma_{Wt}(\tau) < \gamma_{Wt}(\tau')$, where $\gamma_{Wt}(\tau')$ corresponds to the balanced growth rate with $\tau' > 0$, by Lemma 10. It also follows, by Lemma 7, that even if $\tau < 0$, $\gamma_{Wt}(\tau)$ increases as the absolute value of $\tau = \tau'$ decreases.

hence decreases the "size of the pie", but also accelerates income inequality, and thereby increases "the riskiness of individual slices" of that pie.

It is noteworthy that the welfare function defined in (37) has a built-in bias towards a less unequal distribution of consumption due to the concavity of the utility function. Nevertheless, consideration of the negative effect of redistribution on economic growth surpasses its built-in positive effects via less unequal income distribution. This result follows from two facts that hold on a balanced growth path with non-zero progressivity. First, a higher income inequality due to a lower progressivity reduces TFP only along its transition paths towards its new stationary state at a lower level, while the positive effect of lower progressivity on the growth rate of per capita income continues as long as growth continues. Second, welfare benefits from lifting up the non-stationary path of income per capita outweigh the welfare loss from increasing the stationary income-inequality to a higher level.

This striking result, however, stands or falls on the potency of parental nurturing in producing endogenous growth. The following lemma makes that point clear.

Lemma 11 *If the increasing returns are not strong enough such that* $a \equiv (\alpha + \beta - 1) < (\frac{1}{\mu} - 1)(1 - \alpha)$, *or equivalently,* $\frac{\beta}{1-\alpha} < \frac{1}{\mu}$, *there would be no endogenous growth and a positive value of* τ *could maximize the long run output.*

Proof See Appendix.

Thus, we conclude that increasing returns to scale in human capital accumulation at the family level neither ensures endogenous growth nor implies the dynamic inefficiency and welfare retarding effect of increasing progressivity. Instead, if, and only if, increasing returns in the intergenerational production of human capital are large enough to sustain perpetual growth, a policy of progressive redistribution would be dynamically inefficient and a greater progressivity would lower the balanced growth rate of economic welfare.

We now explore some extensions of our model environment to check the robustness of our key result as described in Proposition 2.

4.5 Robustness of the model's finding: some extensions

In this section we examine the robustness of Proposition 2 and, in particular, explore if zero progressivity would remain dynamically efficient if we consider valued leisure, parental time input for developing the child's human capital and a scheme of progressive redistribution that does not tax income.

4.5.1 Valued parental time

First, if agents get disutility from work, a lower progressivity in the income tax schedule would necessarily increase labor supply and hence output and income growth.

Second, if parental interaction is essential to intergenerational transfer of human capital then the limited time endowment for such interaction would impose a constraint on how fast a generation can grow their human capital. We, therefore, explore what progressivity would make the date t time allocation by each parent i between the labor supply (l_t^i) and the parental interaction (p_t^i), such that $l_t^i + p_t^i = 1$, consistent with a dynamically efficient growth path for the economy. Suppose that the modified human capital accumulation process satisfies:

$$h_{t+1}^i = \kappa \xi_{t+1}^i \left(h_t^i\right)^\alpha \left(\hat{e}_t^i\right)^\beta (p_t^i)^\eta, \eta > 0, \kappa > 0, \alpha, \beta \in (0, 1), \alpha + \beta > 1, \quad (38)$$

where $\hat{e}_t^i$ denotes the post-subsidy expenditure on education as discussed in Sect. 4.2.

By (38), the potential growth rate of human capital decreases if parents spend less time with their children and more time in their work. A greater progressivity, on one hand, encourages parents to spend additional time interacting with their children and that promotes human capital growth. On the other hand, that same act also generates an additional tax burden for their children in future, which has a negative impact on the parent's utility because of intergenerational altruism.

In our model with logarithmic utility, it turns out that the discounted present value of the loss of utility due to the additional tax burden of the child, exactly offsets the gain from reduced tax obligations due to substitution of work time for parenting time. Consequently, a redistributive policy would fail to provide additional incentives for parents to spend time with their children and thereby to promote economic growth, making the optimal parenting time independent of progressivity. Applying the above condition to modify Lemma 6 we find that the non-stationary growth rate of per capita income satisfies

$$\gamma_t = \mu \left(\ln \kappa - \sigma^2/2 + \beta \ln s(0)\right) + \mu\eta \ln p + (1-\alpha)\varepsilon \ln(1-p) \\ -(1-\alpha-\beta\mu)\ln y_t + (1-\Upsilon(\tau)L)\Lambda_{t+1}, \tag{39}$$

where $\Upsilon(\tau) \equiv 1 - \beta\mu\tau(2-\tau)$ and $s(0) \equiv \frac{\rho\beta\mu}{1-\rho\alpha}$.

Note that like parental time, by assumption, parental investment in education is also essential to the accumulation of her child's human capital (i.e., $\beta > 0$). It follows, therefore, from (39) that the necessary and sufficient condition for endogenous growth continues to be the same as the condition described in Proposition 1. Once we apply that condition of endogenous growth into (39), the growth rate of per capita income satisfies (28), and converges to a stationary growth rate, given by (29), except for the term involving p. Consequently, by Lemmas 7 and 8, it follows that the growth-maximizing degree of redistribution is still zero, and thereby, it extends our result described in Proposition 2.[16] In other words, even if parental nurturing is mostly time, redistribution continues to affect growth adversely and continues to make zero progressivity dynamically efficient.

4.5.2 *Education finance*

In our next extension, we consider an alternative scheme of redistribution with zero income tax, coupled with a scheme of income-tested progressive education subsidy, as discussed in Benabou (2002), such that

$$\hat{y}_t^i = y_t^i, \qquad \hat{e}_t^i = (1+d)\left(\tilde{y}_t/y_t^i\right)^\tau e_t^i \tag{40}$$

where $0 \leq \tau < 1$ continues to measure progressivity and $\int_0^1 \hat{e}_t^i di = (1+d)\int_0^1 e_t^i di$ and the condition (34) to switch off microeconomic disincentive of redistribution on growth, discussed earlier, determines the subsidy rate d as a function of progressivity τ.

[16] If we consider utility that is not logarithmic, as in Glomm and Ravikumar (1998), the optimal parenting time would be either an increasing or a decreasing function of the parents' human capital, depending upon the elasticity of intertemporal substitution. If it is a decreasing function, there would be an inherent tendency of convergence of human capital and income and the issue of growth-promoting redistribution would lose its relevance. Alternatively, if it is an increasing function, as the empirical supports presented by the authors indicate, the more educationally productive parents would be spending a longer period of time parenting. Consequently, by Proposition 2, zero progressivity would continue to be dynamically efficient.

Lemma 12 *Under the education finance scheme (E), the optimal saving rate s^E and the sequence of balanced growth rate γ_t^E satisfy*

$$s^E(\tau) = \frac{\rho\beta\mu}{1-\rho\alpha+\rho\beta\mu\tau} \equiv \frac{s(0)}{1+\tau s(0)}, s'(\tau) < 0 \tag{41}$$

where $s(0) \equiv \frac{\rho\beta\mu}{1-\rho\alpha}$ denotes the laissez-faire saving rate and

$$\gamma_t^E = \mu\left(\ln\kappa - (1-\mu)\sigma^2/2 + \beta\ln s(0)\right) - \alpha\beta\mu\tau^2\Lambda_t(\tau). \tag{42}$$

Proof See Appendix.

Note that if $0 < \tau < 1$, $\Lambda_t(\tau)$ approaches $\Lambda(\tau)$, which is given by (18), and γ_t^E approaches a time invariant balanced growth rate $\gamma^E(\tau)$.[17]

Lemma 13 $\frac{\partial\gamma^E}{\partial\tau} < 0$.

Proof As shown in Lemma 7 that $\gamma^E(\tau = 0)$ exists. Differentiating (42) with respect to τ yields

$$\frac{\partial\gamma^E}{\partial\tau} = -\frac{(1-\beta\mu)\,\mu^2\sigma^2}{(2-\beta\mu\tau)^2} < 0. \tag{43}$$

It follows, therefore, that for all $-1 < \tau < 1$, $\frac{\partial\gamma^E}{\partial\tau} < 0$. □

It follows, therefore, that zero progressivity would continue to be dynamically efficient even under the alternative scheme of redistribution that avoids income tax altogether to eliminate its negative effect on economic growth.

Thus we establish the validity of Proposition 2 also within reasonable extensions of our model environment.

4.6 Proportional tax with lump-sum subsidy

An issue arises concerning the generality of our result in terms of alternative forms of redistribution. In particular, suppose that we consider a proportional tax with a lump-sum subsidy. We suggest that even with such a policy, redistribution continues to retard growth. The key insight that drives our result stems from the fact that a redistribution of resources from a more educationally-productive parent to a less educationally-productive one, however that might be done, has a cumulative negative effect on the average human capital in all subsequent periods. This growth-retarding effect on the average human capital holds because of the non-convex curvature of the human capital accumulation technology. Alternative methods of redistribution do not change that fact. Moreover, we note from the condition for endogenous growth that the long-run balanced growth rate of per capita income is a 1-1 function of the growth rate of average human capital. In other words, the endogenous growth condition makes transitory improvements of TFP with various types of redistribution irrelevant, as long as those policies negatively influence the growth rate of average human capital.

Also, the size of the subsidy relative to one's income, even when it comes as an education voucher, turns out to be not important. The model's growth-promoting channel that works through relaxing the credit constraints of the poor, only enhances TFP in discrete jumps. The

[17] If, however, $-1 < \tau < 0$, by (17), $\Lambda_t(\tau)$ increases over time causing γ_t^E to decline forever.

per capita income growth rate does not increase, as it depends completely on the human capital accumulation rates which cannot be improved by lump-sum subsidies, for the following reasons. First of all, a larger subsidy to a low-income adult, even if it comes in the form of a voucher, would not raise the unique unconstrained optimal rates of saving for education without redistributive taxes. Also, those income-tested vouchers would be likely to discourage private investment in schooling and, hence, the subsidy receivers would not contribute to the additional growth. Secondly, by (34) or (41) we achieve those laissez-faire rates of saving and investment with our proportional subsidies as a part of our experimental design. If we replace them with lump-sum subsidies, they would not fully offset the distortions in saving and investment decisions of the tax-payers or, equivalently, the subsidy providers and, that would further slow down growth. Finally, the macroeconomic cost of redistribution of resources from a more educationally-productive parent to a less educationally-productive parent would continue to outweigh the macroeconomic benefit from TFP enhancement, no matter how large the subsidy is relative to the income of the poor. In this context, it is also noteworthy that Momota (2009) demonstrates how an education subsidy may sometimes decrease the average school enrolment ratio and future economic growth.

Thus, our main results apply to a general policy of redistribution from high income to low income people. However, we make our model's tax structure progressive to keep it close to the common practice observed in most countries, as argued by Benabou (2002) and others. We now discuss how we can use the model's hypothesis of a negative effect of progressivity on growth to organize data systematically to interpret the existing empirical work on this topic.

5 A guiding hypothesis for interpreting data

Regarding the effect of progressivity on growth, in the first instance, the empirical literature provides mixed results. Easterly and Rebelo (1993) conclude that the effects of taxation are difficult to isolate empirically. We offer a hypothesis based on the key proposition of our model as a guiding principle for organizing the data to discern a consistent narrative among all seemingly conflicting observations.

5.1 Hypothesis

By Proposition 2 of our model, zero progressivity yields the maximal growth path for the economy. It implies that if parental nurturing constitutes the sole engine of output growth in a country, a lower progressivity would cause a higher rate of growth in that country. The following discussion provides the rationale for the case that the poorer and less-developed countries would be more likely to rely predominantly on parental nurturing for sustaining economic growth than their more-developed counterpart. Consequently, it would be more likely to find a significantly negative relationship between progressivity and growth among the poorer countries.

Alternative theoretical channels for producing endogenous growth in economic models of a well-developed country may include public education (see, e.g., Saint-Paul and Verdier (1993), or knowledge spillover externality (see, e.g., Perotti 1993, and Benabou 1996b), or innovations founded on public good, such as non-rival research ideas (see, e.g., Garcia-Penalosa and Wen 2008). Progressive redistribution typically plays a positive role in fostering growth in those models, making the optimal progressivity rate positive. If both parental nurturing and those alternative channels contribute to endogenous growth simultaneously, the relative contribution of each channel would be a critical factor in determining the optimality of progressivity of income taxes. Also, it is likely that the parental nurturing mechanism of growth accounts for only a relatively small proportion of endogenous growth in a developed country.

On the other hand, developing countries, especially those in their early stages of development, may rely primarily on parental nurturing to sustain economic growth. They may lack appropriate institutions such as effective patenting agencies, for various reasons including those discussed in Acemoglu et al. (2006) and Acemoglu and Robinson (2006), which may limit the applicability of the innovation-led growth models.[18] Moreover, the early stages of development may correspond to a low marginal product of human capital due to the absence of a sufficiently large amount of knowledge-intensive capital stock and the predominance of agrarian based technology, which typically requires only unskilled labor. Galor et al. (2009) explain how a low marginal product of human capital due to the above economic conditions, coupled with a highly unequal distribution of landownership, may adversely affect the emergence of institutions for public education. Also, low literacy rates, rigid societal structures and other barriers to communication, including those discussed in Stewart and Ghani (1991) and Foster and Rosenzweig (1995), may limit the potency of knowledge spillover as the prime engine of growth in less-developed countries.

Consequently, barring the very poor countries where no endogenous growth may be present, our model predicts a pattern of a stronger negative effect of progressivity on the growth rate of income per capita in a sample of poorer countries. Next, we examine if, consistent with the model's prediction, a clear narrative emerges from various empirical findings reported in the literature.

5.2 Empirical findings

An influential study by Perotti (1996) concludes that progressivity, measured by the average marginal income tax rate (AMTR), exerts a positive impact on the growth rate of per capita income. In his small sample of 49 countries, Perotti (1996) also notes a significantly larger positive impact on the growth rate in the richer countries than in the poorer countries. In a recently conducted study, Li and Sarte (2004) find that a decrease in progressivity following a major tax reform in the U.S. (Tax Reform Act of 1986), led to a small but non-negligible increase in the growth rate. Engen and Skinner (1996) make a similar claim noting that, in the U.S., major tax reform reducing all marginal rates by 5% may only lead to an increase in growth in the order of 0.2–0.3%. Thus the existing body of evidence supports only a weakly negative or positive impact of progressivity on growth.

On the other hand, from the empirical study of Padovano and Galli (2002), who use a panel of 25 middle-income, industrialized countries from 1970 to 1998, we discern a much stronger negative effect of progressivity on economic growth than what the above studies report. We also note similar findings in Rabushka (1987), who focuses exclusively on a group of 63 developing countries and in Koester and Kormendi (1989), who review a sample of primarily developing countries. In a recent study, Gray et al. (2007) also find a strong negative effect of progressivity on growth in the sample of East European and Central Asian countries in transition.

It is noteworthy, in the present context, that Trostel (2004) reports strong evidence of increasing returns in the intergenerational production of human capital especially in low income countries. In light of that evidence, the observed stronger negative effect of progressivity

[18] Acemoglu and Robinson (2006) conclude that political elites in countries with low average human capital coupled with large political rents would block formation of institutions that promote innovation.

on the growth rate of income per capita in poorer countries, as discussed above, points towards a narrative that is consistent with the model's prediction. The task of identifying that narrative exactly with the model's hypothesis requires additional data which currently do not exist. So we leave that task for future work.

6 Concluding remarks

We argue that if parental nurturing is a dominating force in human capital formation, then income redistribution may not promote economic growth. The premise is that redistribution shifts resources towards the less educationally-productive families and, thus, in the presence of credit markets imperfections and increasing returns in the intergenerational production of human capital, it reduces the aggregate level of investment in human capital. If the degree of increasing returns is large enough to bring sustained growth into the economy, then the macroeconomic cost of redistribution may be large enough to make a redistribution scheme, progressive or regressive, dynamically inefficient.

The main hypothesis extends to models that include a credit market and tradable capital goods as they reduce the potential benefits of redistribution by lowering the variance of marginal products among the production units in the economy. Similarly, it extends to those models that allow typical disincentives of redistribution, on work effort, saving and investment in education, all of which increase costs of redistribution. Also, it is robust against considerations of alternative schemes of redistributive policy involving lump-sum educational vouchers, coupled with a proportional or a progressive average tax rate.

The existing literature provides empirical support for our model's key assumption of increasing returns to the intergenerational production of human capital especially in the context of developing countries. Thus, we conclude that increasing progressivity as a part of an equity-promoting redistributive policy has an additional important effect that may operate toward the reduction in the growth rate of per capita income in countries that are in the early stages of development. Note, however, that the above conclusion holds, if, and only if, countries are not stuck in a poverty trap of the kind described in Galor and Zeira (1993) but, instead, are emerging from the early stages of development, predominantly by benefiting from increasing returns to human capital accumulation at the family level, rather than by benefiting from spillover of information, public education and innovation. We leave the task of determining the relative empirical relevance of various channels of growth in a country to future research noting, however, the importance of that analysis in view of the conflicting policy implications.

Acknowledgements We are grateful for comments from three anonymous referees, the Editor, Oded Galor, Roland Benabou, Dilip Mookherjee, Richard Rogerson, and Debraj Ray. We also benefitted from the comments made by participants at the 5th Annual Conference on Economic Growth and Development in Delhi ISI as well as at the 2009 Southern Workshop in Macroeconomics (SWIM) in Auckland. Bandyopadhyay acknowledges partial support from a PBRF grant from the Economics Department, University of Auckland. Tang acknowledges financial support from a Developing Researcher Grant from the School of Accounting, Economics and Finance, Deakin University.

Appendix

Proof of Lemma 1 Denoting the saving rate $s_t^i \equiv e_t^i/\hat{y}_t^i$ and using (5), we rewrite (7) as follows:

$$\ln U\left(h_t^i, k_t^i, M_t; T\right) = \max_{s_t^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln\left(1-s_t^i\right) + \ln \hat{y}_t^i\right] \\ +\rho E_t\left[\ln U\left(h_{t+1}^i, M_{t+1}; \tau\right)\right] \end{array} \right\}, \tag{A.1}$$

and continuing with the same notation after substituting (2) and (3) into (6) yields

$$h_{t+1}^i = \kappa\left(s_t^i\right)^{\beta} \xi_{t+1}^i \left(h_t^i\right)^{\alpha+\beta\mu(1-\tau)} \left(l_t^i\right)^{\beta(1-\mu)(1-\tau)} (\tilde{y}_t)^{\beta\tau}. \tag{A.2}$$

The dynastical agent solves (A.1) subject to (2) and (3) and (A.2).

We guess the value function as: $\ln U\left(h_t^i, M_t; \tau\right) = Z \ln h_t^i + B_t$. Then by substituting this value function into (A.1), we get

$$Z \ln h_t^i + B_t = \max_{s_t^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left(\ln\left(1-s_t^i\right) + (1-\mu)(1-\tau)\ln l_t^i + \tau \ln \tilde{y}_t\right) \\ \quad + ((1-\rho+\rho\beta Z)\mu(1-\tau) + \rho\alpha Z)\ln h_t^i \\ +\rho\left(\begin{array}{l} Z\left(\begin{array}{l} \ln\kappa + \beta\ln s_t^i - \sigma^2/2 \\ +\beta(1-\mu)(1-\tau)\ln l_t^i + \beta\tau\ln\tilde{y}_t \end{array}\right) \\ +B_{t+1} \end{array}\right) \end{array} \right\}. \tag{A.3}$$

Taking partial differential with respect to $\ln h_t^i$ gives

$$Z = \frac{(1-\rho)\mu(1-\tau)}{1-\rho\alpha-\rho\beta\mu(1-\tau)}. \tag{A.4}$$

The first-order condition of (A.1) with respect to the saving rate is

$$\frac{1-\rho}{1-s_t^i} = \rho\left(\frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i}\frac{\partial \ln h_{t+1}^i}{\partial s_t^i}\right) \tag{A.5}$$

where $\partial \ln h_{t+1}^i / \partial s_t^i = \beta / s_t^i$.

The above optimization problem (A.3) is strictly concave. Consequently, (A.5) is sufficient for the optimization exercise, and Lemma 1 follows immediately after we substitute (A.4) into (A.5). □

Proof of Lemma 2 By assumption, at the initial date $t = 0$, the human capital is lognormally distributed. By (12), it follows that h_t^i remains lognormally distributed over time, and hence, by (2), y_t^i is lognormal and hence, the income per capita y_t is[19]

$$y_t = \int_0^1 y_t^i di = \exp\left(\int_0^1 \ln y_t^i di + \frac{1}{2}\text{var}\left[\ln y_t^i\right]\right). \tag{A.6}$$

The median income is

$$y_{t,median} = \exp\left(\int_0^1 \ln y_t^i di\right). \tag{A.7}$$

In line with Benabou (2002), therefore, the inequality index is

$$\Lambda_t \equiv \log\left(\frac{y_t}{y_{t,median}}\right) = \frac{1}{2}\text{var}\left[\ln y_t^i\right] = \mu^2\Delta_{ht}^2/2. \tag{A.8}$$

[19] See Crow and Shimizu (1988) for details about properties of lognormal distribution.

This proves Lemma 2. □

Proof of Lemma 3 To derive the expression for the break-even point defined in (4), we note that the mean of y_t^i in logarithm, according to Eq. (A.6), satisfies

$$\ln y_t = \mu m_{ht} + \Lambda_t, \tag{A.9}$$

and the mean of $\left(y_t^i\right)^{1-\tau}$ in logarithm is

$$\ln \int_0^1 \left(y_t^i\right)^{1-\tau} di = (1-\tau)\mu m_{ht} + (1-\tau)^2 \Lambda_t. \tag{A.10}$$

Taking the difference between the income before and after tax yields

$$\ln \tilde{y}_t = \mu m_{ht} + (2-\tau)\Lambda_t. \tag{A.11}$$

Combining (A.9) with (A.11) yields (16). This proves Lemma 3. □

Proof of Lemma 5 From the proof of Lemma 2, we know that human capital is lognormally distributed. Therefore, it follows that

$$\ln h_t = m_{ht} + \Delta_{ht}^2/2. \tag{A.12}$$

Substituting (A.12) into (A.9) yields (19). Then we can get the analytical expression of TFP for each date t as follows:

$$\begin{aligned} \ln TFP_t &= (\mu - 1)\mu \Delta_{ht}^2/2 \\ &= -\frac{1-\mu}{\mu}\Lambda_t, \text{ by } \Lambda_t \equiv \mu^2 \Delta_{ht}^2/2. \end{aligned} \tag{A.13}$$

Thus, the proof of Lemma 5 is completed. □

Proof of Proposition 1 and Lemma 6 Integrating both sides of (13) across all agents i we get,

$$\begin{aligned} \int_0^1 \ln y_{t+1}^i di &= \mu \ln \kappa + \mu\beta \ln s(\tau) - \mu\sigma^2/2 \\ &\quad + (\alpha + \beta\mu(1-\tau)) \int_0^1 \ln y_t^i di + \beta\mu\tau \ln \tilde{y}_t. \end{aligned} \tag{A.14}$$

By Lemma 2, y_t^i is lognormally distributed. Therefore, it follows that

$$\int_0^1 \ln y_t^i di = \ln \int_0^1 y_t^i di - \frac{1}{2} \operatorname{var}\left[\ln y_t^i\right]. \tag{A.15}$$

Combining (A.15) with (A.14) yields

$$\ln \int_0^1 y_{t+1}^i di - \frac{1}{2}\text{var}\left[\ln y_{t+1}^i\right] = \mu \ln \kappa + \mu\beta \ln s(\tau) - \mu\sigma^2/2 + \beta\mu\tau \ln \tilde{y}_t$$
$$+(\alpha + \beta\mu(1-\tau))\left(\ln \int_0^1 y_t^i di - \frac{1}{2}\text{var}\left[\ln y_t^i\right]\right). \tag{A.16}$$

By substituting (16) into (A.16), we get

$$\gamma_t = \mu\left(\ln \kappa - \sigma^2/2 + \beta \ln s(\tau)\right)$$
$$-(1-\alpha-\beta\mu)\ln y_t + (1-\Psi(\tau)L)\Lambda_{t+1}, \tag{A.17}$$

where L denotes the lag operator, and $\Psi(\tau) \equiv 1-(1-\alpha)\tau(2-\tau) > 0$. By (A.17), the growth rate γ_t declines to zero as long as $1-\alpha-\beta\mu > 0$. Consequently, the necessary and sufficient condition for long-run endogenous growth is: $\frac{\beta}{1-\alpha} = \frac{1}{\mu}$. Then applying the endogenous condition for growth, we get (25). Thus, the proofs of Proposition 1 and Lemma 6 are completed. □

Proof of Lemma 10 $W_t \equiv \ln \int_0^1 U\left(h_t^i\right) di - \frac{1}{2} var\left[\ln U\left(h_t^i\right)\right] = \int_0^1 \ln U\left(h_t^i\right) di$ (since h_t^i is lognormally distributed). The solution to (A.1) implies $W_t = Zm_{ht} + B_t$ and

$$\gamma_{Wt} = Z\left(m_{ht+1} - m_{ht}\right) + \left(B_{t+1} - B_t\right). \tag{A.18}$$

By incorporating the first order condition into (A.3) yields,

$$W_t - \rho W_{t+1} = Z\left(m_{ht} - \rho m_{ht+1}\right) + \left(B_t - \rho B_{t+1}\right), \tag{A.19}$$

where,

$$B_t - \rho B_{t+1} = (1-\rho)\ln(1-s(\tau))$$
$$+(1-\rho+\rho\beta Z)\tau \ln \tilde{y}_t$$
$$+\rho Z\left(\ln \kappa - \sigma^2/2 + \beta \ln s(\tau)\right). \tag{A.20}$$

It follows, therefore, from (14), (A.4) and the condition for endogenous growth that

$$\frac{W_t}{(1-\rho)} = \frac{\mu m_{ht}}{1-\rho} - \frac{1}{1-\rho}\frac{\mu}{1-\rho}\frac{\rho\sigma^2}{2}$$
$$+\frac{1}{1-\rho}\left(\ln(1-s(\tau)) + \frac{\rho\mu\left(\ln\kappa + \beta \ln s(\tau)\right)}{1-\rho}\right)$$
$$+\frac{1-\rho\alpha}{1-\rho}\tau(2-\tau)\sum_{n=0}^{\infty}\rho^n \Lambda_{t+n}. \tag{A.21}$$

Consequently, by (A.9), at each date t, the growth rate γ_{Wt} of welfare satisfies,

$$\gamma_{Wt} = \gamma_t - \Phi_t, \tag{A.22}$$

where $\Phi_t \equiv (\Lambda_{t+1} - \Lambda_t) - (1-\rho\alpha)\tau(2-\tau)\sum_{n=0}^{\infty}\rho^n\left(\Lambda_{t+1+n} - \Lambda_{t+n}\right)$.

If $\tau > 0$, by (17) and (28), Λ_t and γ_t will converge to a constant level in the long run. Consequently, on the balanced growth path, $\Phi_t = 0$ and, hence, $\gamma_{Wt} = \gamma_t = \gamma(\tau)$, where, $\gamma(\tau)$ is given by (29). Thus, the proof of Lemma 10 is completed.[20] □

Proof of Lemma 11 If $1-\alpha-\beta\mu > 0$ then the equilibrium sequence M_t monotonically converges to a unique steady state as a function of τ, and the steady state inequality $\Lambda_t = \Lambda(\tau)$ is given by (18). Moreover, by (18), we can get $\Lambda'(\tau) < 0$. It implies that redistribution helps to reduce the static inequality $\Lambda(\tau)$. Since $1-\alpha-\beta\mu > 0$, by (A.17) income per capita, $\ln y_t(\tau)$, converges to a constant in the steady state as shown below,

$$\ln y(\tau) = \frac{\mu\beta \ln s(\tau) + \mu \ln \kappa - (1-\mu)\mu\sigma^2/2 - \Omega\Lambda}{1-\alpha-\mu\beta}. \tag{A.23}$$

Equivalently, we can rewrite the above as follows

$$\ln y(\tau) \equiv \frac{\Gamma_1(\tau) - \Omega\Lambda}{c_1}, \tag{A.24}$$

where $\Gamma_1(\tau) \equiv \mu\beta \ln s(\tau) + \mu \ln \kappa - (1-\mu)\mu\sigma^2/2$ and $c_1 \equiv 1-\alpha-\mu\beta$, and rewrite inequality as below

$$\Lambda(\tau) \equiv \Gamma_2(\tau)\sigma^2. \tag{A.25}$$

Differentiating both sides of (A.24) with respect to τ yields

$$c_1 \frac{\partial \ln y(\tau)}{\partial \tau} = \Gamma_1'(\tau) - \Omega'\Gamma_2(\tau)\sigma^2 - \Omega\Gamma_2'(\tau)\sigma^2 \equiv \Gamma_1'(\tau) - F(\tau)\sigma^2, \tag{A.26}$$

where $F(\tau) \equiv \Omega'\Gamma_2(\tau) + \Omega\Gamma_2'(\tau)$. Note, $F(\tau)$ does not depend on σ^2. Thus, this feature can help find a critical minimal threshold of the variance of innate ability $\sigma^* \equiv \frac{\Gamma_1'(0)}{F(0)} = f(\alpha,\beta,\rho,\mu) \geq 0$ such that the long-run output maximizing degree of redistribution is $\tau^* > 0$ if, and only if, $\sigma^2 > \sigma^*$. Thus, the proof of Lemma 11 is completed. □

Proof of Lemma 12 Agent solves (A.1) subject to (2) and (40) as below

$$\ln U\left(h_t^i, M_t; \tau\right) = \max_{s_t^i, l_t^i} \left\{ \begin{array}{l} (1-\rho)\left[\ln\left(\left(1-s_t^i\right)/(1+\theta)\right)\right. \\ \left. +\mu \ln h_t^i + (1-\mu)\ln l_t^i\right] \\ \\ +\rho E_t\left[\ln\left(U\left(h_{t+1}^i, M_{t+1}; \tau\right)\right)\right] \end{array} \right\}, \tag{A.27}$$

where h_{t+1}^i is given by (33). We guess the same value function as: $\ln U\left(h_t^i, M_t; \tau\right) = Z \ln h_t^i + B_t$. Substituting that value function into (A.27) yields

$$Z \ln h_t^i + B_t = \max_{s_t^i, l_t^i} \left\{ \begin{array}{l} \rho\left(Z\left(\ln\kappa - \sigma^2/2\right) + B_{t+1}\right) \\ \\ +(1-\rho)\ln\left(\left(1-s_t^i\right)/(1+\theta)\right) + \rho\beta Z \ln\left((1+d)s_t^i\right) \\ \\ +\rho\beta Z\tau \ln \tilde{y}_t + (1-\rho+\rho\beta Z(1-\tau))(1-\mu)\ln l_t^i \\ \\ +((1-\rho+\rho\beta Z(1-\tau))\mu + \rho\alpha Z)\ln h_t^i \end{array} \right\}. \tag{A.28}$$

[20] If $\tau < 0$, $\Phi_t > 0$. Also, by (28), there exists a progressivity rate $\tau' = -\tau > 0$ such that $\gamma_{Wt}\left(\tau'\right) - \gamma_{Wt}(\tau) = \Omega(\tau)\left(\Lambda_t(\tau) - \Lambda_t\left(\tau'\right)\right) + \Phi_t > 0$. In other words, for any given regressivity, one can find a progressive scheme of redistribution under which a society's welfare would grow at a faster rate.

Taking the partial differential of (A.28) with respect to $\ln h_t^i$ yields

$$Z = \frac{(1-\rho)\mu}{1-\rho\alpha-\rho\beta\mu(1-\tau)}. \tag{A.29}$$

The first-order condition for the saving rate is unchanged and is still given by Eq. (A.5). Substituting (A.29) into (A.5) gives (41).

The equilibrium dynamics of human capital and income for the dynasty i are the same as (12) and (13). Thus, by the proof of Proposition 1, we can get the exact same growth function as (29). The function of Λ is the same as (18) because the dynamics of human capital and the variance of $\ln h_{t+1}^i$ have the same format as under income tax scheme. Thus, the proof of Lemma 12 is completed. □

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