HERENGA DELTA 2021

VALUES AND VARIABLES

The 13th Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics

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KIA ORA

Ngā mihi aroha ki ngā tangata katoa and warm greetings to you all. Welcome to Herenga Delta 2021, the Thirteenth Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics.

It has been ten years since the Volcanic Delta Conference in Rotorua, and we are excited to have the Delta community return to Aotearoa New Zealand, if not in person, then by virtual means. Although the limits imposed by the pandemic mean that most of this year's 2021 participants are unable to set foot in Tāmaki Makaurau Auckland, this has certainly not stopped interest in this event. Participants have been invited to draw on the concept of *herenga,* in Te Reo Māori usually a mooring place where people from afar come to share their knowledge and experiences. Although many of the participants are still some distance away, the submissions that have been sent in will continue to stimulate discussion on mathematics and statistics undergraduate education in the Delta tradition.

The conference invited papers, abstracts and posters, working within the initial themes of *Values and Variables*. The range of submissions is diverse, and will provide participants with many opportunities to engage, discuss, and network with colleagues across the Delta community. The publications for this thirteenth Delta Conference include publications in the International Journal of Mathematical Education in Science and Technology, iJMEST, (available at https://www.tandfonline.com/journals/tmes20/collections/Herenga-Delta-2021), the Conference Proceedings, and the Programme (which has created some interesting challenges around time-zones), by the Local Organizing Committee. Papers in the iJMEST issue and the Proceedings were peer reviewed by at least two reviewers per paper. Of the ten submissions to the Proceedings, three were accepted.

We are pleased to now be at the business end of the conference and hope that this event will carry on the special atmosphere of the many Deltas which have preceded this one. We hope that you will enjoy this conference, the virtual and social experiences that accompany it, and take the opportunity to contribute to further enhancing mathematics and statistics undergraduate education.

Ngā manaakitanga,

Phil Kane (The University of Auckland | Waipapa Taumata Rau) on behalf of the Local Organising Committee

Stephanie Budgett (The University of Auckland | Waipapa Taumata Rau)
Rosie Cameron (The University of Canterbury | Te Whare Wānaga o Waitaha)
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PAPERS

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PAPERS

EMBEDDING CONCEPT MAPPING INTO UNIVERSITY MATHEMATICS: COMPARISON AND VALIDATION OF MARKING RUBRICS

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ABSTRACT

Concept mapping is a visual way of presenting a group of related abstract concepts and identifying relationships between them by connecting related concepts with directed arrows that specify relationships. In the last few decades, concept mapping has become a popular research and educational tool. However, despite its extensive usage, not much research has been done in designing methods to evaluate concept mapping tasks and their validation. Moreover, very little has been reported about concept mapping usage in mathematics education. In this study, university students (N=260) in a large undergraduate mathematics course (for non-mathematics majors) were assigned to construct a concept map for *Vector Space*, which they had studied in the course. This research investigated various ways to evaluate students' concept mapping activity by comparing four rubrics. Using multiple linear regression to predict final exam outcomes, we were able to identify the best rubric for assessing student concept mapping. We found that the most important aspect in assessing concept mapping tasks is the inverse ratio between the number of concepts and the number of relationships between them presented in student work. This finding informs practical recommendations for implementing concept mapping activity in mathematics courses that we present at the end of the paper, together with a call for future research to investigate the causal relationship between the use of concept mapping and learning outcomes. It is of great interest to find out whether increasing the amount of concept mapping activity in a mathematics course would enhance student conceptual understanding.

INTRODUCTION

Origin and description of concept maps

Concept maps were first introduced by Novak and Cañas in research undertaken at Cornell University in the 1970s (Novak & Cañas, 2008; Novak & Musonda, 1991). Concept mapping is a visual representation tool to organise information about a chosen concept (Nesbit & Adesope, 2006). The concepts are often presented in enclosed shapes, and lines are used to connect any two concepts which are related. The most general concept that embraces all other concepts is presented at the top, and more specific concepts should be derived in descending order, hence reflecting a hierarchy of the presented concepts (Novak & Cañas, 2008; Schroeder et al., 2018). "Linking words or linking phrases" (Novak & Cañas, 2008, p. 1) are short descriptions on the lines that specify how the connected concepts are related. Some examples of such words/phrases include "is an example of", "generalises to" and "contains".

Novak (Novak & Cañas, 2008), the pioneer of concept maps describes the key features of concept mapping as the following:

- Hierarchy: The order in which the concepts are presented should imply a hierarchy from general concepts to specific concepts in a descending manner. However, the hierarchical relationship between concepts may differ with respect to the context. As a guide to deciding

the hierarchy between concepts, it is useful to have a "focus question" (Novak & Cañas, 2008, p. 2), a specific question that the concept mapper wishes to answer.
- Cross-links: Cross-links can be thought of as connections that are found between concepts that are derived from different strands developed from the main concept. The discovery of cross-links relies heavily on the creativity of the concept mapper.
- Examples: Having specific examples as a part of the concept map allows a clear understanding of a concept that may be abstract on its own. However, it is important to note that examples are not considered as concepts.

Benefits of concept mapping

Concept mapping is a great way for learners to make their internal understanding explicit (Croasdell et al., 2003; Kinchin et al., 2000). It provides an opportunity for the mappers to think in a critical and a complex (non-linear) way (Gul & Boman, 2006; Lee et al., 2013).

There are two major ways students can use concept mapping for their learning. It can either be given as a study guide completed by an expert for students to study with, or they can create a map independently. However, the Brod (2020)'s review showed that concept maps have a much greater impact when students create their own. This is because when a student creates their own concept map, the mapping process requires a meaningful engagement from the concept mapper, which involves higher-order learning activities such as organising and synthesising and, as a result, the task is very likely to enable high quality of learning (Nesbit & Adesope, 2006; O'Day & Karpicke, 2020; Schroeder et al., 2018). According to Moorf and Readence (1984), this may be a possible reason as to why concept mapping has a greater effect when done at the end of learning a topic rather than at the beginning (Moorf & Readence, 1984; Nesbit & Adesope, 2006).

The nature of concept mapping promotes the mapper's skill of showing their understanding in an organised way (Novak & Cañas, 2008; O'Day & Karpicke, 2020). Such skill is beneficial for "free recall" (O'Day & Karpicke, 2020, p. 2) of information (Hunt, 2012; Kahana, 2017; Raaijmakers & Shiffrin, 1981). Moreover, in the process of creating a concept map, students are expected to "select and isolate key pieces of information, organize that key information in a graphical form, and integrate those pieces of information together with relationship links" (Fiorella and Mayer (2015) as cited in O'Day and Karpicke (2020, p. 10). Such explanation is supported by the results of a recent experimental study conducted by O'Day and Karpicke (2020), which established a positive impact on learning when concept mapping and retrieval practice are incorporated into learning practice.

In certain contexts concept mapping has been shown to be more effective than conventional instructional practices (Jegede et al., 1990; Novak, 1990). Croasdell et al. (2003) have provided a specific comparison between learning through concept maps (abbreviated as CM) and linear note-taking (abbreviated as NT) to conclude that:
- CM is more effective for retrieving information compared to NT.
- Spotting important information is more easily done using CM than using NT.
- Relationships between concepts are easily noticeable in CM.
- Reviewing is less time-consuming and has a greater effect when using CM.
- The structure of CM is more flexible for adding new ideas.

However, when concept mapping is used as a form of assessment, there may be some disadvantages. Firstly, irrespective of the students' actual understanding of the chosen concept, the student may not have the required skill to make their understanding explicit or turn it into a given template. Secondly, because concept mapping is such a subjective activity that is highly dependent on the individual, consistency in the marking process cannot be guaranteed (McClure et al., 1999). In fact, the second point served as a foundation for the design, development and implementation of our study, which is reported in this paper.

Ways to evaluate concept mapping

Concept maps are not only a good tool for learning, but also for assessing students' understandings (Croasdell et al., 2003; McClure et al., 1999). However, as unique as concept maps can be, it is also very difficult for teachers to evaluate students' works.

Kinchin et al. (2000) define three different categories for considering the structural quality of concept maps. (Refer to Figure 1 below.) The first type (A in Figure 1) is "spoke" (Kinchin et al., 2000, p. 47), where the main concept is placed at the centre, and the only connections found in the map are between each derived concepts and the central concept, resembling the shape of a spoke. The second type (B in Figure 1) is "chain" (Kinchin et al., 2000, p. 47). A chain type of concept map shows a linear structure, developing only a single strand of a specific aspect of the main concept. Both "spoke" and "chain" are not considered good examples of concept maps. The last type (C in Figure 1), "net" (Kinchin et al., 2000, p. 47), shows a hierarchy by developing more specific concepts of each strand from the main concept and also reveals connections between concepts developed from different strands.

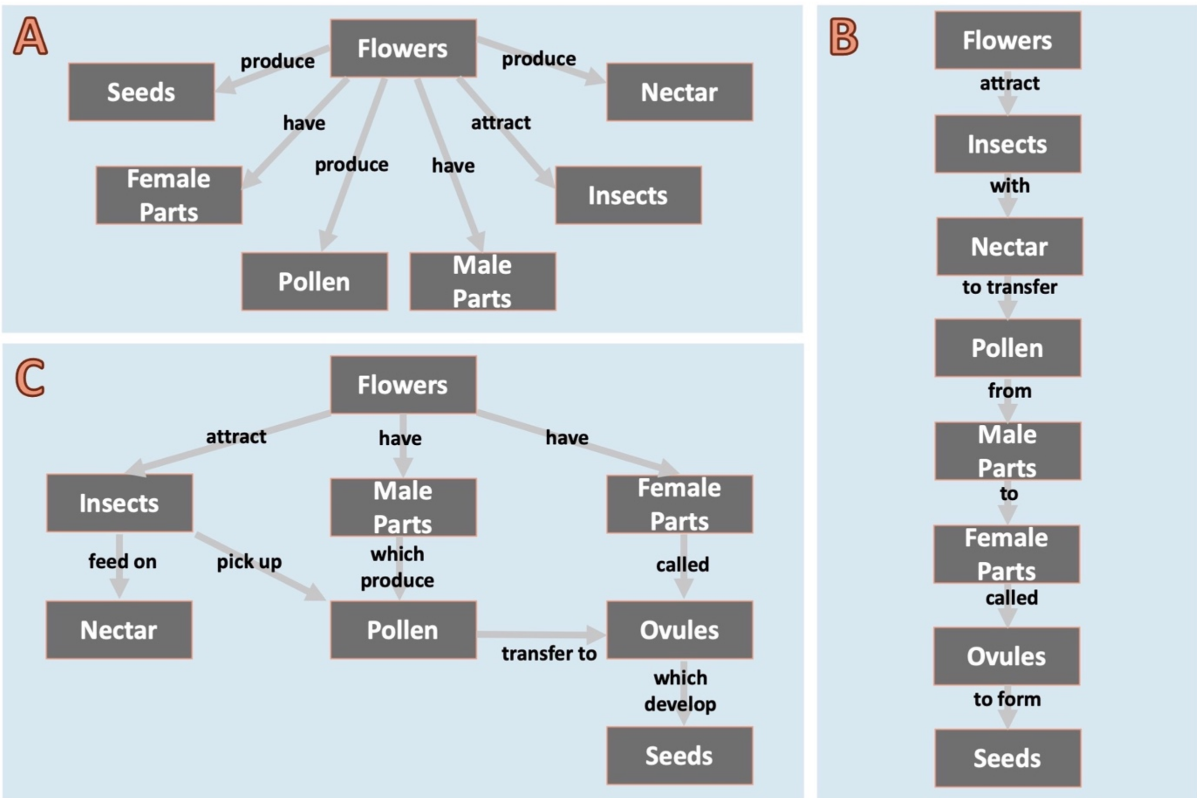


Figure 1: Structural types of concept maps, adapted from Kinchin et al. (2000)

Croasdell et al. (2003) provide very specific methods of evaluating concept maps. The components to be considered are:

- The total number of concepts identified;
- The total number of relationships found;
- The complexity of the map (subtract the minimum number of possible relationships when placed linearly from the total number of relationships used by the mapper);
- Comparison between the student's map and an expert's concept map;

- The progress made by the student during the course by comparing their concept mapping at different time points.

These components were explicitly defined by researchers of concept mapping for the purpose of research investigations to be used as an evaluation tool. However, no research provides a systematic way to evaluate learners' concept mapping activity that can be used in practice for mathematics university education. Addressing this gap, this study reports on the design, development, and implementation of concept mapping as part of a university mathematics course and reports on validation to identify an optimal method for assessing learners' concept maps.

Research questions

The main goal of this study was to find an optimal way that can be used in practice to evaluate concept maps. To achieve this goal, we devised a comparative analysis of students' scores when assigned according to the different rubrics. Specifically, we aimed to answer the following research questions:

- Which of the marking methods is the best predictor of learners' performance, measured as scores on the final exam?
- Which of the chosen marking methods provides the best way to assess student concept mapping activity?

METHODS

Research site

The study was conducted at a large research-intensive university (University of Auckland, New Zealand) in an undergraduate mathematics course covering Calculus II, Linear Algebra II, and Introduction to Ordinary Differential Equations, serving the needs of students majoring in a variety of disciplines. For a large proportion of non-mathematics majors taking this course, a lack of interest in the subject contributes to low intrinsic motivation, and suboptimal engagement with the course. An additional challenge is the size of the course: the enrolment numbers range from 350 to 550 students per semester. The course is delivered over 12 teaching weeks with the following weekly structure: three 1-hour lectures and one 1-hour tutorial (25 to 30 students per room working on problems).

This study was conducted in Semester 1, 2021 (March - June), when the COVID-19 pandemic has affected many places internationally. However, it should be noted that in New Zealand, due to the elimination strategy with closed borders, most educational institutions were functioning as normal from late 2020, with a few exceptions. Specifically, in Semester 1, 2021 at the University of Auckland, most courses were delivered face-to-face except for the first two weeks of the semester.

Participants

In the trial semester, 355 students were enrolled in the course, with 35 students studying overseas and completing the course online. An important component of this course is tutorials, which are practical sessions where students work on provided mathematics problems. All students are required to attend a tutorial each week for ten weeks. In addition, students are expected to submit their solutions to a "marked problem" weekly. A concept mapping task was given in each tutorial as a part of the question set and assigned twice as a marked problem.

Intervention: Knowledge Organisers and concept maps in tutorials

The intervention design was informed by the findings from experimental educational psychology pertaining to the learning-enhancing effect of the use of concept mapping in educational contexts. In each tutorial, students were expected to complete a Knowledge Organiser, starting from Week 2. Moreover, two concept mapping tasks were assigned as “marked problems”: in Week 3, students were asked to create a concept map on Series and in Week 7 on Vector Space. A template for Knowledge Organisers was provided (see Figure 2 on the next page). For a given concept, a Knowledge Organiser tasks students to state the definition of a given concept, provide at least two examples of the concept, state at least one non-example and create a concept map of the concept. The Knowledge Organisers were designed by the first author, with the help of the second author, who had been teaching this course for 15 semesters. The students were given a week to complete each Knowledge Organiser. Out of ten Knowledge Organisers that the students were expected to complete during the course, only two were collected for marking.

A short introductory session on Knowledge Organisers was provided in the first lecture by both authors, and an example of a Knowledge Organiser was uploaded on Canvas (Learning Management System) so that students could access it anytime. However, after receiving the first set of Knowledge Organisers submitted by students, the quality of the submissions revealed that some students did not understand the task clearly. Therefore, another explanation session was provided by a lecturer of the course (second author), during a lecture.

DATA COLLECTION

Data was collected from the learning management system (Canvas), providing marks for course assessment together with scores assigned by a researcher (first author) according to four different rubrics for evaluation of concept mapping activity.

Ethics approval was granted by the University of Auckland Human Participants Ethics Committee on 25/02/2021 for three years (reference Number UAHPEC21976).

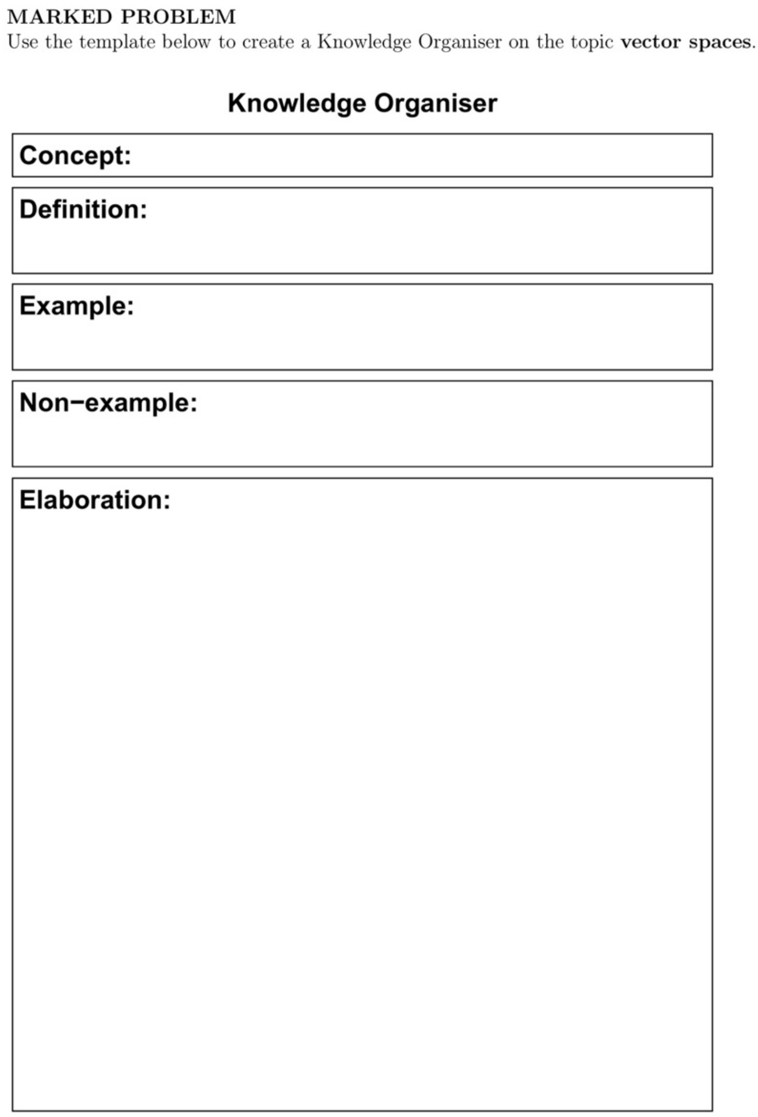
Coursework marks were taken into account for this study. The course assessment structure comprised the following:

- 1 Final exam (50%)
- 1 Mid-semester test (20%)
- 30 Quizzes (15%)
- 10 Marked problems (10%)
- Tutorial participation (5%)

The final exam, the largest assessment component at the end of the semester, was held online. However, the mid-semester test was held on campus in-person (invigilated) for most students except for 35 students who could not enter the country due to the COVID-19 pandemic and had to take the test online. The mid-semester test was excluded from the data due to a significant difference in the performance of the two groups and concern that violation of academic integrity could translate into misleading results. Similarly, tutorial participation marks were excluded from the data due to substantial difference in getting credit for the two groups of students (face-to-face students gained marks for participating in tutorials, whereas online students had to submit written solutions).

The 30 quizzes were short online assessments that were due before each lecture from the second week of teaching. The lowest four marks were dropped. Hence, the data used in this study contains the top 26 quiz marks only.

Finally, the ten marked problems were short written assignments due weekly. Two of the marked problems were on concept maps, one on *Series* and the other on *Vector Space*. The guideline for Knowledge Organisers given to students is shown in Figure 3 below.


MARKED PROBLEM

Use the template below to create a Knowledge Organiser on the topic **vector spaces**.

Knowledge Organiser

Concept:

Definition:

Example:

Non−example:

Elaboration:

Figure 2: Knowledge Organiser template provided to students (Elaboration for concept mapping)

- **Concept**: Name the concept.
- **Definition**: Provide a definition of the concept.
- **Example**: Give two or more examples of the concept that are NOT in the course book.
- **Non-example**: Give at least one example of something similar but not the same as the concept given (NOT from the course book).
- **Elaboration**: Draw a diagram (concept or mind map) about the given concept using other concepts that are known to you, identifying the relations between them to organise and visualise the information.

Figure 3: Guidelines for completion of a Knowledge Organiser given in tutorials

A model solution was provided to the students with a disclaimer that there are many variations of a ‘correct’ concept map.

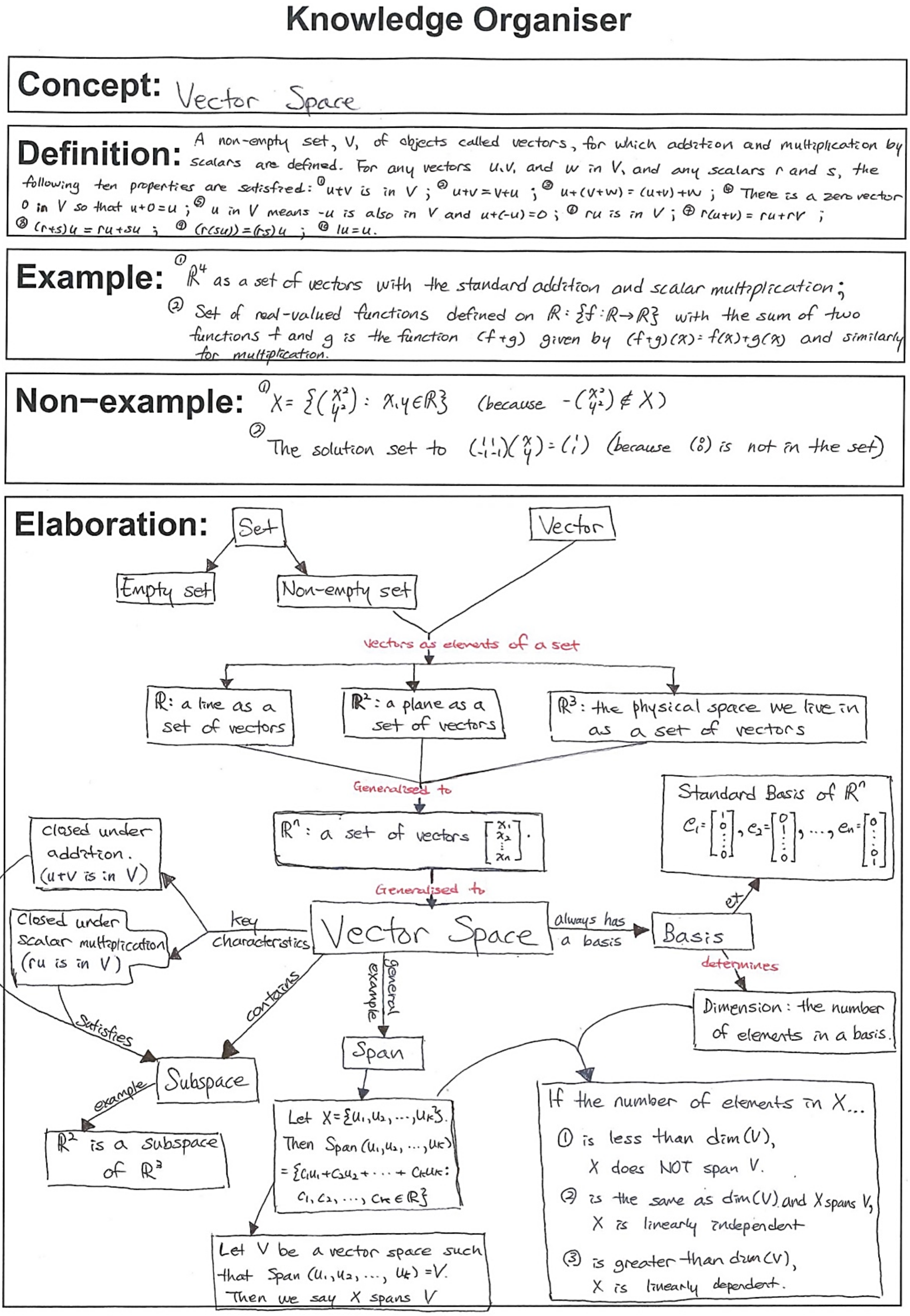


Figure 4: Model solution to a Knowledge Organiser (including concept mapping)

DATA ANALYSIS

As this research focused on finding an optimal way for analysing students' work in concept mapping, four different evaluation methods were selected. Specifically, the following methods suggested by other researchers were adopted with minor alterations to suit the practicalities of our educational context.

Method of Kinchin et al. (2000)

Firstly, the ideas of spoke, chain and net by Kinchin et al. (2000) were manipulated. The maximum marks allowed were 3. It was suggested by Kinchin et al. (2000) to award marks as follows:

- 0/3: Nothing was provided, or the work produced is not in the form of a concept map.
- 1/3: A chain or spoke type of concept map was produced.
- 2/3: A net type of concept map was produced with weakly developed connections.
- 3/3: A net type of concept map was produced with highly developed connections.

However, considering that the students may not have experienced such tasks before, we were concerned that a highly strict marking scheme may cause students to lose motivation. Hence, including "effort" into consideration, a modified marking scheme was produced, which was used in the course:

- 0/3: Nothing was provided, or the work produced is not in the form of a concept map.
- 1/3: A chain or spoke type of concept map was produced with weak evidence of effort.
- 2/3: A chain or spoke type of concept map was produced with strong evidence of effort. OR A net type of concept map was produced with weak evidence of effort
- 3/3: A net type of concept map was produced with sufficient evidence of effort.

Methods of Croasdell et al. (2003)

As previously mentioned, Croasdell et al. (2003) proposed many different aspects to consider in evaluating the quality of concept maps. Out of the five components listed in the Introduction section, only the first three (the number of concepts, the number of relationships, and the map's complexity) were chosen. The complexity is measured by a numeric value, which is equal to the inverse ratio of the number of concepts and the number of relationships between them.

We did not include the last two aspects outlined by Croasdell et al. (2003) (listed in the Introduction section) because of the following reasons; first, comparing the students' maps to a map of an expert seemed to be contradicting the fact that the concept mapping task should be a creative and idiosyncratic process. Also, the last aspect (tracking development through different time points of the course) was not suitable for our study because the concept maps were collected for marking only twice during the course.

The four rubrics used in the study: Structure, Concept Count, Relationship Count and Ratio methods

For convenience, the methods introduced in this section will be replaced by concise descriptions to label the four rubrics compared in this study. The modified method of Kinchin et al. (2000) will be described as the *Structure* rubric. Out of the methods introduced by Croasdell et al. (2003), the evaluation method with a focus on the number of concepts will be referred to as the *Concept Count* rubric, and the method counting the number of relationships will be labelled as the *Relationship Count* rubric. Lastly, the method of analysing the complexity as the inverse ratio between the concepts and the relationships used in concept mapping will be described as the *Ratio* rubric.

Exam concept map question

The final exam comprised 30 multi-choice questions, with one question focusing on concept mapping. The concept map question had a form of a completed concept map with a few missing concepts and relationships for students to figure out. Here is the question:

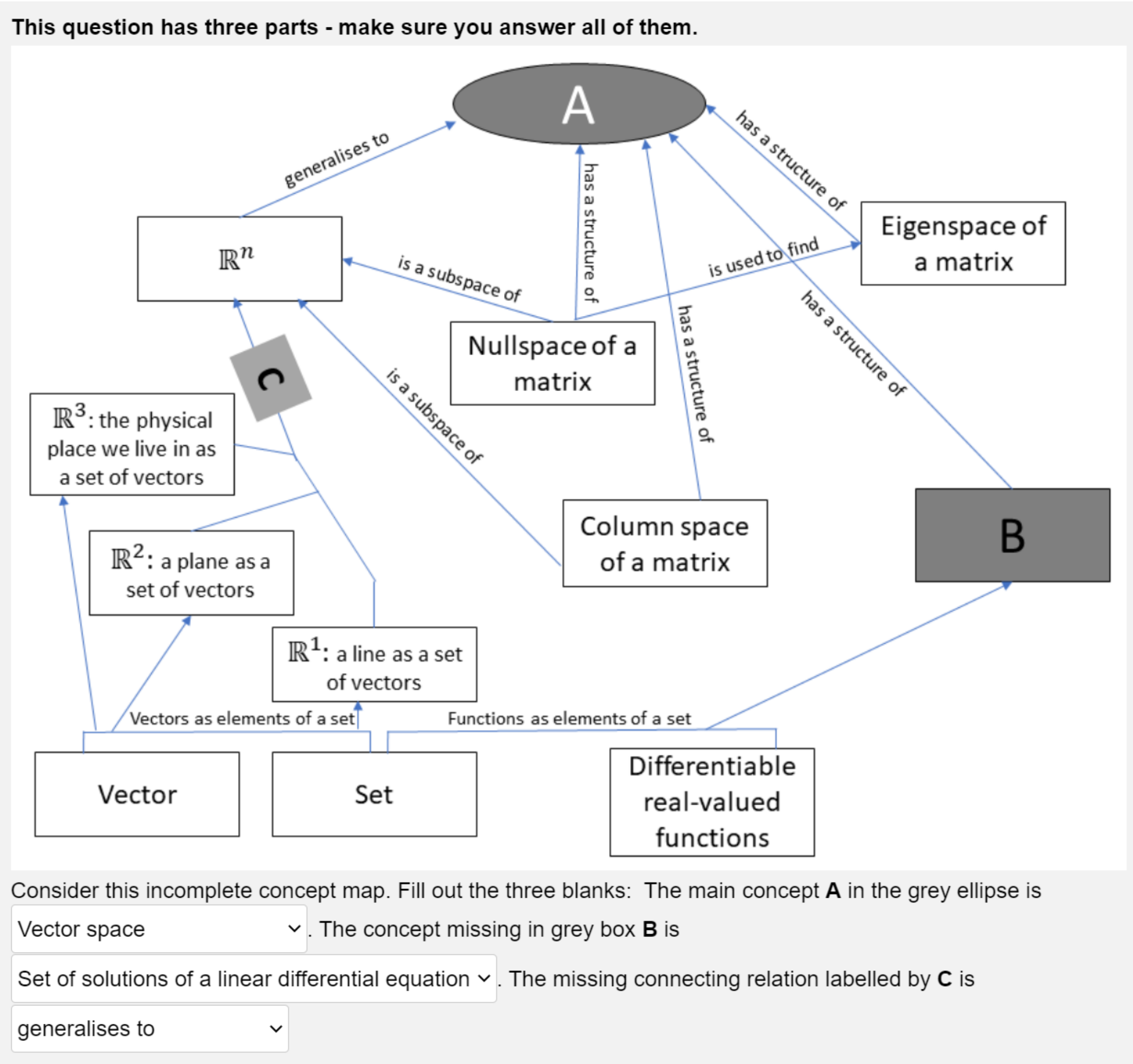


Figure 5: Concept map question in the final exam

Students were presented with three *in-line choice* questions as part of the question with multi-choice options to replace objects A, B and C in the concept map. The multi-choice options given for each of A, B, and C are shown in Figure 6, with the correct answers at the bottom of the lists:

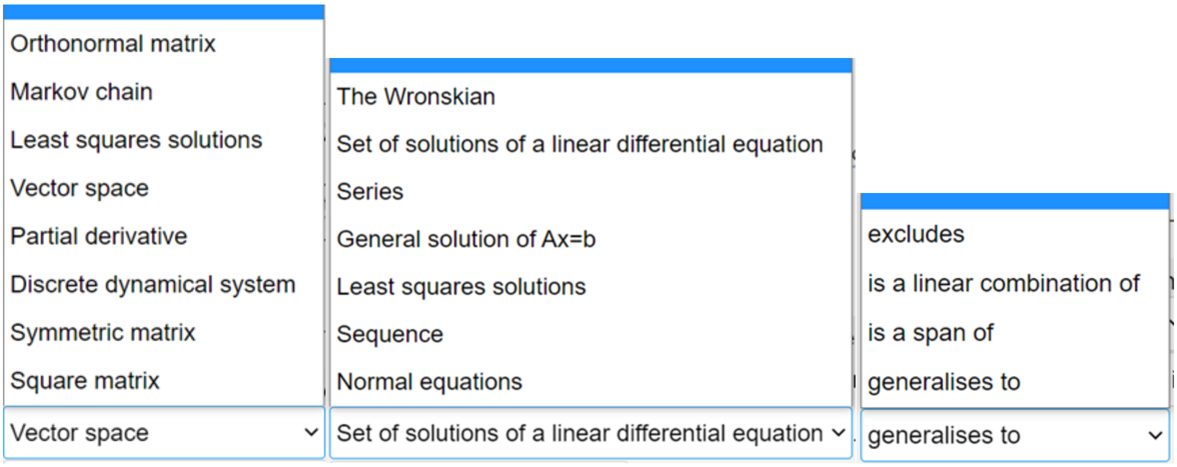


Figure 6: Options for A, B and C

Considering that it may be the first time that students would have encountered such a question in an exam context, a mock exam was provided containing a similar question, allowing for formative practice.

RESULTS AND DISCUSSION

This section compares the four rubrics for the evaluation of concept maps produced by students as part of the coursework assessment by comparing multiple regression models to predict their final exam outcomes. The final exam scores were chosen for this analysis because by constructing one's own concept map, a meaningful engagement will be required from the concept mapper. From research, it is known that concept mapping involves higher-order learning activities such as organising and synthesising and, as a result, it is expected to enable high quality of learning (Nesbit & Adesope, 2006; O'Day & Karpicke, 2020; Schroeder et al., 2018).

The main goal of this analysis was to identify whether the students' performance of the concept mapping task was an appropriate predictor of their overall performance. Thus, the final exam scores were chosen as a measure of their overall performance with separate consideration of student performance on the concept map exam question.

In order to identify the most appropriate marking rubric, a multiple regression was done using scores from different marking rubrics, the total quiz score and the total marked problem scores (excluding the two concept map problems) as the independent variables. The final exam outcomes were selected as the dependent variable. Specifically, two dependent variables were considered separately: the score on the concept map question only and the total final exam score.

Predicting Exam Concept Map Question Score

Table 1 shows the results of the first four tests with the score on the concept map exam question as the dependent variable. After data cleaning, four multiple regressions were run to predict the exam concept map question score from the concept map scores using each evaluation method, the total quiz score and the total marked problem score. The assumptions of linearity, independence of residuals and homoscedasticity were satisfied. There was no evidence of multicollinearity. There was one unusual point, identified by its studentised deleted residuals. However, no problem was found with the values of the data point. Hence, there was no reason to delete the data point. The assumption of normality was met.

Table 1: Multiple regression results for Exam concept map question score (Models 1-4)

Exam concept map question score	*B*	95% CI for *B*		SE *B*	β	R^2	ΔR^2
		LL	UL				
Model 1						.044	.032*
Constant	1.642***	.736	2.547	.460			
Structure method	.100*	.023	.177	.039	.161*		
Total quiz score	-.001	-.013	.011	.006	-.008		
Total marked problem score	.010	-.005	.025	.007	.112		
Model 2						.030	.018
Constant	1.678***	.765	2.591	.464			
Concept count method	.012	-.002	.027	.007	.106		
Total quiz score	.000	-.013	.012	.006	-.006		
Total marked problem score	.011	-.003	.026	.007	.124		
Model 3						.037	.026*
Constant	1.693***	.783	2.602	.462			
Relationship count method	.015*	.002	.028	.007	.139*		
Total quiz score	-.001	-.013	.011	.006	-.009		
Total marked problem score	.011	-.004	.025	.007	.119		
Model 4						.045	.034**
Constant	1.623***	.718	2.528	.459			
Ratio method	.264**	.067	.461	.100	.164**		
Total quiz score	-.001	-.013	.012	.006	-.007		
Total marked problem score	.010	-.004	.025	.007	.117		

Note. Model = "Enter" method is SPSS Statistics; *B* = unstandardized regression coefficient; CI = confidence interval; LL = lower limit; UL = upper limit; SE *B* = standard error of the coefficient; β = standardized coefficient; R^2 = coefficient of determination; ΔR^2 = adjusted R^2. *$p < .05$. **$p < .01$. ***$p < .001$.

Models 1, 3 and 4 were statistically significant in predicting the exam concept map question score, but Model 2 was not. (Model 1: $F(3, 256) = 3.890$, $p = .010$; Model 2: $F(3, 256) = 2.617$, $p = .052$; Model 3: $F(3, 256) = 3.321$, $p = .020$; Model 4: $F(3, 256) = 4.010$, $p = .008$)

In each of models 1,3, and 4, only one of the variables added statistically significantly to the prediction. Those variables were the concept map scores using the *Structure* method ($p = .011$), the *Relationship count* method ($p = .027$) and the *Ratio* method ($p = .009$), respectively. In Model 2, none of the variables added statistically significantly to the prediction.

In conclusion, Model 4 has the best fit with the highest adj. R^2 value, and with the highest value of the unstandardised regression coefficient (B=0.264), thus suggesting that the *Ratio* method of evaluating student concept mapping activity is the most accurate.

Predicting Exam Score

Table 2 shows the results of the four multiple regression tests with the total exam score as the dependant variable, using each of the four evaluation rubrics, the total quiz score and the total marked problem score as independent variables. The assumptions of linearity, independence of residuals and homoscedasticity were satisfied. There was no evidence of multicollinearity. There were no unusual points, and the assumption of normality was met.

Table 2: Multiple regression results for Total Exam score (Models 5-9)

Total exam score	*B*	95% CI for *B*		SE *B*	β	R^2	ΔR^2
		LL	UL				
Model 5						.192	.183***
Constant	6.730***	1.398	12.061	2.707			
Structure method	.560*	.108	1.013	.230	.140*		
Total quiz score	.060	-.011	.131	.036	.126		
Total marked problem score	.167***	.081	.252	.043	.292***		
Model 6						.191	.182***
Constant	7.025*	1.685	12.366	2.712			
Concept count method	.102*	.017	.187	.043	.136*		
Total quiz score	.057	-.014	.129	.036	.120		
Total marked problem score	.172***	.087	.257	.043	.300***		
Model 7						.195	.186***
Constant	7.084**	1.757	12.411	2.705			
Relationship count method	.103**	.026	.179	.039	.152**		
Total quiz score	.057	-.014	.128	.036	.120		
Total marked problem score	.169***	.084	.255	.043	.296***		
Model 8						.201	.192***
Constant	6.604*	1.300	11.907	2.693			
Ratio method	1.738**	.583	2.892	.586	.168**		
Total quiz score	.059	-.012	.130	.036	.124		
Total marked problem score	.168***	.083	.253	.043	.294***		

Note. Model = "Enter" method is SPSS Statistics; *B* = unstandardized regression coefficient; CI = confidence interval; LL = lower limit; UL = upper limit; SE *B* = standard error of the coefficient; β = standardized coefficient; R^2 = coefficient of determination; ΔR^2 = adjusted R^2. $^*p < .05$. $^{**}p < .01$. $^{***}p < .001$.

All of the models 5, 6, 7 and 8 were statistically significant in predicting the total exam score (Model 5: $F(3, 256) = 20.311$, $p < .001$; Model 6: $F(3, 256) = 20.171$, $p < .001$; Model 7: , $F(3, 256) = 20.734$, $p < .001$; Model 8: $F(3, 256) = 21.455$, $p < .001$).

In models 5, 6, 7, and 8, two of the variables added statistically significantly to the prediction. The total marked problem score was significant in all four models ($p < .001$ in all four models). The other significant variable in models 5, 6, 7, and 8 were the concept map scores using the

Structure method (p = .015), the *Concept count* method (p = .019), the *Relationship count* method (p = .009) and the *Ratio* method (p = .003), respectively

Out of the four models, Model 8 reported the highest unstandardised regression coefficient of the evaluation method (Model 5: B=.560; Model 6: B=.102; Model 7: B=.103; Model 8: B=1.738) and the highest adj. R^2, thus suggesting that the *Ratio* method is better suited for marking student concept mapping tasks.

Discussion

Out of the eight tests that were conducted, all models except Model 2 were statistically significant. In models 1, 3 and 4, the concept map scores obtained from different marking rubrics turned out to be the only significant independent variable. In models 5, 6, 7 and 8, the concept map scores from different marking rubrics as well as the total marked problem score were significant independent variables. This is an expected result as the students' concept mapping performance is most likely to be directly related to their ability to recognise correct concepts and relations in the given concept map on the exam. On the other hand, the total marked problem score consists of marks from mathematical problems on different topics. Hence, it is very likely to be a significant predictor of the overall performance on the final exam.

The total quiz score was not a statistically significant predictor in any of the tests. A possible reason for this is that the quizzes were designed to provide an impetus for revision after every lecture so that the frequency of student engagement is increased in order to improve student self-efficacy (Evans et al., 2021; Riegel & Evans, 2021). The questions in the quizzes are academically less demanding compared to creating a concept map or completing a marked problem. Moreover, students were allowed 2 attempts at each quiz, which gave them a higher chance of scoring full marks and the lowest four scores were dropped. Therefore, it is less likely to reflect students' understanding of the mathematical content accurately.

According to the results of these models, the concept map scores from the *Structure* and the *Ratio* rubrics seem to be the best predictors of the students' performance on the exam concept map question. The models using these two marking rubrics showed the highest adjusted R^2 values (Model 1: adj. R^2 = 0.032; Model 4: adj. R^2 = 0.034). Comparing the effects of the different marking rubrics as a predictor of the final exam performance (total), we observed that the *Ratio* method showed the highest adjusted R^2 value of 0.193, with the highest B value (B=1.674).

This shows that the rubric utilising the ratio of the number of concepts and the number of relationships best reflects students' overall mathematical performance. Hence, the most important factor to consider when evaluating students' concept maps is the proportion and not just the count of concepts and connections that a learner comes up with.

Then, why might the *Structure* method scoring also be a significant predictor of the exam concept map question with the second-best indicators? Unlike the concept mapping tasks that the students were given as a marked problem in the tutorials, the exam concept map question required the students to comprehend a concept map that was already nearly completed by another person. Therefore, in this process, it was necessary for students to comprehend the structure of the concept map. Hence, the scores from the *Structure* method were likely to factor heavily in predicting the students' performance in this particular exam question.

CONCLUSION

In this study, we demonstrated how concept mapping activities can be incorporated in a mathematics course and found an optimal method to evaluate concept maps created by students, thus providing recommendations for implementation in practice. Out of the methods tested in this study, the *Ratio* method was identified to be the most comprehensive way of evaluating students' concept mapping. However, in predicting performance on a ready-made concept map on a final exam, two methods appear adequate for assessment: the *Structure* method as well as the *Ratio* method.

The structure method's advantage is that it is not just a formulaic way of evaluating as it captures the various outputs of the concept maps made by the students, such as examples and explanations. However, this method may result in inconsistency in the marks given out since the marking process may be subjective

Taking this consideration into account, we conclude that the *Ratio* method is the most optimal evaluation method. As Novak, the creator of concept maps, outlines in his study, cross-link is one of the main features of concept maps (Novak & Cañas, 2008). This shows that only having many concepts with no cross-links overlooks the major aims of creating concept maps. On the other hand, because the cross-links connect two concepts, it is impossible to have a "relationship" that stands alone without linking any concepts. That is, relationships cannot exist without the presence of concepts. Hence, the *Ratio* method, which considers both features is most likely to evaluate the concept maps accurately. Most importantly, an optimal concept map focuses on finding maximal relationships within the selected concepts. In particular, if there were two concept maps with the same number of concepts, the map with more relationships identified is considered to be of higher quality. In this sense, the *Ratio* method perfectly captures this key feature of concept mapping. However, there is a caveat to consider when a concept map is minimalistic yet has a high inverse ratio of concepts and relations. For example, a concept map with two or three concepts only. In this case, we recommend including evaluation of effort as part of the marking rubric so that students do not use this loophole.

In summary, this study demonstrated successful incorporation of concept mapping activities as part of a large undergraduate mathematics course. It showed promising results pertaining to the utility of concept mapping as a learning-enhancing tool. Students' performance on concept mapping task was a significant predictor of their exam scores. This suggests that further implementation studies could be conducted with a focus (1) on establishing causal relationships between the use of concept mapping and learning outcomes, (2) investigating to what extent an increase in concept mapping activity throughout the semester would enhance student conceptual understanding, (3) researching different ways to incorporate concept mapping activity in a course, for example, as a group task in face-to-face tutorials, enabling collaborative interactions.

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FACILITATING UNDERGRADUATE RESEARCH IN MATHEMATICS ON A VIRTUAL PLATFORM

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KEYWORDS: high-impact practices, undergraduate research, diversity issues

ABSTRACT

All students in mathematics should have access to excellent undergraduate education in a supportive environment. Providing students diverse opportunities early in their career to study mathematics, as practiced by researchers and scientists, is critical. The Summer Undergraduate Research Experience (SURE) program at Kent State University is designed to fund promising undergraduate researchers for eight weeks over the summer to engage in faculty-supervised research. The SURE program supports students with either a $2,800 stipend (40 hours/week) or $1,400 stipend (20 hours/week). In mathematics, statistics, and computer science, selected scholars complete research projects involving Number Theory, Matrix Theory, Probability Theory, Big Data, Artificial Intelligence, Biological Modeling, Graph Theory, and Differential Geometry. During the academic year similar undergraduate research projects continue to be supported through departmental Undergraduate Research Assistant programs as well as Choose Ohio First scholar programs. The reimagined approach to virtual undergraduate research is unique and presents many opportunities and challenges for SURE, URA, and COF scholars. The presentation will highlight ways to build a successful model for undergraduate research with a built-in virtual component. Women and ethnic minorities are underrepresented in mathematics. We posit that properly developed outreach and enrichment programs such as SURE will result in attracting and retaining students across the totality of the population. The virtual environment provides a platform for building a strong model for research using the right technological tools. In the presentation, we will feature methods helpful in creating similar enrichment programs, share sample projects, discuss dissemination of undergraduate research, summarize student feedback, and reflect on ways to implement meaningful evaluation metrics.

INTRODUCTION

Teaching is an enterprise that delivers the current state of knowledge to students. We expend significant amounts of time, resources, and talent to ensure that undergraduates in mathematics do well in beginning mathematics courses and are successful in sequential courses. Academic support programming, bridge programs, effective assessment and evaluation methods, reformed delivery of curriculum, initiatives that engage students, and extensive research on how students learn, are just a handful of scholarly efforts that form the foundational fabric of most undergraduate programs in mathematics across the globe. In the spectrum of tertiary education, these efforts are focused on the lower end to freshman and sophomore students, and rightfully so. Do we have similar vibrant efforts that promote excellence and offer research opportunities to students in the upper end of the spectrum? We believe that the evidence is indefinite.

Research pursues knowledge at the boundaries of our current understanding, constantly venturing into the unknown yet to be discovered. The undergraduates we serve in mathematics benefit from research that faculty pursue, bringing in quality and depth into curriculum. Given the critical interplay between research and undergraduate education, universities that have created dedicated units to promote faculty-student research, have been able to flip the focus to offering research opportunities to upper level mathematics students.

At Kent State University, the Office of Student Research (OSR) was established to honor the institution's commitment to support research endeavors of both undergraduate and graduate students. The work of the office is funded through the university and private donors. Projects capable of sustaining quality research between faculty and students require funding on a steady scale. Thus, internal funding of research is justified.

The goal of this article is to address two fundamental questions that are critical to developing and sustaining undergraduate research programs in mathematics. The first idea is how to create a learning environment in which undergraduate research is valued as a high-impact practice that unfolds inclusiveness and promotes excellence. The second idea is how to design outcomes that measure the effectiveness and sustainability of the research programs. These two fundamental questions need to be raised and addressed in a cyclical continuum even for well-founded programs (Gallian, 2007). On the first idea, the importance of creating high-impact practices, such as undergraduate research experiences, is well established (Kuh, 2008; Zhao & Guh, 2004). A sure way to promote excellence is to intentionally create a learning community with a robust support system (Treisman, 1992, Kasturiarachi, 2004). A niggle that remains lurking, one which we as mathematicians try to resolve, is the lack of representation of women and minorities in our endeavor. While addressing this issue is critical, it is not easy to do, for it requires efforts for reclaiming cognitive bandwidth in students lost most often due to poverty, racism, and social marginalization (Verschelden, 2017). This is where the second idea of establishing vigorous outcomes enters, providing necessary space for research programs to improve and become engrained in university culture.

The article is organized as follows. We begin by describing the structure of programs we wish to highlight followed by important elements in recruiting students. Next, we share three sample projects and end with a discussion of outcomes and student feedback.

STRUCTURE OF PROGRAMS

There are many excellent external summer research opportunities for undergraduates. One of the best funded and highly successful programs is the Research Experiences for Undergraduates (REU) supported by the National Science Foundation (NSF). There are approximately 1800 REU programs nationwide, of which about 180 are newly funded (NSF REU Programs, n.d.). The REU programs are highly competitive in nature and the best students in STEM programs enter into the small cohorts at a site to complete a specific research experience for 10 weeks in summer. Mathematics curriculum across institutions varies in depth and emphasis, yet there is a high degree of agreement on how to present and integrate concepts in mathematics courses (National Research Council, 2013). In most mathematics programs students are required to complete a research or experiential learning unit, or both, prior to graduation. Competitive external undergraduate research opportunities, while meeting the needs of the top echelon of undergraduates, is far from sufficient to meet local needs of the university community. Institutions must be proactive. At Kent State University there are three programs designed to support undergraduate research. The Summer Undergraduate Research Experiences (SURE) and the Undergraduate Research Experience (URA) programs are funded through the university. The Choose Ohio First (COF) scholar research is a requirement built into a scholarship grant program funded by the Ohio Department of Higher Education (ODHE). All three programs have similar goals: promoting, supporting, and sustaining undergraduate research.

SURE Program

The SURE program is funded by the university through the Office of Sponsored Programs with additional funding through donor support. The strategic goals of the SURE program include building research capabilities both local and global, expanding the economic impact, creating a culture of student research by expanding outreach, and fostering partnerships with local industry and government entities. The university has a dedicated 41,000 square-foot Centennial Research Park that provides space and support for specialized start-up companies to conduct research. With such dedicated space acting as an incubator, undergraduate research opportunities have increased. The SURE program provides a $2,800 stipend to students who work on a research project with a faculty member for eight weeks during summer. A subset of students is chosen to complete a shorter research experience and are funded at $1,400. Additionally, faculty facilitators can request up to $400 to cover expenses related to the project. Being a student-centered program, faculty are usually not remunerated, although merit points are awarded through the respective department for their effort.

Each academic year a call for proposal is sent out to all faculty in the university who engage in undergraduate research. A faculty member is expected to write the proposal with the nominating student. The faculty member usually provides the outline of the proposed research and the student completes a statement of interest as part of the proposal. Proposals are evaluated by a team of expert faculty and staff members. Established as a response to the University's Strategic Priorities and its commitment to "student first," the Office of Student Research (OSR) continues to support the research endeavors of undergraduate and graduate students at all campuses of Kent State University. Although the program is relatively new it has seen a steady growth over several years as depicted in Figure 1. The program is currently engaging 70 students who represent 35 majors from five campuses of Kent State University. Of the participants, 36% are first generation and 75% have federal financial need. Under-represented students make up 13% of the cohort. Approximately 62% of the students are engaged full-time undergraduate research and 38% part-time working both on and off-campus.

All students who engage in the summer research experience are expected to present their research in a 3-minute presentation in the following semester. This mini-conference is held both in-person and virtually. Three-minute thesis presentations are popular in many parts of the world (e.g., Three Minute Thesis, University of Queensland, n.d.). During the Covid-19 period the virtual presentations have proven to be a success with more students and faculty being able to join the discussions. Each student presentation is scored by the judges on communication (10 points), engagement (5 points), content (20 points), and comprehension (10 points). Thirty monitory awards are given out in 12 disciplinary categories for the best presentations during a follow up luncheon. The 3-minute faculty advisors work with their mentees to present at the annual Undergraduate Research Symposium sponsored by the university. In this forum undergraduates from all disciplines and programs across the university present their research in a formal conference setting. The fruitful faculty-student partnerships usually result in student-led publications, further enhancing the value of this high-impact practice.

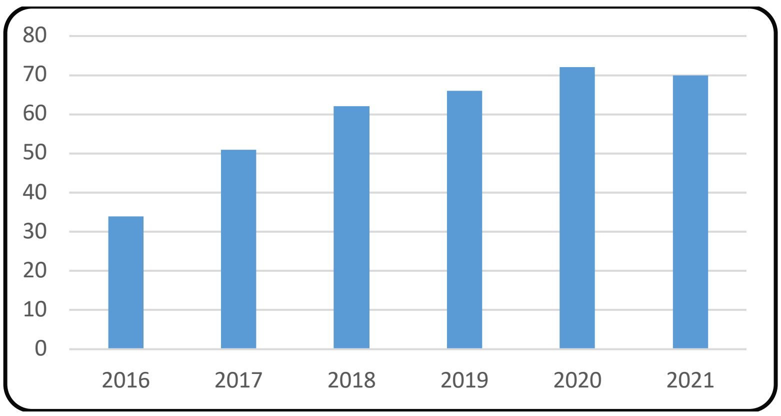

Figure 1: SURE student count 2016-2021

URA Program

The Undergraduate Research Assistant program is a university wide initiative that every department and academic unit is expected to support through internal funding during the academic year, including summer. Typically, each semester a department funds five to ten 10-week research projects. The number of awards and the participating departments vary depending on available funding in that unit. The selection process is competitive. Both the faculty sponsor and the student have to submit applications that outline the proposed research, a timeline, and a plan to measure outcomes. The proposals are evaluated by a faculty committee and recommendations are forwarded to the department chair or director of the unit. The funding for a URA can vary between $500-$1000 for the 10-week period. Facilitating faculty members are expected to submit a final report at the conclusion of the research project. Several research projects result in publications that focus on undergraduate research. All URA scholars are expected to present their research at the annual Undergraduate Research Symposium or at local departmental seminars.

COF Program

The Choose Ohio First (COF) scholarship program is a competitive grant program sponsored by the Ohio Department of Higher Education (ODHE) designed to significantly strengthen STEM education by providing funding to undergraduate and graduate students enrolled in competitive academic programs. The COF program is a conduit that integrates statewide regional economic needs with innovation educational programs. Recruiting underrepresented STEM student groups including women and students of color is a priority of the COF program (Choose Ohio First, n.d.).

Kent State University's eight campus system, with its large footprint in the state of Ohio, has served as a leading university in the COF program. It has been a partner in the effort to bolster Ohio's economy by contributing towards a stronger workforce in STEM related industries.

By producing the greatest number of graduates of any of the universities in Northeast Ohio, Kent State University's collaboration with the COF program has provided life-changing experiences for its signature "student first" core value. As part of the requirements of the COF program, we have implemented the following initiatives.

- Student support programs to improve retention and graduation rates
- Enhance learning through participation in research and business internship practices
- Refined the learning outcomes in key courses of targeted programs

Undergraduate research plays a key role in helping COF scholars make the right connections with industry. Faculty members are encouraged and supported to produce research proposals that appeal to area industry who partner with Kent State University through the Centennial Research Park. There are many research projects that are accessible to mathematics and computer science students (Friedman & Littman, 1994). Designing a good project can be challenging. If the design is done with students in mind, with a virtual networking component, a good project can turn into a better one. During the research semester COF scholars work with a faculty mentor on a selected research topic. They learn the basic principles for conducting successful research in their major field of study. Speakers are invited from Ohio industry, including COF alumni, who advertise internship positions at their companies as well as share their own experiences in securing such opportunities. The COF research project culminates with a presentation (short talk or poster) at the annual COF Conference, which was held virtually for the first time in 2021 (COF Conference, 2021). The COF conference attracts students from seven universities and covers presentations in all STEM fields. Through this conference scholars present their research projects to the university community and industrial partners. The number of presentations at the conference have grown steadily over the past five years and in 2021 saw the highest number of participants (442) engaged in 148 presentations in all STEM areas covered by COF. Of these 40% were in mathematics and computer science.

Conducting research and disseminating results can be accomplished on two quite distinct platforms. The first is the traditional setting where research is conducted in-person with weekly meetings and presentation of results following a live format. This modality is ideal for localized research projects that capture the best internal resources derived from many sources within the university. These projects should, in the best of all possible worlds, serve to showcase the university's research strength, justify rewards and incentives, and underlie future budget allocations that express the values of the institution.

The second is the new, subtle, and equally valuable, virtual platform. By moving to a virtual research platform out of necessity, we have identified components that can remain as permanent features of all three programs, allowing for the blending of traditional experiences with virtual opportunities. The most striking features that we wish to maintain in all three programs are:

- ability to share and edit documents on a virtual server;
- build an additional weekly virtual meeting as an extra point of touch;
- provide students the flexibility of working off-site;
- cultivating important virtual presentation skills; and
- include outside experts who can contribute to the research project virtually

STUDENT RECRUITMENT AND SELECTION

All three undergraduate research programs described in the previous section have a rigorous application and selection process. Students who are in their second or third year are encouraged to apply for undergraduate research opportunities. The mathematics program at Kent State University has a Writing Intensive Course (WIC) requirement and an Experiential Learning Requirement (ELR) built into the curriculum. Figure 2 depicts the organization of both requirements.

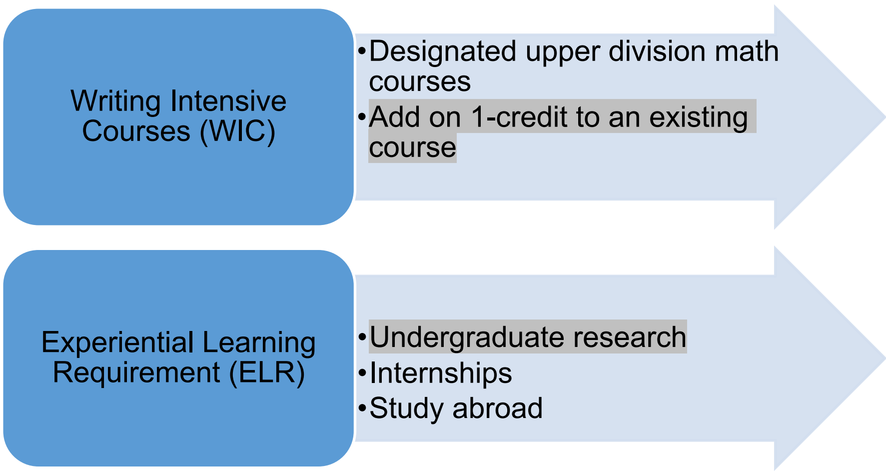

Figure 2: Credit bearing undergraduate research

In Table 1 we list the specific courses that have the WIC or ELR designation. These courses are often springboards from which students launch into a research project in mathematics.

Table 1: ELR and WIC courses

Course	Type	Course	Type
Math 19099 Field Exp. in Math	ELR	Math 21092 Computer practicum	ELR
Math 40055 Actuarial Math	ELR & WIC	Math 41001 Modern Algebra I	ELR & WIC
Math 41002 Modern Algebra II	ELR & WIC	Math 42001 Analysis I	ELR & WIC
Math 42002 Analysis II	ELR & WIC	Math 42039 Modeling projects	ELR & WIC
Math 49992 Internship in Math	ELR	Math 49998 Research in Math	ELR

When designing an undergraduate research initiative, it is important to know the size of your potential pool. For purposes of comparison we have provided, in Figure 3, the three-year enrollment data for mathematics and its companion computer science.

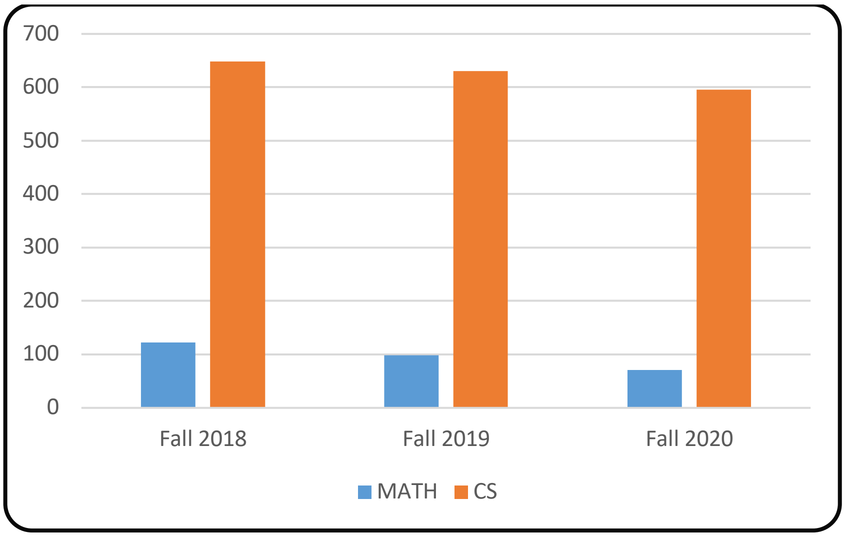

Figure 3: Students majoring in Mathematics and Computer Science

Kent State University has been a leader in promoting multicultural initiatives across every unit of the institution. All three undergraduate research programs have a history of engaging low-income, first generation, and under-represented students. The SURE program has implemented a process to solicit and recognize projects that best address success and excellence for traditionally marginalized populations. By providing culturally aware and appropriate research mentoring support to under-represented faculty we have been able to attract more diverse students into our research programs. Workshops have been offered to prepare SURE mentors to more adequately address the needs of first-generation, under-represented and low-income students.

In Figures 4 and 5 we have shared the demographic breakdown of mathematics and computer science students. It is clear from this data that our approach to diversify both faculty mentors and research scholars is necessary and timely. The virtual platform has enabled us to be innovative in tailoring our research programs to attract and support women and underrepresented minorities in mathematics and computer science. The work is ongoing.

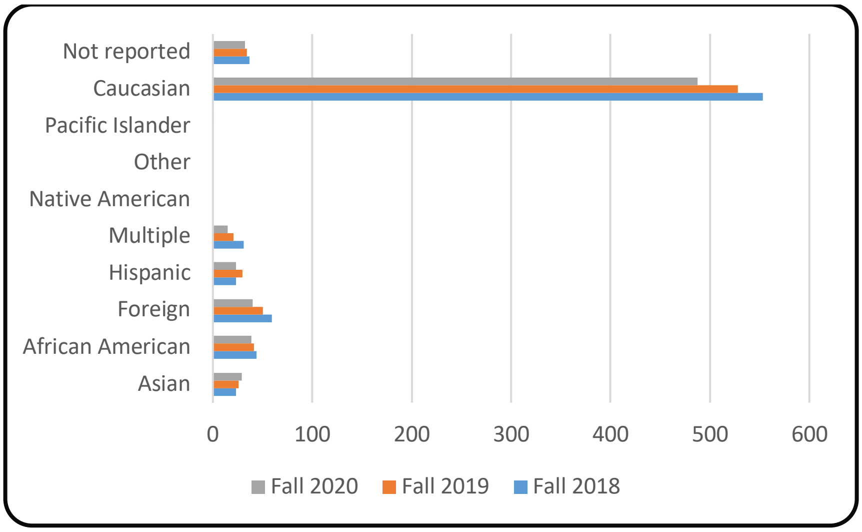

Figure 4: Combined Demographics in Mathematics and Computer Science

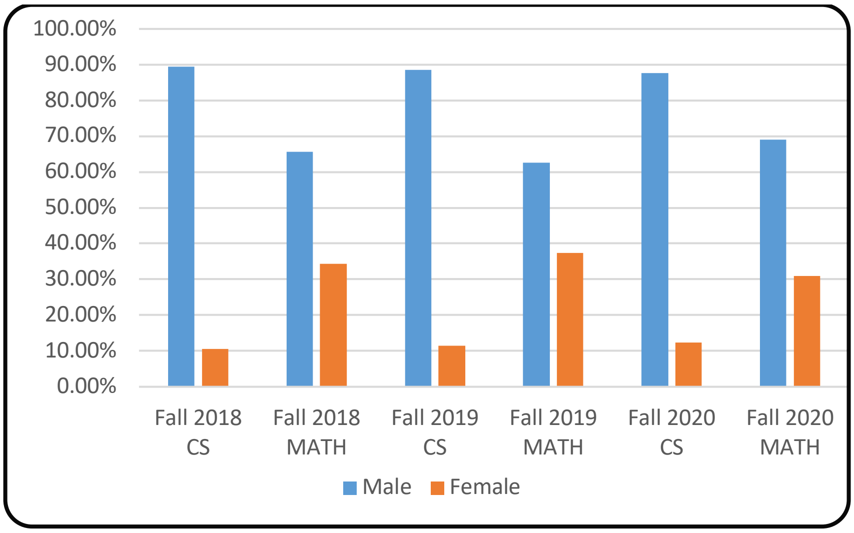

Figure 5: Gender distribution in Mathematics and Computer Science

SAMPLE PROJECTS

The undergraduate research programs we have highlighted cover multiple disciplines. The SURE program is open to STEM, social sciences, and fine arts students. The local URA programs are similarly open to all disciplines served by a department or unit such as a regional campus. The COF program is limited only to STEM students. A large percentage (34%) of COF research projects in the seven-consortium university partnership in northeast Ohio are in mathematics, mathematics education, probability & statistics, and computer science. In these areas of study, technology plays a dominant role in supporting inquiry and affecting virtual research. Most mathematics and computer science research projects include programming, computer algebra systems, mathematical writing software such as LaTeX, and statistical programming in *R*©. Even in pure mathematics research projects, students are encouraged to be industrious to experiment with simpler cases, explore counter examples, produce visual representations, and write simple code, just so they gain a deeper understanding of abstraction. This style of influencing contemporary lines of inquiry is deeply embedded in mathematics education research according to Dubinsky and Tall (1991), giving the novice researcher a wide entry to journey through abstraction.

We present three sample projects successfully completed by undergraduates during the academic year 2020-2021.

Excursions in huddle numbers

It is known that the sum equals product equation $a_1 + a_2 + \cdots + a_n = a_1 a_2 \cdots a_n$ has integer solutions. This equation is also known as the huddle equation. For example, $a_i = 1$ *for* $1 \leq i \leq n-2$, $a_{n-1} = 2$, $a_n = n$, is a solution to the equation (Guy, 1994). Many interesting results exist for integer solutions. Of course, if we allow for non-integer solutions, there are an array of solutions one can consider. For example, in the $n = 3$ case, a simple trigonometric identity for a plane triangle provides another non integer solution. Let the interior angles of the triangle be α_1, α_2, and α_3. The following identity connects the sum of tangents to their product. Namely, $\tan\alpha_1 + \tan\alpha_2 + \tan\alpha_3 = \tan\alpha_1 \tan\alpha_2 \tan\alpha_3$. Motivated by work completed along this thread (Kasturiarachi, 2021), a SURE student was assigned the task of investigating if results similar to integer and non-integer solutions could be extended to Gaussian integers. In Figure 6 we have depicted a few new results to the huddle equation with Gaussian integers that will soon be published in an undergraduate research journal.

Huddle Equation

For $n > 2$, the positive integer solutions to the huddle equation,

$$a_1 \cdot a_2 \cdot \cdots \cdot a_n = a_1 + a_2 + \cdots + a_n,$$

are known. Several results have been extended to real numbers as well. In a SURE project (summer 2021) the following new results were proven by a student for Gaussian integers.

Theorem

If n is odd, there are infinitely many Gaussian integer solutions to the huddle equation.

Theorem

If n is even, there are at least two non-unit Gaussian integer solutions to the huddle equation. Moreover, there are finitely many solutions in this case.

Figure 6: Research project: huddle equation

Geometric portraits of infinite series

Leonardo da Vinci once said of the infinite “What is that thing which does not give itself, and which if it were to give itself would not exist? It is the infinite!” The study of infinite series is fascinating. Infinity is where things happen that don’t. Infinite series offer a window into the study of infinity. Even though the terms of the harmonic series get smaller and smaller, the series itself diverges because its terms do not get smaller faster, so the series refuses to converge. If the harmonic series is the most notable among divergent series, the geometric series gets the distinction as the most common convergent series when the ratio r satisfies $|r| < 1$. There are many other interesting convergent series that appear throughout calculus and number theory. The practice of modern mathematics is to study these convergent series from a purely algebraic point of view. The primary goal of this research project was to look at convergent series from a "geometric" viewpoint (Kobayashi, 2014). The basic question we raised was, does a convergent series have a geometric interpretation? To state it differently, can we create a geometric portrait for a given convergent series? The research project created geometric portraits, along the lines of "proof without words," for standard convergent series as well as several intriguing series that arise in number theory. An example of a geometric portrait is given in Figure 7 for the standard geometric series with initial value $a = 1/4$ and ratio $r = 1/4$ (and whose sum is $1/3$). In Figure 7, $n = 4$ refers to four circles necessary to capture the ratio $r = 1/4$.

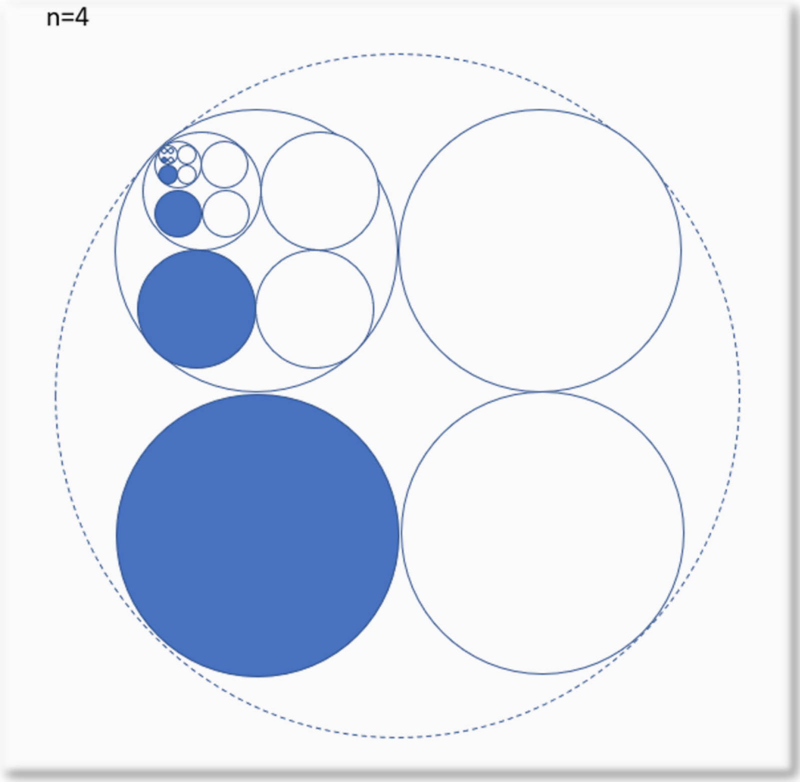

Figure 7: Research project: Portrait of geometric series with *a = ¼ and r = ¼*

Heat-mapping Covid-19 data

In summer 2020 a COF scholar in Computer Science was assigned a research project through the SURE program with the author as the supervising faculty member. As Covid-19 positivity rates were coming in and the death rates were being reported, the task was to design a real time heat-map that tracked the Covid-19 infections and death rates in all 88 counties in Ohio initially and all 50 states eventually. The data was reported daily by the Department of Health in Ohio and was similarly done for all other 49 states daily by the Center for Disease Control (CDC). Accessing this daily public data set was easy. We used a script written in *R*® to capture and filer the data and covert it to a heat-map with colors ranging from white to shades of red. The white representing low or no infections while the shades of red indicating the severity of infections and deaths. As the days, weeks, and months dragged on and Covid-19 infections and death rates mounted, we added a slider to each state's heat map so the end user could see the time evolution of the disease, both by county and state (Figure 8). This website was shared with the Department of Health in Ohio.

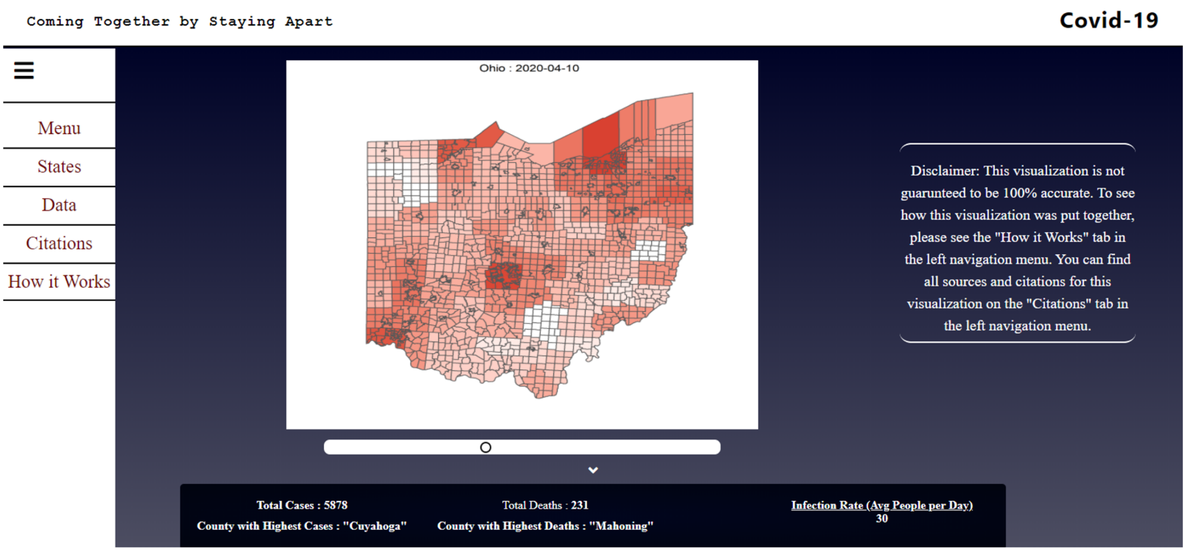

Figure 8: Heat-mapping of Covid-19 infection and deaths by county and state

OUTCOMES AND DISCUSSION

Undergraduate research in mathematics is considered a high-impact practice proven to be an invaluable part of mathematics education programs. Faculty who engage in mathematics undergraduate research and directors who manage such programs have a responsibility to provide evidence of effectiveness. Students who engage in research have high grades and matriculate on time. Because these traditional measures of success are a given, we need to reimagine ways to capture concrete evidence to assess the impact of research programs.

In order to measure outcomes of research programs we need to create an environment that evaluates the mathematics itself. This can be done at two levels. The first is gauging the social and societal impact and the second is the mathematical output.

Social and societal impact

All three programs highlighted in this article have information on student expectations as part of the initial research proposal as well as post surveys that measure the change in students' perspectives over the duration of the research experience. The SURE and COF programs are designed to ensure that each cohort is operating as a learning community. This affords students to network with each other and share resources. It also provides opportunities for faculty and administrators to establish connections with industry through internships, provide resources and support for graduate school applications, and engage the students in a culture of research. These characteristics form the core of social and societal impact of undergraduate research programs. It is worth noting that sustaining undergraduate research and making it a bedrock of the student experience requires steadfast support from the university, as has been established at Kent State University for SURE and URA programs. On a broader scale, external partnerships are vital to cultivate societal connections. The COF program is designed with the intention of creating a workforce that can inject talent to industry in Ohio. The COF scholarship grant funds require a one to one match from the university, bolstering the necessary synergy that improves outcomes.

Mathematical output

Assessing the direct impact of undergraduate research project can be done by measuring the mathematical output. Any activity that is directly linked to the research program needs to be documented. This includes articles submitted to journals that focus on undergraduate research, presentations at conferences such as MathFest-- Mathematical Association of America's annual conference, local undergraduate research symposia, and internships that are generated through industrial partnerships. The final report that is required to be submitted for each project encapsulates the mathematical output.

The COF program has most of the above benchmarks built into its structure. The undergraduate research completed through the COF program almost always leads to a presentation at the annual COF scholar showcase. In 2021, there were 442 scholars from seven institutions that gave 148 presentations or posters at the conference.

Similarly, the SURE program has seen a significant growth in sharing of undergraduate research and creative activities. There was a drop in the number of presentations in 2021 due to Covid-19. However, since we had already moved to a blended platform in 2020, the all virtual 2021 symposium has shown promise. Having the option for presentations to be virtual increases synchronous audience participation and allows for post conference asynchronous access. Figure 9 depicts the growth of undergraduate research and creative activity presentations at Kent State University.

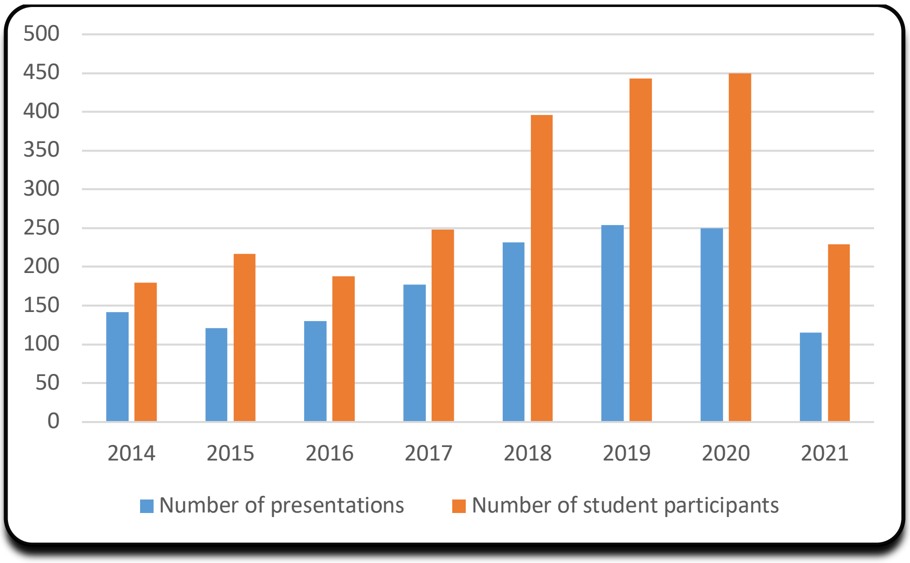

Figure 9: Undergraduate research and creative activity

Student feedback

Student evaluation of the research programs are conducted at the end of their experience. We also have extensive surveys administered to all freshman and seniors in STEM fields. Few questions are set on a Likert scale while most are open ended. A sample of student feedback (verbatim) is provided in Table 2 for SURE and Table 3 for COF.

Table 2: Sample student comments: SURE

SURE program comments	
I am incredibly thankful for the SURE program. Through my participation as an undergraduate, I found my passion for research. I loved the work I did so much that I ended up applying for a PhD program. I love the path I am on and I wouldn't be here today if it wasn't for the SURE program.	*I now have confidence to apply for internships and jobs and feel like I have some experience to make me a good candidate.*

Being paid to do research during the summer. I love learning, but being paid made it so I could actually devote time to research instead of work and school and that was awesome.	*Learning about the research topic and about the research process. It was challenging and a great test of my cognitive capabilities.*

Table 3: Sample student comments: COF

COF program comments	
This program allowed me to complete research on topics that will help me in my future career as a teacher. I also developed friendships with my peers that were of the similar major as I was.	*COF not only helped to fund my undergraduate education, it provided me with the research experience needed to be successful in graduate school and beyond.*
COF has always been a great environment, not only for exploring my extracurricular research interests, but also for improving my skills in communication, team building, work ethic, leadership, and for helping me network with other academics and professionals in my field. Not to mention that our COF organizers have always be extremely helpful and awesome!	*This program has really helped me out financially. Without it, I think I would have dropped out of school or I would have had so many loans to pay off. Not only that but I've always had a passion for math. I started out as math education and realized right away that I didn't want to be a teacher. This scholarship motivated me to get a degree in Pure Mathematics which challenged me to do something I never thought I was capable of doing.*
Choose Ohio First gave me the chance to work with likeminded peers and to learn from those experienced in navigating and contributing to my desired field. It has made me feel like what I am studying is important to the world and like it is productive and worthwhile.	*This scholarship has really done so much for me. Because of COF, I will be graduating Spring 2022 debt free. It has helped me to make some very close friends and has allowed me to explore different careers paths for myself. It is also the reason that I was able to obtain my Summer internship.*

Measuring outcomes in undergraduate research programs is developing and evolving. Much needs to be done. As an institution we will focus on the following ideas as future goals.

- Explore ways to expanding funding sources for undergraduate research, both internal and external
- Provide opportunities to engage and disseminate research utilizing available virtual options
- Expand the outreach to include more women and underrepresented group to STEM undergraduate research
- Create a cohort of student research ambassadors who will promote the benefits of research experiences to rising juniors
- Increase social presence and maintain an asynchronous depository of research presentations and resources for student access

CONCLUSION

Undergraduate research programs play a vital role in creating opportunities for students majoring in mathematics to engage in a culture of inquiry. In the design of undergraduate research programs, creating inclusive learning environments that promote excellence, and measuring outcomes effectively, are necessary for the growth and sustainability of the programs. Using virtual engagement as an integral part of the research platform, students are able to perform, engage, and thrive in a university culture that values discovery.

ACKNOWLEDGEMENTS

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RANDOM VARIABLES IN STUDENTS' DEVELOPMENT OF PROBABILISTIC THINKING IN A MODELING SETTING

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KEYWORDS: Probabilistic Thinking, Random Variables, Modeling, SOLO taxonomy

ABSTRACT

This study is part of a long-term project whose aim is to investigate the development of students' mathematical thinking about probabilistic models by examining the interaction between their intuitive reasoning and the mathematical models they construct in given modeling contexts. We are particularly interested in how students learn to incorporate random variables into their models. The purpose of this study is to develop an empirically supported theoretical framework for classifying student responses during modeling activities into levels and to develop hypotheses regarding how transitions between different levels of response can be facilitated. The framework we develop for analyzing levels of student responses is based on the Structure of Observed Learning Outcome (SOLO) taxonomy. We apply this framework to data from a modeling activity implemented in an introductory graduate probability course that is also taken by advanced undergraduates. We show that our classifying framework is easy to apply in this setting and the results of doing so show that increases in the levels of responses may be precipitated by instructor prompts suggesting the inclusion of randomness in the model and the use of random variables (as opposed to distributions) as the primary representation of randomness in the model.

INTRODUCTION

This study is part of a long-term project whose aim to investigate the development of students' mathematical thinking about probabilistic models by examining the interaction between their intuitive reasoning and the mathematical models they construct in given modeling contexts. Our focus is on a population of students that we will call "prospective probabilistic modelers", in analogy with "prospective mathematics teachers". Just as prospective mathematics teachers are on a career trajectory that will lead to them teaching professionally, "prospective probabilistic modelers" are on career trajectories that will lead to them using probabilistic models professionally and they need specialized knowledge about probabilistic models. Probabilistic modeling in the professional setting, which heavily relies on the use of random variables in models, is well developed and in many areas there are standard models and modeling techniques that one must be familiar with to work in those areas. For example, the standard models in actuarial science in Australia are prescribed by the Institute of Actuaries of Australia. Our overarching goal is to investigate students' development and use of specific probabilistic constructions and heuristics that many of them will need to use in professional settings after graduation.

The purpose of this study is to develop an empirically supported theoretical framework for classifying student responses during modeling activities into levels and to develop hypotheses regarding how transitions between different levels of responses can be facilitated and the role of random variables in these transitions. To accomplish this goal, we implemented an infectious disease modeling activity in a probability course at a midsized university in the United States that is taken by graduate students and advanced undergraduate students. The activity we implemented guided students to reinvent a common method for agent-based modeling of infectious disease.

The following research question guided our study:

RQ: How does students' probabilistic thinking about real-world models develop at the undergraduate and graduate levels?

To address this question, we adopted the Structure of Observed Learning Outcome (SOLO) taxonomy (Biggs & Collis, 1982 & 1991), which has used in many existing studies about the development of students' probabilistic thinking (Mooney et al., 2014). We operationalized levels in SOLO taxonomy in the context of our study and our classification of students' responses throughout the modeling activity showed how they progressed through levels of probabilistic thinking by incorporating random variables in their models as they proceeded with the modeling activity.

THEORETICAL BACKGROUND

Probabilistic Knowledge

The study of probabilistic knowledge has a long history in the education literature generally and the mathematics education literature specifically, see Chernoff and Sriraman (2014); Batanero and Chernoff (2018). Probability and statistics play ever increasing roles in modern society as big-data driven probabilistic and statistical approaches become incorporated, beneficially or harmfully, into every-day life (O'Neil, 2016). Moreover, unique features of probability as a domain of knowledge (Batanero & Chernoff, 2018, p. vi) make it a rich environment for educational studies.

How prospective probabilistic modelers construct probabilistic models is an understudied topic in the research literature. Rather than being focused on how to construct and analyze probabilistic models, which is a core activity of professional probabilistic modelers, as noted in (Pfannkuch et al., 2016) "the main focus and concern in the literature has been on the ways that one can think about probability and on probability conceptions and misconceptions'" (p. 12). There has been some compelling recent work looking at prospective probabilistic modelers (e.g., Pfannkuch et al., 2016; Pfannkuch & Budgett, 2016; Budgett & Pfannkuch, 2018) which focus on introducing ideas about modeling, developed based on interviews with professional modelers, into the teaching of isolated probabilistic ideas that form a standard part of the undergraduate probability curriculum (Poisson processes and Markov chains).

Understanding how students learn probabilistic modeling is tremendously important due to the real-world applications of probabilistic modeling. For example, in Taleb (2007), published the year before the global financial crisis of 2008, Taleb argued that probabilistic financial models did not sufficiently account for the risk of rare, extremely adverse events and, as a result, traders were significantly underestimating the risk of their positions. Probabilistic models also play important roles in quantitative approaches to epidemiology, medicine, industrial design, and computer science. By studying how students approach probabilistic modeling in the final stages of their education before they enter professions that rely on these models, this project hopes to build foundational knowledge necessary to improve the workforce readiness of graduating students entering these fields.

Modeling

Existing studies have established the importance of modeling in students' development of mathematical thinking for not only learning of mathematics but also for developing critical thinking for decision making in daily life (e.g., McNair, 2000). Real-life modeling contexts enable the formation of mathematical concepts and models, and thus different levels of formalized knowledge, provide wide applications, and promote collaborative learning through students' interactions and contributions (Dapueto & Parenti, 1999; De Lange, 1987 &1996). Specifically, modeling experiences help students build mathematical thinking through their iterative and emergent development of complex ideas in which they make mathematical arguments and critical judgements based on their work (Confrey, 2007).

The approach to modeling that we will take can be characterized as theory-driven (Pfannkuch & Ziedins, 2014) emergent (Gravemeijer, 2007) modeling. According to Pfannkuch and Ziedins (2014), in theory-driven modeling students are given a model and “students work within the closed world of the model” (p. 111). This approach was developed, for example, in Borovcnik (2006); Borovcnik and Kapadia (2011). We remark that, in this setting, the closed world of the model is often given by providing a simplified version of reality to be used as the modeling context. This point of view has been applied to studying educational issues around several advanced topics in probability, such as Markov processes (Pfannkuch & Budgett, 2016) and connections between Poisson processes and exponential distributions (Budgett & Pfannkuch, 2018). This point of view is also common in the statistical modeling literature, which is closely related to probabilistic modeling, see e.g., Pfannkuch et al. (2016) and the references therein. We remark that, although one works within the closed world of the model, real or synthetic data can be incorporated in terms of fitting the given model to the data.

We briefly remark that the theory-driven approach stands in contrast to the data-driven approach, in which students work in an empirical situation and try to develop a model based on the empirical data, see e.g., Lesh and Lehrer (2003); Pfannkuch and Ziedins (2014); Garfunkel and Montgomery (2019) among many others. We adopt the theory-driven approach primarily because, for prospective probabilistic modelers, the specific models and modeling ideas are important and our activities introduce students to these models and ideas. For example, when someone becomes an actuary, they will be expected to apply and understand existing actuarial models, not start the modeling process from an informal context.

METHODS

Analytical Framework: Methods of Analyzing Student Responses

According to the recent overview by Mooney et al. (2014), all of the most prominent frameworks (e.g., Jones et al., 1997; Tarr & Jones, 1997; Watson et al., 1997; Watson & Moritz, 2003) for characterizing the development of students' probabilistic thinking are based on the Structure of Observed Learning Outcome (SOLO) taxonomy developed in Biggs and Collis (1982, 1991). See also Shaughnessy (2007) for extensive discussion of the use of this taxonomy in probability and statistics education research. We develop our own adaption rather than using a common existing one such as Jones et al. (1997) as those are focused on the K-12 level and do not address the more sophisticated constructions needed at the level we are considering.

The SOLO taxonomy has five modes of response and, within each mode, five levels of reasoning in student responses. For our purposes, the modes are less essential because they will not help us distinguish responses. This is because all prospective probabilistic modelers operate at what is called the formal mode when they encounter our activities and are trying to, eventually, transition to what is called the postformal mode, where professional probabilistic modelers operate (Biggs & Collis, 1982 & 1991). The levels of reasoning within each mode are

(a) *prestructural*, where “the task is engaged, but the learner is distracted or misled by an irrelevant aspect belonging to a previous stage or mode” (Biggs & Collis, 1991, p. 65),

(b) *unistructural*, where “the learner focuses on the relevant domain, and picks up one aspect to work with” (Biggs & Collis, 1991, p. 65),
(c) *multistructural*, where “the learner picks up more and more relevant or correct features, but does not integrate them” (Biggs & Collis, 1991, p. 65),
(d) *relational*, where “the learner now integrates the parts with each other, so that the whole has a coherent structure and meaning” (Biggs & Collis, 1991, p. 65),
(e) *extended abstract*, where “the learner now generalizes the structure to take in new and more abstract features, representing a new and higher mode of operation” (Biggs & Collis, 1991, p. 65).

Our operationalization of these in the context of probabilistic modeling is given in Table 1.

Table 1. Levels of Probabilistic Thinking

Level	SOLO Classification	Description of Level
0		Responses based on intuitive or non-probabilistic reasoning, which may not be connected to constructs in probability mathematically
1	Unistructural	Responses based on reasoning about a single feature of modeling context using probabilistic formalism
2	Multistructural	Responses based on reasoning multiple aspects of the modeling context using probabilistic formalism, but incomplete specification of the relationship between them
3	Relational	Responses based on systematic reasoning about multiple aspects of the modeling context using probabilistic formalism with complete specification of the relationship between them
4	Extended Abstract	Responses take new (to the student) aspects from the modeling context that are not necessary to completely specify a model and incorporate them into a completely specified model in order to explain the behavior of the model

Our Level 0 does not exactly correspond to *prestructural* responses, so we have not labeled it as such. In the population of students we are considering, we have not observed *prestructural* responses, but students occasionally start by trying to reason their way to an answer making informal, or non-probabilistic, mathematical arguments. Similar observations have occurred previously in the probability and statistics education literature, for example the lowest level of stochastic reasoning in Shaughnessy (1992) is “non-statistical” reasoning. Level 1 responses are *unistructural* responses because they isolate a single aspect of the modeling context and use that to provide an answer without considering how other aspects of the context may impact the one they focus on. Level 2 corresponds to *multistructural* responses because they include multiple aspects of the modeling context, but the relationships between them have not been fully given, so they cannot be considered as integrated into a coherent structure. Level 3 corresponds to *relational* responses because in these responses a fully explained, coherent structure, describing multiple aspects of the context and their relationships is given. Level 4 corresponds to *extended abstract* responses, as these responses involve incorporating new abstract, context specific, features into the model, such as deterministic approximations as a method to understand stochastic processes or ideas of arbitrage to price derivatives. This represents a transition from the formal mode of response, where theories students know are used, to the postformal mode, where new (to the student) theories are developed (Biggs & Collis, 1991). Specific examples of those levels will be presented in our results.

Participants

Participants were students from an introductory graduate probability course at a midsized university in the U.S. that is also taken by advanced undergraduate students. This course treated probability from an abstract perspective, focusing on theoretical properties of random variables. Prior to the modeling activity from which data was collected, the course focused on a (measure theoretic) definition of expected value, weak and strong laws of large numbers, and types of convergence of sequences of random variables (e.g. almost sure convergence, convergence in probability, and convergence in distribution). The modeling activity from which data was collected was the first activity in which students were tasked with using the theoretical tools developed in the course to model and analyze a real-world situation.

Eleven students in the course volunteered to participate in the study. Eight of the participants were graduate students and three were undergraduate students. Students' backgrounds ranged from no prior exposure to probability at the undergraduate/graduate level to having prior exposure to advanced probability (e.g. stochastic calculus).

Activities

Participants worked in groups (two groups of four and one group of three) on a flu modeling activity for three days as part of the class. Most of the activity was provided as a pdf handout with the scenario and questions in Figure 1. Before starting the activity, the instructor presented the following additional details about the scenario: Every day each infected person independently picked a member of the class (possibly themselves) uniformly at random and if the person they picked was not already infected then they would become infected starting the next day. There was no recovery from infection so, once someone became infected, they remained infected for all subsequent days. Students were told to assume the flu spread according to these rules in their responses to questions 1-5. Students worked on these activities as total of three 75-minute class periods.

Flu is starting to spread throughout the country. So far your class has remained flu-free, but one of your classmates came in today with the flu and is infecting other students at this very moment! From now on, each infected student will have a chance to infect another member of the class each day. How will the flu spread through the class in the coming days?

Initial Thoughts

1. Think about what a graph of the number of infected students (Y) over time (t : # of days from today) might look like. Sketch a curve and share with your group.

HOW WOULD YOU MODEL AND ANALYZE THE SPREAD OF FLU THROUGH A POPULATION?

2. Suppose that the size of the population is N and at that at time t = 0 there are A infected people in the population. How would you describe the number of people who will be infected at time t = 1?
3. What is your best estimate for the number of infected people at time t = 1?
4. Suppose that we let the size of the population tend to infinity and that the proportion of infected individuals at time t = 0 tends to $a \in (0, 1)$. What can you say about the proportion of infected individuals at time t = 1 as the size of the population tends to infinity?
5. In the setting of the previous problem, what do you think will happen over a multi-day period?

Figure 1: Flu Activity Handout

RESULTS

In this section we will present our analysis of our student data according to the levels in Table 1, and how response levels related to the use of random variables. It should be noted that our focus was on identifying levels of probabilistic thinking that are reflected on students' written responses instead of evaluating their answers based on mathematical correctness. As discussed above, students worked in groups, which we will refer to as Groups A, B and C. Groups A and B had 4 students each while Group C had 3 students. Students initially worked on Question 1 independently before discussing with their group, so we report their responses individually. Subsequently the groups worked collaboratively so we report the group responses. Table 2 summarizes our classification of responses for each group according to the levels in Table 1.

Table 2: Summary of response classifications (Levels)

Question	Group A	Group B	Group C
Question 1	1, 0, 0, 0	0, 0, 0, 0	1, 0, 0
Question 2 (before prompt/after prompt)	1 / 3	0 / 2	1 / 2
Question 3	3	2	2
Question 4	3	2	2
Question 5	N/A	N/A	N/A

On Question 1, students worked individually so the individual responses of each group member are included in the table. For subsequent questions, the group response was classified. On Questions 2 and 4, students gave responses and then had an opportunity to revise their responses based on prompts from, and discussions with, the instructor, however no group had time completed a response to the prompt on Question 4 before the time for the modeling project ended. Students did not have time for Question 5.

We only observed changes in the level of response following interventions by the instructor. For example, in Question 2, Groups A and C contained members who provided Level 1 responses to Question 1 and subsequently provided Level 1 responses to Question 2 while all members of Group B provided Level 0 responses to Question 1 and subsequently initially provided a Level 0 responses to Question 2. This suggests that students in Groups A and C might have seen the value of their group member's Level 1 response and adopted the higher-level response as part of the group. Even when students encountered questions whose resolution required a higher Level response, as each group encountered in Question 2 and as Groups B and C encountered in Question 4, we observed that, although some students recognized the shortcomings of their responses, no group transitioned to a higher level response without intervention from the instructor.

After analyzing response levels, we examined student responses immediately before and after level transitions to find patterns in their use of random variables. We found that initially students' responses did not make any significant use of random variables (and were incorrect), after being prompted about which random variables might be helpful and discussing what was needed to implement this with the instructor, students were able to introduce those random variables and produce correct models. We note that the instructor only suggested what may be helpful and did not provide details on how to incorporate these into the model. After providing their initial responses to this question_the instructor prompted the groups that it may be easier to represent the number of infected students using random variables to represent who each infected student selected. After being prompted the groups did more work, including additional discussions with the instructor and groups presenting partial progress to the class, and provided new responses. We will demonstrate how students made use of random variables in the rest of this section by analyzing the pre-prompt and final responses to Question 2. We remark that the groups' final answers were similar to what they presented. We did not have any explicit evidence showing groups adopting ideas from other groups' presentations although it could have affected their further refinement of their answers implicitly.

Group A Progression
Group A's initial response to Question 2 was an attempt to find the probability mass function of the number of infected students on day 1. The total number of infected people was the only aspect of reality that was formalized in this response, see Figure 2.

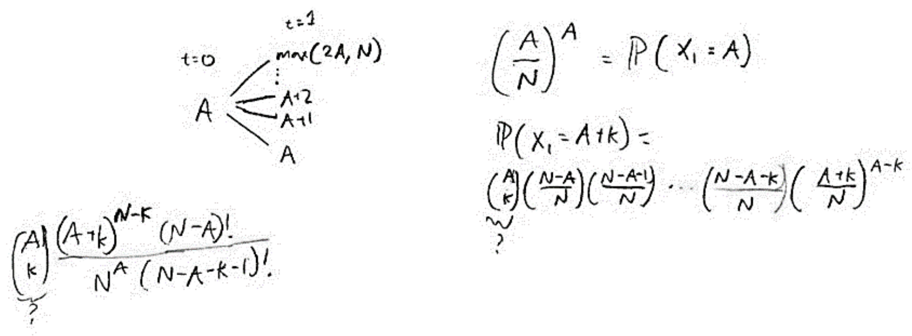

Figure 2: Group A's Initial Response to Question 2 (Level 1)

This is classified as Level 1 because the only element of reality that is formalized using probabilistic formalism is the total number of infected students on Day 1, which has been formalized as the random variable X_1 along with a proposed distribution for this random variable. It is worth noting that the proposed distribution, which is the expression for $P(X_1=A+k)$, is incorrect. In this activity the distribution of the total number of infected students on day 1 is not one of the standard named distributions typically encountered in undergraduate probability (e.g. Binomial, Poisson, Geometric etc.) and is quite difficult to write down exactly and Group A was unable to do this successfully. As indicated by the question marks in Figure 2, Group A was not confident in their response.

After receiving a prompt from the instructor and discussing how to incorporate the prompt among themselves and with the instructor, Group A provided a Level 3 response, see Figure 3.

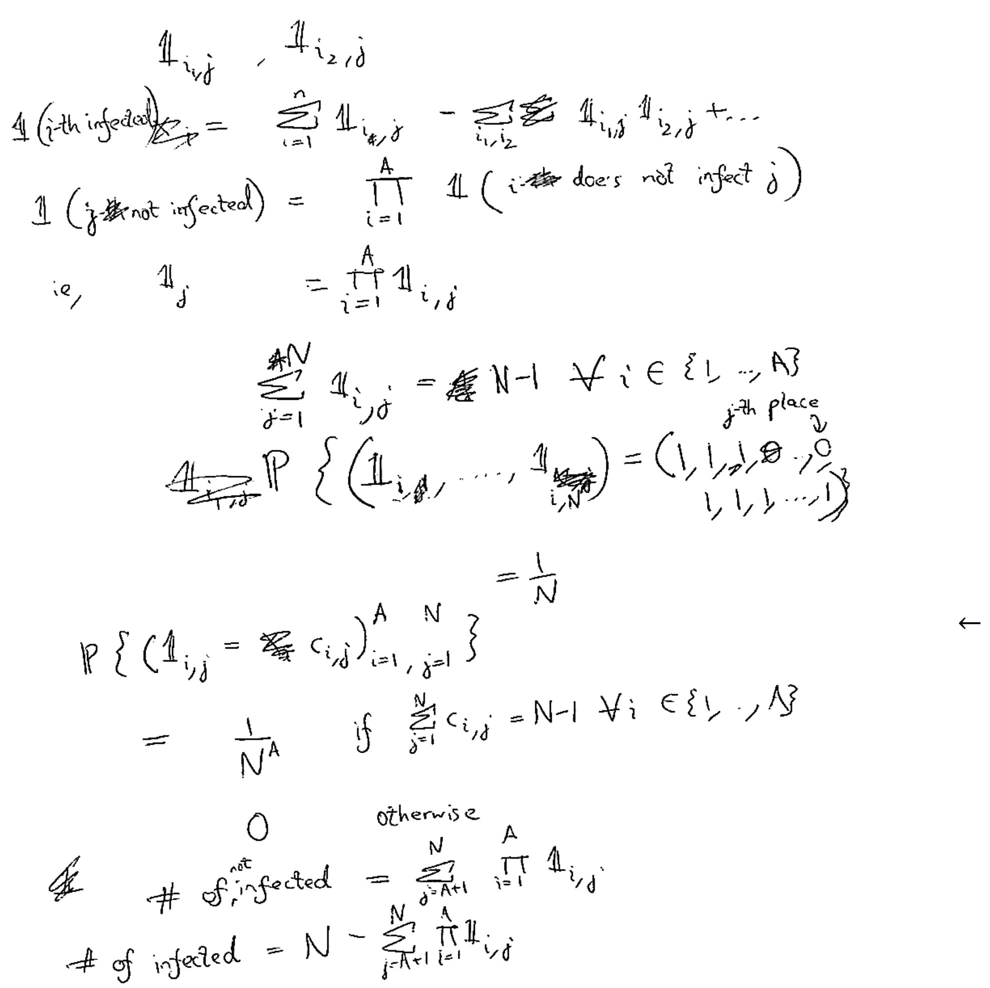

Figure 3: Group A's Final Response to Question 2 (Level 3)

This answer systematically formalizes all aspects of reality, introducing the random variables $\mathbb{1}_{i,j}$ to represent whether or not sick person i infects healthy person j, fully specified relationships between these random variables by providing their joint distribution, (See the arrow in Figure 3), and constructing representations of other aspects of reality from these random variables (e.g., "# of not infected" and "# of infected" in the last two lines of Figure 3).

We remark that the instructor's initial prompt suggested using indicator random variables for who the sick students attempt to infect to help count the number of newly infected students. In subsequent discussion as the students attempted to incorporate this, the instructor suggested thinking about a student not being infected and, later suggested that joint distributions were needed to complete the description. The instructor did not provide details on how to incorporate these features into the model and the students' Level 3 response differs a sample solution prepared by the instructor as part of their lesson plan that was not shared with students, which indicates that it includes the students' own work incorporating the instructor's suggestions beyond transcribing what the instructor said.

Group B Progression

Group B initially responded to Question 2 with heuristic reasoning about what the expected number of infected individuals would be, see Figure 4.

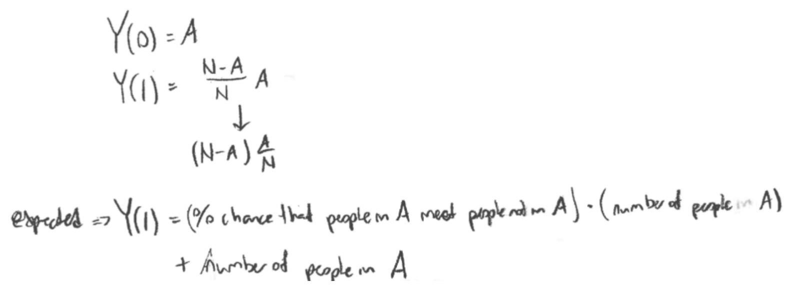

Figure 4: Group B's Initial Response to Question 2 (Level 0)

This was classified as Level 0 because the response did not formalize a random variable to take the expected value of and thus only included an informal notion of expected value and informal notions of chance. We also mention that the quantity Group B settled on (i.e., the last two lines of Figure 4) was not the expected value of the total number of infected students, which could be seen by formalizing the random variable and computing its expected value rigorously.

After being prompted and discussing how to incorporate the prompt among themselves and with the instructor, Group B's response was changed to the following (Figure 5).

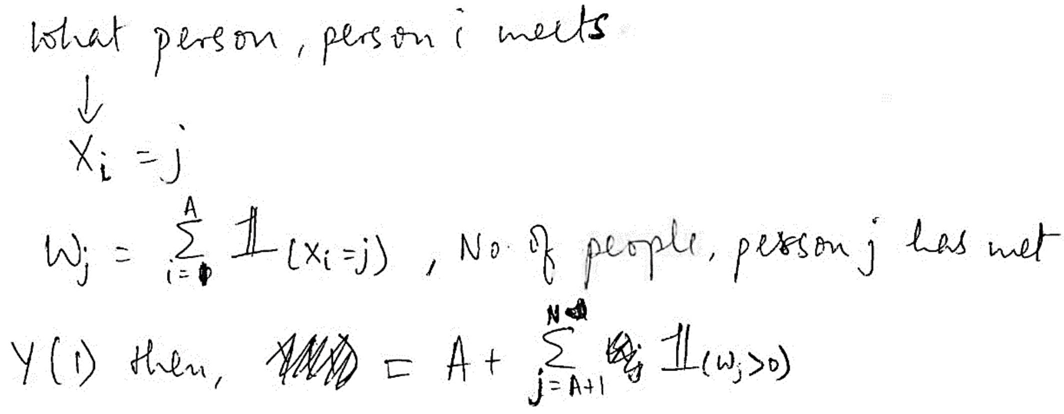

Figure 5: Group B's Final Response to Question 2 (Level 2)

This response was classified as Level 2 because they introduced random variables for determining who each sick person tries to infect, but did not fully specify the relationships between them (e.g. the joint distribution of the X_i for multiple indices i). Students were able to take the X_i and use them to construct Y(1) using intermediary random variables W_j, which were also not included in the instructor's prompt.

Similarly to Group A the instructor's initial prompt suggested using indicator random variables to help count the number of newly infected students. In subsequent discussion as the students attempted to incorporate this, the instructor suggested thinking about who each infected student tried to infect and, later suggested that joint distributions would be needed to complete the description. The instructor did not provide details on how to incorporate these features into the model and, as can be seen from Figure 5, Group B did not incorporate joint distributions into their response. Additionally, the response differs a sample solution prepared by the instructor, which indicates that it is not a transcription of what the instructor said.

Group C Progression
Group C initially responded to Question 2 was similar to Group A's in that they tried to determine the exact distribution, see Figure 6.

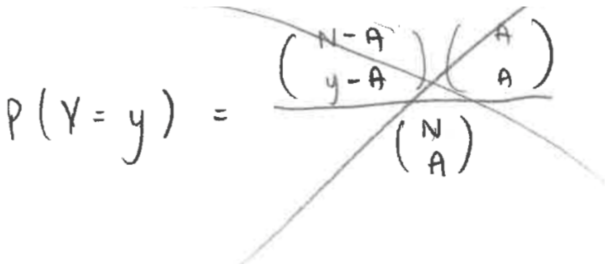

Figure 6: Group C's Initial Response to Question 2 (Level 1)

As indicated by the right-hand-side being crossed out in Figure 6, Group C was not confident in this response.

After being prompted by the instructor and discussing how to incorporate the prompt among themselves and with the instructor, Group C's response was similar to that of Group B, in that they introduced random variables for who each sick person infected but did not specify the joint distribution, see Figure 7.

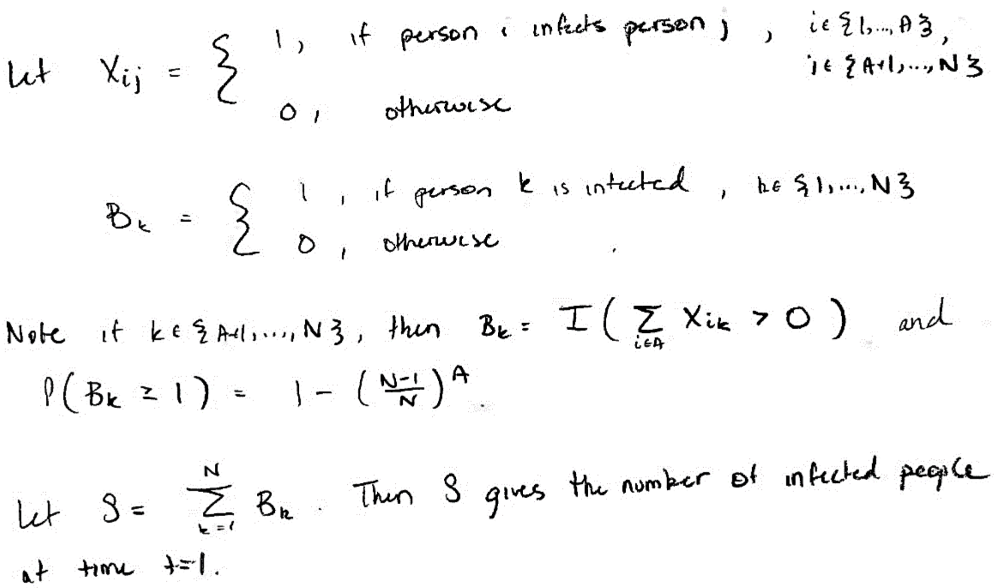

Figure 7: Group C Final Response to Question 2 (Level 2)

Group C's interactions with the instructor were similar to those of Group B.

DISCUSSION AND CONCLUSIONS

In our results, we showed how student responses in a modeling activity can be classified according to an operationalization of the SOLO taxonomy. To the best of our knowledge, this is the first time a SOLO-type taxonomy has been applied to probabilistic modeling in the advanced undergraduate/introductory graduate probability setting. A useful feature of our levels of probabilistic reasoning is that are easy to use in practice. We did not encounter any responses that were difficult to classify. Furthermore, by classifying responses at various stages of the modeling activity we were able to clearly see the progression of students' probabilistic responses. In particular, we saw three phenomena that we think merit further investigation.

In our results, we found that student responses only changed level as a result of prompts from the instructor or when working in a group with a student who had previously given a higher-level response It would be interesting to investigate what prompts from the instructor help facilitate students providing higher level responses and what can be done to help students advance in Level without instructor intervention. We believe this may be possible because in our study students often (but not always) seemed to see that their lower-level responses were incomplete, but needed prompts to see how to improve their responses.

Finally, based on the post-prompt responses to Question 2, we found that students were capable of using random variables to construct models once they had been prompted on which random variables to introduce. However, based on their pre-prompt responses that either did not incorporate randomness or involved looking for the exact distribution of the number of infected students, we conclude that the students in our study were not able to apply their knowledge of random variables in the modeling context without being prompted. This is potentially related to the fact that the modeling activity we observed was the first modeling activity in the course. Interestingly, non-random models and exact distributions were not covered in the course we observed and, therefore, it seems students must have been relying on prior knowledge in their responses. This hints at the idea that in their prior coursework they either did not encounter ideas of probabilistic modeling, or their prior exposure emphasized exact distributions over random variables. We think it would be very interesting to investigate how students' prior exposure to modeling and probability impacts their adoption of random variables in modeling contexts.

We note that our data set was relatively small and, while we did not observe any advances in level without instructor prompts, we expect both of these phenomena will be observed in larger data sets, especially if one uses our classification to analyze data from introductory undergraduate probability courses rather than introductory graduate probability courses as we have. Nonetheless, we conjecture the typical student progression will not involve regressions in level within activities and that instructor prompts will be necessary to increase the level of student responses. Furthermore, we think it would be particularly interesting to investigate how instructors can prompt changes in levels of response in active learning contexts. Additionally, it would be interesting to examine a data set that involves the same students participating in several modeling activities. This would enable an investigation of which modeling ideas students take from one activity to the next and whether or not students are more likely to adopt random variables on their own in subsequent activities.

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PLENARY ABSTRACTS

PER CAPITA, IN MICE

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KEYWORDS: statistics in the media, risk, denominators

StatsChat (statschat.org.nz) is the blog of the University of Auckland Department of Statistics, centered around statistics in the media. The blog was started in 2013, aiming to raise the public profile of the department and to provide useful examples for statistics teachers. Somewhat unexpectedly, journalists have also become an important audience. I had expected the main topics to be uncertainty and confounding; these do come up, but in fact the use of appropriate denominators has been the most important statistical issue. Because I work in medical statistics, I have also written quite a few posts about the over-interpretation of biomedical and health research in the news, and whether this is attributable to reporters or to researchers and their public relations offices (it's a mixture). These posts pursue the statistician's role of being precise about what questions are actually being asked and answered using the data. In this presentation, I will explore what statistics in the media says about public understanding of statistics and science, and what the success of the blog says about interest in these topics.

MATHEMATICS AS FOSSIL OR FUEL? THE ROLE OF UNIVERSITY MATHEMATICS EDUCATION IN NURTURING ETHICAL NARRATIVES ON MATHEMATICS

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KEYWORDS: university mathematics education, commognition, change research

In recent years, university mathematics education research (RUME) has risen to become a fast rising area of mathematics education research (RME). All major RME conferences now have dedicated RUME venues (such as the UME Thematic Working Group in the European Congresses on Mathematics Education (CERME) and the various Topic study groups at ICME). Conferences such as DELTA, RUME and INDRUM have been growing steadily for years and specialist journals such as the *International Journal for Research in Undergraduate Mathematics Education* have also taken off in this period (Durand-Guerrier et al., 2021).

One narrative on RUME studies is that of gradual emancipation from a rather narrow, individualistic focus on cognitive aspects of student learning and their turn towards a richer and grander vista of issues – pedagogical, institutional, affective, social and cultural (Nardi, 2017). As the scope of RUME studies has been growing, so has the need for research that attends to the processes of institutional change in mathematics (and other) departments where mathematics teaching and learning takes place (Reinholz et al, 2020). At the heart of reform that has strength and longevity lie multi-faceted synergies between the communities of Mathematics and Mathematics Education (Nardi, 2015) cover more and more ground that includes research, teaching, university-level mathematics teacher education and professional development as well as public communication about mathematics.

In this lecture, I draw on examples of a particular kind of UME activity that is research informed, values-driven (especially about the role of mathematics educators in nurturing ethical narratives on mathematics) and aspires at longitudinal change grounded in cross-community synergies. All examples demonstrate the capacity of discourse analysis – specifically the theory of commognition (Sfard, 2008) – to support the design, tracing and dissecting of discursive shifts in medium/long term interventions (Nardi et al, 2021).

Example 1 (Viirman & Nardi, 2021) draws on a Norway-based study which engaged biology students with biology-themed Mathematical Modelling activities to challenge deficit narratives about the role of mathematics in their discipline and about their mathematical competence and confidence.

Example 2 (Moustapha-Corrêa et al., 2021) draws on a Brazil-based study which engaged teachers with activities featuring mathematical practices from the past (history of mathematics) and in today's mathematics classrooms (activities from the MathTASK programme) to trigger changes in teachers' narratives about how mathematics comes to be and how its emergence can be negotiated in the mathematics classroom.

Example 3 (Nardi & Biza, in press) draws on the research-informed inception, design, delivery and assessment of an introductory RME course on a BA Education programme which aims to welcome Education undergraduates (most of whom are prospective primary teachers) into RME and invite them to revisit their own, sometimes traumatic, experiences of learning mathematics and reconsider their often sparse or deficit narratives about mathematics and its raison-d'être.

Across the three examples, I will illustrate how the discursive shifts orchestrated by these interventions generated new narratives about mathematics (and its pedagogy in Examples 2 and 3), de-ritualised participation in mathematical routines and, ultimately, meta-level learning – especially about what, and whom, mathematics is for.

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SKEWED THINKING: MOTIVATION AND LEARNING CHALLENGES AMONG FIRST-YEAR STUDENTS IN MATHEMATICS AND STATISTICS

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KEYWORDS: motivation; cognitive bias; fear of failure; educational psychology; learning

In this keynote presentation, Dr Sotardi (she/her) draws on her expertise in educational psychology to answer the question: *why do first-year mathematics and statistics students struggle with motivation and learning*? She highlights five common flaws in judgment that hinder first-year students from academic success. The five cognitive biases to be discussed in this presentation are the overconfidence effect, implicit associations, the imposter syndrome, the Pygmalion effect, and the curse of knowledge. Dr Sotardi will offer real-life examples to illustrate each bias in first-year educational contexts and present how such skewed thinking can lead to anxiety, fear of failure, and under-performance. The focus of this keynote presentation is to help mathematics and statistics educators identify, understand, and mitigate cognitive biases to better support learners.

WHAT HAPPENS TO MĀORI STUDENTS IN NEW ZEALAND'S MATHEMATICS EDUCATION SYSTEM?

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KEYWORDS: Maori, Pasifika, education

Coming from a Māori heritage I have felt sensitivity to the experiences of Māori and Pasifika cultures within our system. The road is not always clear for these students to succeed in mathematics. Not because they are uncapable, but since the delivery of system and environmental factors can unknowingly produce inequalities in different ethnic groups. In this talk I will attempt to paint a picture of the experience of being Māori through sharing my own stories, those of others and some of the solutions currently in place. From 2014 to 2019, I was the Programme leader for AUT's Certificate of Science and Technology programme, alongside playing an ongoing role in AUT's Uniprep programme. Both these courses are specifically designed to address the imbalances experienced from New Zealand's pre tertiary education system for Māori and Pasifika. I will share both these programmes with you and what they mean in terms of equitable access to tertiary STEM education for all.

TECHNOLOGY IN MATHEMATICS EDUCATION: PAST, PRESENT, FUTURE

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KEYWORDS: technology, mathematics education, reflection

Now, more than ever, students are learning mathematics with technology. Mathematics teachers and professors are “becoming digital” in new and relatively radical ways, including within the contexts of delivery, assessment and learning communities. Entire institutions have pivoted to fully online mode of operation due to COVID-19 in a very short amount of time. How did we (finally) get here, and where are we going with technology in mathematics education?

To explore the above ideas, I would like to offer a personal reflection on the past, present and future of technology in mathematics education. My reflection is based on more than 30 years of learning and teaching with technology, initially as a student and then as a university educator (Tisdell, 2016-2021; Tisdell & Loch, 2017).

My reflection draws on critique, deconstruction and problematization concerning the roles of technology in mathematics education at the university level. In particular, I aim to seek out the unspoken and the implicit concerning technology in the many ways we currently learn and teach mathematics. My style of critique aims to position itself as a counterpoint to what I regard as over-simplistic thinking with regards to technology, such as generalizations, unsubstantiated yet dominant discourses, and questionable binaries.

In doing so, I hope to unsettle current digital forms of mathematics pedagogy in ways which open up new perspectives, foster richer understandings and enable action to emerge.

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ORAL PRESENTATION ABSTRACTS

ACTIVE LEARNING GROUPWORK BASED ONLINE TUTORIALS

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KEYWORDS: active learning, online tutorials, online whiteboard

Pre-COVID, we used ‘whiteboard tutorials’ in most of our large undergraduate mathematics and statistics subjects. Whiteboard tutorials take place in a classroom with whiteboards (or blackboards) around all the walls, and students work together in small groups on mathematical tasks at the boards. The classes are a form of active learning, helping students to develop skills such as group work and communication, and provide a valuable social element to students’ University experience. In this presentation, we describe a model for online ‘whiteboard’ tutorials which uses online collaborative whiteboards, and preserves many of the strengths of the pre-COVID whiteboard tutorial model in an entirely online environment. We discuss some challenges with the model and possible mitigations.

THE IMPACT OF COVID-19 ON ACADEMIC OUTCOMES IN UNDERGRADUATE STATISTICS COURSES

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KEYWORDS: COVID-19, academic outcomes, equity groups

This talk presents preliminary findings from a survey administered to students enrolled in first- and second-year statistics courses in Semester 1, 2020 at the University of Auckland. The primary aim of the study was to investigate the impact of the COVID-19 pandemic and the resulting campus closure on the students' academic performance and well-being. Further, we also aimed to investigate whether these outcomes differ based on lifestyle and demographic factors, such as membership in equity groups. The long-term consequences of COVID-19 for university students are unknown. We intend to compare the academic outcomes in the COVID-19 impacted semester to those of previous semesters in order to identify important factors associated with differences in academic performance.

EMBEDDING SUSTAINABILITY INTO A FIRST YEAR BUSINESS STATISTICS UNIT TO PROMOTE GLOBAL CITIZENSHIP

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KEYWORDS: Statistics Education, United Nations Sustainable Development Goals (UN SDGs), Business Statistics, Curriculum Development

Just as we entered into the last Decade of Action to accelerate progress towards achieving the United Nations Sustainable Development Goals (UN SDGs) in the 2030 Agenda, a global pandemic hit us in early 2020. As the pandemic took hold, it impacted all three dimensions of sustainable development: economic, social and environmental. The pandemic highlighted how interdependent we are reminded us that global solidarity is essential to address what United Nations describes as the “world’s biggest challenges”.

The higher education sector plays an indispensable role in championing the UN SDGs agenda. Business schools are pivotal to influence and educate their students to become responsible and sustainable business practitioners. To this end, it requires business degrees to incorporate the UN SDGs into the curricula design, by creating new learning content and methods to develop students' interdisciplinary and transdisciplinary skills. The importance of statistically literate citizens and business people is more obvious than ever before since data is ubiquitous and can be used for evidenced-based decision making.

In this presentation, we will share our experiences of how we embedded sustainability into a large first-year Business Statistics unit in a metropolitan University in Sydney. By aligning learning design and assessments to the UN SDGs, we hope to influence students to take initiatives supporting the UN SDGs. Ultimately, we expect students to become responsible innovators contributing to create an inclusive and sustainable global economy.

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REIGNITING MATHEMATICAL AND STATISTICAL THINKING FOR FINAL YEAR SCIENCE STUDENTS

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KEYWORDS: Sustainability, mathematical thinking, statistical thinking

In a third-year capstone unit for science students, the curriculum is designed around the UN Sustainable Development Goals (UN SDGs). The assessments are structured such that the main assessment is a group based-project proposal that addresses one or more of the UN SDGs with a final written report and corresponding video pitch/presentation.

The majority of these students are enrolled in majors in which they may have completed a first-year mathematics and/or statistics unit, but no further studies in mathematics and/or statistics. The authors have used this opportunity to develop some student-centred activities that specifically address the mathematical and statistical critical thinking skills of these students before they graduate, and at the same time provide additional perspective on the importance of quantitative thinking in issues of sustainability.

The mathematical thinking component is based on Al Bartlett's celebrated lecture Arithmetic, Population, and Energy (Bartlett, 1978), and evolves around several simple in-class activities that relate to linear and exponential growth, embedded into a minimum of theory to provide the conceptual framework. The statistical thinking is based on Wild and Pfannkuch (1999) and uses additional examples to emphasise how statistics should be reported.

In this presentation, we will provide more detail about these activities which might be adopted/adapted by others for their classes.

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DESIGNING A FOUNDATION MATHEMATICS COURSE FOR TODAY'S STUDENT

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KEYWORDS: foundation mathematics, self-efficacy, course design

Foundation mathematics courses play a key role in today's tertiary education system, allowing mathematically underprepared students to engage with their chosen STEM degrees. In this talk I reflect on the process of reimagining a foundations of mathematics course that has no pre-requisite but adequately prepares students for further calculus courses.

There are several key issues that needed to be addressed in the course design. For example, there is strong evidence in the literature to suggest that self-efficacy has an impact on student motivation, persistence, and chance of success. Another target issue is to provide students with opportunity to learn and revise core numeracy concepts such as a symbolic understanding of the equals sign, and fraction arithmetic.

I will give an overview and motivation for the redesigned course structure, and outline the main elements of the course. These include a co-requisite structure to cover core numeracy skills; weekly homework delivered online to allow for instant feedback and interleaving; collaborative problem solving workshops; and an option to take the course at a slower pace. I will also reflect on lessons learned throughout the first implementation of this course and some promising initial outcomes.

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BOALER'S AND BLOOM'S TAXONOMIES TO GUIDE MATHEMATICS QUESTIONS

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KEYWORDS: question design, mindset, Bloom's taxonomy

Growth mindset (Dweck, 2006) is the belief that our intelligence is not set at a predetermined maximum level. A person on the fixed mindset end of the mindset scale believes that most students who are accepted to study university mathematics have a high set level, and that those who have to work hard to do what others may find easy have a low set level. Growth mindset is important for motivating students to seek challenges, try alternative approaches, and use feedback to improve. Boaler (2015) describes six principles for designing learning tasks that should promote growth mindset in mathematics classrooms. We call these principles Boaler's taxonomy. While Boaler's work has predominantly been applied in school mathematics contexts, we have established that these principles can be applied to university mathematics assessment tasks (Campbell et al., in press). We consider Boaler's taxonomy in relation to Bloom's taxonomy as a guide for developing mathematics assessment tasks for first-year mathematics courses.

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FORMING AN ONLINE COMMUNITY OF PRACTICE: WHAT WE LEARNT AND WHAT COMES NEXT

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KEYWORDS: community of practice; online assessment; open book; verbalizing; feedback

With a cumulative experience of 100 years of teaching higher education mathematics students, we are four academics at three different higher education institutions over two continents. Our common history is that we were all members of the 2015 Elephant Delta organising committee. From February 2021 we held fortnightly Zoom meetings to share ideas, discuss challenges and explore the potential for collaborating on research.

We place high importance on our teaching and the learning of our students. Forming an online community of practice (CoP) forged by our experiences and thoughts on assessment during the pandemic led us to identify five propositions to guide our university mathematics teaching: (1) Open book assessment has value; (2) Verbalising helps learning; (3) Assessment can promote self-directed learning; (4) Assessment can develop higher order thinking skills (HOTS); and (5) Assessment as learning and for mastery learning benefits students. Our meetings and homework pushed us to think about why we teach what we teach, assess how we assess, and how we can make both more relevant to a changing world. We learnt more deeply about assessment by interrogating each other's work, observing and identifying misconceptions or errors (made by ourselves and others), and learning different ways of solving problems through discussion. We noted that sustaining the CoP required comfort in being confronted and criticized. The next aim of our CoP is to research moving away from written feedback on assessment in favour of voice or video comments.

STUDENT AND STAFF APPROACHES TO ACADEMIC INTEGRITY IN CALCULATION-BASED ASSESSMENTS

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KEYWORDS: academic integrity, assessment, online

Face-to-face examinations have long been a cornerstone of university mathematics assessment (Iannone & Simpson, 2011; Thoma & Nardi, 2016), but with the advent of Covid-19 universities are increasingly opting for online assessments. The difficulty in monitoring students taking large-scale mathematics assessments in an online setting creates a significant challenge in maintaining assessment integrity. Students have the potential to seek assistance from friends, family, tutors, online ‘tutoring’ sites such as Chegg, and online calculators such as Wolfram Alpha that not only present the solution to a problem, but the working as well. While a huge amount of literature exists about academic misconduct more broadly, research specific to mathematics/calculation-based assessments has been lacking to date (Seaton, 2019).

Our research project seeks to investigate student and academic staff perceptions in relation to behaviours that exist in a 'grey area' separating clear misconduct from appropriate/reasonable use of available resources and technology. In a recent survey of Australian university staff and students, we asked participants to rate various academic misconduct scenarios on a scale of “No misconduct” to “Clear misconduct”. These scenarios range from more traditional sources of help, such as friends and tutors, to newer forms of help such as online calculators and online forums.

This talk will discuss the methodology of the survey instrument and present preliminary findings from the survey data, seeking to explore the consistency with which students categorise specific scenarios as clear misconduct, and the differences between staff and student attitudes towards academic misconduct.

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PEER CURATION OF SUPPLEMENTAL MATERIALS

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KEYWORDS: learning community; student-driven learning; knowledge network; resources; student agency

Resources for the teaching and learning of mathematics include those provided or recommended by the teacher supplemented by others available elsewhere, such as knowledgeable friends and family, the library and effectively limitless online resources. Many online resources available as supplemental material can be helpful for learning yet teacher curation takes a great deal of time and risks missing important student perspectives. Student curation would be preferable to teacher curation in certain respects, yet a traditional course provides no means for students to share information about these opportunities with one another. In two Calculus courses a “knowledge network” initiative was launched to provide a platform for students to share supplemental materials with one another. Image hotspots located on a course concept network offered embedded links to student-selected videos, tools and websites. Other than a check for relevance and accuracy by a teacher the initiative was student-driven; students voluntarily selected the resources, shared them within their learning community and reported benefit within the calculus courses and beyond.

CODEX ONLINE MODULES @ UC

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KEYWORDS: online, learning design, mathematics, CODEX, exposition, STACK.

In May of 2020 Christopher Sangwin and George Kinnear published a paper called: Coherently Organised Digital Exercises and Expositions (CODEX). In this talk I will outline what that paper is about and show you the resources that we in the Department of Mathematics and Statistics at the University of Canterbury were inspired to build after reading the paper. I will present the work of many people across our department to show you the tools we used and the modules we have built.

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ARE ALL BLANK ANSWERS WORTH THE SAME MARK OF ZERO?

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KEYWORDS: Instant Feedback, SOLO taxonomy, learning moments

When marking test papers as a mathematics educator, what to do when confronted with blank spaces where the answers should have been written. Are all blank spaces worthy of the same mark?

Did students have no knowledge of what the topic; or did students just not understand the question but has knowledge; or anxiety issues taking place or did students just have the 'Blank Moment'? These situations would all be awarded the same mark but the reasons for the blanks are not equal. Recently, to combat this problem, a system of exchange of marks for clues on starting or proceeding further in a question has been introduced. The benefits are two-fold as the students are provided instant feedback when in the 'thought-zone' of the question and the educators may use these questions as a guide to improving 'learning moments' in class. The use of SOLO taxonomy will give both educator and student, a guide to where their learning lies and where it can improve.

This presentation will describe how the current system is being applied for online tests throughout the recent trimesters. It will be an interactive session to show how the 'clue-giving' will arouse the little grey cells as Hercule Poirot often said when solving a mystery.

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HELPING BRIDGING MATHEMATICS STUDENTS MAKE THE CONNECTION IN COVID 19

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KEYWORDS: Assessment, Annotative, Zoom

This paper provides a brief overview of the preparatory programs offered at Central Queensland University and a detailed explanation of how a video teleconferencing tool (Zoom) is currently used to enhance the delivery of the suite of mathematics bridging units during Covid 19.

While there is the belief that technology breaks down boundaries and enables us to connect to our students more easily, regardless of all the available technology, mathematics instruction is still best given in a “talk and chalk” format (Adams & Hayes, 2019). Instructional videos that allow the student to watch handwritten instruction are almost part of the standard subject design. They allow students to learn at a convenient time that fits within their lifestyle and commitments but videos lack the interactive component that makes face-to-face teaching preferable. During the COVID 19 pandemic this preferred method of mathematical instruction was unavailable. Therefore, online lectures were conducted using a combination of Zoom, a Tablet PC and PDF Annotator, providing students with a comparable experience much closer to that of face-to-face.

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EFFECTIVE QUESTIONING IN "INTERACTIVE LECTURES": AN ALTERNATIVE APPROACH

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KEYWORDS: question design, active learning, engagement

Many statistics educators are making use of readily available technologies to incorporate interactive questioning within more traditional lectures, such as PollEverywhere and Kahoot!. These recent technologies allow efficient participation for large numbers of students in real-time and simultaneously by allowing anonymous or crowd-sourced answers, minimising the embarrassment for asking "stupid" questions or giving "wrong" answers.

Past research has focused on the different modes of questioning (open-ended, multiple choice, continuum, visual and short answer); I propose an alternative classification based on the intended purpose of the question (understanding, discussion, participation, self-evaluation and feedback). Through better understanding of the purpose of a question, it is possible to improve the phrasing to foster more engagement and productive interaction within lecture environments. Increased engagement is supported by tracking student participation data in lectures; this is also supported qualitatively by student responses to surveys conducted both within and outside the lecture context.

This work draws primarily on experience from a second-year introductory statistics course (Analysis of Biological Data) which is taught using a flipped classroom model, with one-hour interactive lectures each week. These question development methods have also been applied successfully in more traditional lecture environments for large (200+) undergraduate and postgraduate statistics classes.

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WHAT MOVEMENT COUNTS AS STUDENTS' MATHEMATICAL KNOWING

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KEYWORDS: body movement, dynamic movement qualities, Laban movement elements, problematising, students

Within mathematics education research, the importance of body movement in students' thinking is increasingly being recognized. As Yoon et al. suggested, at this conference in 2011, students' movements during mathematical activity are more than a means of communication, or an adjunct to understanding: movement can provoke new mathematical knowing. As body movement provides a new variable for mathematics education research, a variety of methods for observing and analysing movement have arisen. Movement analysis has a long history across many fields including performing arts, industrial work studies, and computer interface technology. One well-established movement framework, Laban's movement elements, classifies how the body moves in space. Laban's elements also recognize, and pay careful attention to, the often omitted, but inherent, dynamic qualities of movement, thus providing a link between affect and cognition, two modes of knowing that are often separated. Privileging a variety of ways of knowing mathematics was proposed by Tang et al., at this conference in 2017, as a way to improve equity and engage a wider variety of students in the mathematics classroom. By employing Laban's movement elements, this study investigates the movements of a group of bridging education students as they engage with a mathematical task. This study demonstrates how the dynamic qualities of movement contribute to students' emerging mathematical knowing and suggests students' movements as an important resource in the mathematics classroom.

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MOTIVATION AND ENGAGEMENT: INSPIRING FOUNDATION ART AND DESIGN STUDENTS TO CREATE MATHEMATICAL ART

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KEYWORDS: mathematics, art and design, motivation, coding

At the Swan Delta Conference in 2019, I spoke on the value of teaching mathematics and coding to foundation art and design students, based on my experience over two semesters. Today I will discuss interventions aimed at improving motivation and engagement.

My goal is to motivate foundation art and design students to enjoy creating mathematical art. Mathematics is beautiful. I want my students to agree with me and to eagerly anticipate creating their own mathematical art. How do I achieve this? My students arrive with weak backgrounds in mathematics, wondering why they are studying it. Based on my experience, motivating mathematics students to explore mathematical art and coding is relatively easy compared to motivating art and design students to explore mathematics and coding. So how do we go uphill?

I have experimented with two investigation assignments, handed out in the first class. The first task is to research and report on the life and work of a mathematical artist, contemporary or from the past. The second task is to research and report on repeating patterns from the student’s own culture. In the first two offerings of this course, the research on a mathematical artist was done towards the end of the teaching programme. Moving it to the beginning has provided definite benefits with respect to engagement and motivation. The two assignments are complementary and work well when done together. Introducing Islamic geometric design based on arcs early has also caught interest. Insights based on questionnaire responses and observations are shared.

COURSE REDESIGN FOR FLEXIBLE DELIVERY AND INCREASED ENGAGEMENT IN FIRST YEAR MATHEMATICS

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KEYWORDS: first year, mathematics, online teaching, collaborative, workshop

The UQ2U program at The University of Queensland aims to redevelop UQ's large courses to deliver flexibility and high value on-campus activities. In 2018, MATH1051 (Calculus and Linear Algebra I), UQ's largest first year mathematics course (yearly enrolment exceeds 1500) was selected for the UQ2U program. The project has resulted in the development of online resources delivered through the edge.edX platform, and the subsequent re-design of MATH1051. We describe the MATH1051 journey, from the development of resources in 2018 to implementation in 2019. We share challenges encountered and lessons learned. Pass rates, course evaluation data and student and tutor feedback indicate that the redesign was a success.

QUANTITATIVE INTERVIEWS: EXTENDING TRANSCRIPT ANALYSIS USING NATURAL LANGUAGE PROCESSING

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KEYWORDS: mathematics support, COVID-19, Natural Language Processing

When the COVID-19 pandemic hit the world, it affected education at all levels. The shift to wholly online delivery has been challenging for higher education students and staff. However, it has also led to opportunities for new methods to deliver learning and teaching online. During this period, mathematics and statistics support also shifted their services online.

In this study, students and tutors were interviewed to understand the opportunities and challenges they encountered with wholly online learning, teaching and support. Twenty-three participants were selected from University College Dublin, Ireland, and Western Sydney University, Australia and one-on-one interviews were conducted in late 2020. While interviews are an excellent way to gather detailed information, analyzing them usually requires qualitative techniques which can be time-consuming and result in a small sample size.

In this study, we aim to identify common themes around online mathematics and statistics support by Natural Language Processing (NLP). Interview transcripts were converted to numerical data using text mining techniques and classification and topic modelling methods were applied to identify common themes in the transcripts via the R programming language. These findings were compared to the previous qualitative study results to investigate how software-based models perform versus human-based models. Implementing NLP techniques can help to increase sample size, reduce project time and costs.

X=X+1

Alastair Jamieson-Lane

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KEYWORDS: coding, first year experience, tutorial design

Over the past century, mathematics and programming have become closely interlinked, mathematics informing computational methods, and computers acting as essential tools for mathematical discovery. Plenty of mathematics departments now have one or two courses which give a taste of programming- often folded in between matrix algebra, and differential equations.

Students coming into coding bring with them various assumptions and mindsets directly from mathematics; many of these are helpful, but some act as stumbling blocks and pitfalls. In this presentation, I look in on a few of the places where underlying assumptions common to mathematical classrooms can actively hinder students when it comes to coding, and a couple ways to construct lessons, tutorials and curricula in order to avoid potential pitfalls. I'll be focusing on results from first year mathematics courses where students are introduced to Matlab, but the ideas are applicable in many programming contexts.

LECTURER PERSPECTIVES ON THE TRANSITION FROM SECONDARY TO TERTIARY MATHEMATICS

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KEYWORDS: transition, perspectives, calculus

The transition from secondary to tertiary mathematics has been the subject of increased research in recent years. What is still lacking though is research into the perspectives of university lecturers on this transition.

This talk focuses on two areas of the transition: lecturer perspectives on the reasons students choose or don't choose higher level mathematics in the last two years of secondary school, and lecturer perspectives on Year 12 students' calculus skills and understanding. Twenty lecturers from across Queensland completed two online surveys. Fisher's exact test was used to determine statistical significance.

With regard to subject selection reasons, the results show that lecturers thought friends played an important role in influencing one's decision to choose or not choose higher level mathematics. Students, however, said otherwise. When investigating students' calculus skills and understanding, it was apparent that some lecturers not only had little idea of what was taught at school but also how it was taught. Klymchuk (2011) had similar findings. In addition, there were also statistically significant differences between lecturer and teacher perspectives on how difficult the calculus questions would be for students. These results show the clear need for more dialogue between school teachers and lecturers to help improve the transition from secondary to tertiary mathematics.

This talk reports on one part of a two-year longitudinal study into the transition from secondary to tertiary mathematics from the perspectives of students (n=1000), school teachers (n=60), and university lecturers (n=20).

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BRIDGING MATHEMATICS STUDENTS AND THE CHALLENGES OF LEARNING DIS/ABILITIES

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KEYWORDS: foundation, bridging, learning dis/abilities (LDs)

Students without university entrance are often diverted into a six-to-twelve month bridging or foundation programme. Successful completion usually leads to a degree programme which they would not otherwise have been able to enter (Benseman & Russ, 2003). In the programme each student must pass a mathematics course to satisfy academic numeracy requirements. However, there are a small number of students who struggled with mathematics at school and who once again find themselves in an uncomfortably familiar situation where they had poor learning experiences. These students have known or unknown mathematical learning dis/abilities (MLDs) which often accounts for their continuing frustrations with little progress in mathematics and subsequent anxieties. Little is known about how many of these students are assessed for MLDs, nor how many have missed out for whatever reasons.

In this study, students who repeated bridging mathematics, were invited to share their experiences, in semi-structured interviews. Early findings suggest that these students are all too familiar with repeating mathematics courses, with almost all being held back in their school years. They also shared a dread of having to do more mathematics, particularly workplace numeracy or academic numeracy once they are into their degree studies. Some gave accounts about familial or outside assistance with school mathematics, but most regarded progress from these resources as minimal at best. As well as providing stories about the challenges they met with mathematics particularly at school, the participants also offered some ideas about what a useful learning environment for adult learners (with LDs) might look like.

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SYSTEMATIC REVIEW OF STUDENTS' MISCONCEPTIONS IN LEARNING DOUBLE INTEGRALS

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KEYWORDS: Misconceptions, double integrals, systematic review

Misconceptions and a poor understanding of the concept of the double integral lead to difficulties in learning Vector Calculus. The main objective of this research is to investigate undergraduate engineering students' misconceptions when learning double integrals at a South African university. To frame this research and which forms the focus this presentation, systematic literature review was carried out using Scopus, Science Direct, EBSCOhost and Engineering village databases. The search was restricted to published English language articles relating to misconceptions in double integrals and functions of two variables in the timeframe 1 January 2010 to 31 December 2020. The search yielded a total of 53 publications.

From these studies, the findings reveal that students struggle with concepts that are generally considered to be evident during teaching. We also noted that misconceptions in double integrals were related to students' lack of understanding of prerequisite concepts and advanced mathematical thinking because of the hierarchical nature of mathematics and the independence of mathematics concepts. These prerequisite concepts include trigonometric substitution and algebra. It was also discovered that the frequent misconceptions are generated by errors that are indicative of a misunderstanding or misinterpretation of a double integral question. The findings of this review will inform researchers, teachers and other decision makers on student's understanding of double integrals and may contribute to the development of proactive plans to support teaching and learning of undergraduate mathematics. The findings could also be drawn on when planning curriculum and continued professional development activities for mathematics educators, lecturers, and students.

THROUGH COVID AND BEYOND: A SCOPING REVIEW OF UNDERGRADUATE MATHEMATICS THROUGH REMOTE LEARNING DURING THE COVID-19 PANDEMIC

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KEYWORDS: Online Learning, Undergraduate Mathematics, Scoping Review

Approaches to online education have evolved much within the past two decades with increasing research exploring the incorporation of active learning techniques (Irani & Denaro, 2020) and contemporary pedagogical methods (LeSage et al., 2021). With the increasing integration of online delivery methods, the discussion arises of how effective these delivery methods are. Online mathematics education has cultivated itself as an essential element of many higher institutions, often being met with criticism regarding student satisfaction and varying completion rates. Common barriers to online learning involve a lack of access to technology or the internet, low interaction, and a lack of motivation and support (Muilenburg & Berge, 2005). While the uptake of online learning can be attributed to new innovations such as the inception of MOOCs, the COVID-19 pandemic has helped to usher rapid transitions to deliver courses remotely.

With the emergence of a substantial number of case studies of undergraduate mathematics courses delivered during the COVID-19 pandemic, a scoping review was conducted. In this presentation I will report on a preliminary qualitative analysis on common themes and illuminate potential limitations of online delivery, where efforts can be delegated for future research. The case studies used in this scoping review were sourced from Scopus, ProQuest, and Google Scholar databases. The findings revealed the importance of preparation of such delivery methods and informs how this pandemic is an opportunity to reshape our approach to effective transitions to online learning.

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INVESTIGATING THE EFFECT OF PUZZLE-BASED INTERVENTIONS ON THE INTUITION OF STEM TERTIARY STUDENTS

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KEYWORDS: Puzzle-Based Learning, Intuition, Convergent thinking

This presentation provides results from the large national project "Investigating the impact of non-routine problem solving on creativity, engagement and intuition of STEM tertiary students" dealing with the intuition aspect of the project. The study was conducted over 2018-2020 by a team of researchers and practitioners from four tertiary institutions in New Zealand. The pedagogical theory informing the project was the Puzzle-Based Learning approach developed by Michalewicz and Michalewicz (2008) that has been adopted in many educational settings worldwide. A mixed-method methodology was used with comprehensive pre- and post-test questionnaires and interviews. Although the results indicated that there were no significant changes in students' intuitive thinking before and after the puzzle-based intervention, there were some interesting findings related to gender.

MATHEMATICS LEARNING THROUGH A PROGRESSIVE TRANSFORMATION OF A PROOF: A CASE FROM A TOPOLOGY CLASSROOM

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KEYWORDS: Commognition, proof and proving, topology, university mathematics education

In the mathematics community, proving a theorem is a collective endeavor that has a role in generating mathematical knowledge and understanding (e.g., Rav, 1999). This role is often actualized through social interactions that unfold in research collaborations, seminars, paper peer-reviews, and other structured situations where a proof undergoes a series of transforms. This approach to proofs is very different from typical university classrooms, where a proof quickly reaches the point of endorsement by the classroom community.

We report on an ongoing project in a cross-level topology classroom, where students have been provided with opportunities to construct proofs in pairs, share them with the whole class at the board, and receive feedback from their peers and the course teacher. The overarching aim of the project is to explore opportunities for generating mathematical knowledge and understanding that emerge on individual and classroom levels through progressive transformations of a proof. In this presentation, we mobilize the commognitive framework (Sfard, 2008) to explore students' learning in this process. Specifically, we analyze an interaction between two students as they collaboratively constructed a proof, and the subsequent public re-proving of the same statement by one of them at the classroom board.

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AUTOMATED GRADING SYSTEM: USING DOMJUDGE TO GRADE AND TO PROVIDE FEEDBACK

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KEYWORDS: Programming Education, DOMjudge, automated grading system

DOMjudge is an open-source automated grading system widely used to run programming contests. Here, we present the implementation of this tool to facilitate tutorial exercises and practical assessment for our Foundation Programming paper in the Certificate of Science and Technology. The paper provides students with an introduction to coding using C#.

This online automated grading system was successfully employed following the emergency switch to online learning during the COVID-19 lockdown, and it continued to be used post-lockdown as a blended delivery approach.

Arithmetic exercises[1] were used to guide students through the programming concepts such as input and output. Subsequently, students were taken through the concepts of decision making, and iteration using statistical exercises[2] to calculate maximum, minimum, and average values of data input, and to count number of data that fulfilled a given requirement.

Clear instructions and expected output were prescribed for each exercise, pre-defined test cases were compared against the output of a submission, and students were given instantaneous feedback on their submission. Although feedback is limited, it gives students an indication if the code is correct or not. The record of submission was used to track progress of students. DOMjudge has also proven to be useful in conducting practical assessment, it showed students the correctness of submission. This allows better assessment of a student's programming skills including trouble-shooting skills.

Students' feedback on the use of DOMjudge are positive. They gained greater autonomy without having to wait for tutors' feedback. This helps students to build self-efficacy in their learning.

[1, 2] Exercises link available here.

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CHALLENGES AND STRATEGIES: COPING WITH THE NEW REALITIES OF A COVID-19 CLASSROOM ASSESSMENT

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KEYWORDS: physics, mathematics, statistics, online assessment, academic integrity, foundation education

With the transition to online learning during the COVID-19 pandemic, online assessments enable various possibilities to facilitate the breach of academic integrity. A high percentage of breach of academic integrity was observed after the initial transition online. This presentation discusses the strategies and challenges in a Foundation Physics paper to prevent and to discourage students' unethical behaviour, whilst meeting the university's Authentic Assessment standards.

In the subsequent semester, a new initiative highlighting academic integrity was introduced and the assessment structure modified, which shared similarities with foundation mathematics and statistics papers. A programme-wide initiative was cross-promoted in different classes to turn students' attention toward the values of academic integrity. Weekly problem sets and myimaths exercises aimed to change the purpose of assessment to enhance students' learning. Only a small percentage of mark was given for each problem set, so the cost and effort to cheat in this form of assessment does not retribute, and students were incentivised to correct wrong answers. Practical experiments were replaced by inquiry-based online simulations and video-based analysis. Different measures were also implemented in the final assessment so that each student gets an equivalent but different version of the assessment to prevent collaborative cheating.

Designing a better assessment system is preventative implies that students will still try to cheat but it may be more difficult. Motivating students to take control of their own learning by removing the incentive to breach academic integrity is an idealistic (and unrealistic) goal but virtuous in its attempt and communicates desire for knowledge and learning.

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LECTURE CAPTURE IN UNIVERSITY MATHEMATICS EDUCATION: A SYSTEMATIC REVIEW OF THE RESEARCH

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KEYWORDS: lecture capture, attainment, university mathematics

Lecture capture (LC), the process of recording face-to-face lectures for future viewing, has become a common technology in Western universities in the twenty-first century, yet research on its effectiveness has lagged behind its implementation. In the wake of the Covid-19 pandemic, the urgency of obtaining clear answers about the impact of LC is paramount. The recent worldwide shift to online teaching as an emergency response to the COVID-19 pandemic has resulted in an unprecedented use of LC at scale. In this presentation we report on a systematic review of the current literature that we conducted on the efficacy of LC in tertiary mathematics education. The literature is consistent in the opinion that students and administrators positively view LC for its utility and flexibility despite the moderately strong evidence that most institutions face attendance drops. However, most students do tend to see attending lectures/watching recordings as an “either-or.” The literature predominantly reports a negative association between attainment and the use of LC as a substitute to live lectures. The proportion of students who choose to skip live lectures has steadily increased over the last decade as the student campus culture adjusts to LC. Within this group, LC is used imperfectly, providing false benefits and promoting surface learning strategies. There is evidence that regular use of LC by this large group of students may diminish the quality of their learning. We offer research-informed, evidence-based recommendations to mitigate the unplanned and counterproductive impact of LC implementation.

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"I CAN'T BE ALONE FOREVER" – COLLABORATIVE MATHEMATICS LEARNING AS A FIRST-YEAR INTERNATIONAL STUDENT

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KEYWORDS: international students, mathematical identity, secondary-tertiary transition

The challenges of transitioning from secondary school to university mathematics are widely recognised. For international students studying in a host country where language, culture and educational systems may differ substantially from home, these challenges may be experienced in unique ways. This study forms part of a wider exploration to understand international student experiences during the mathematical transition from school to university.

Detailed accounts of the first-year experiences of two international students were gathered in semi-structured interviews. The students, from Malaysia and Sri Lanka respectively, had schooled largely in other countries before completing their first university mathematics course in New Zealand. Using identity as a lens for interpretation, an analysis of their accounts highlighted the importance of collaborative strategies as they sought to make sense of new mathematical content. Preliminary findings revealed differences in how collaborative groups were formed and in the types of interaction within these groups.

The preliminary findings promote understanding of how international students might engage with others as they grapple with first-year mathematics. This knowledge will be relevant to all who teach undergraduate mathematics to a culturally diverse student body.

UNDERSTANDING STUDENTS' SPONTANEOUS QUESTIONING IN FIRST YEAR UNDERGRADUATE MATHEMATICS ONLINE TUTORIALS

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KEYWORDS: students' spontaneous questions, mathematical inquiry, Cultural-Historical Activity Theory, Maslow's hierarchy of needs

Students' questions have been the area of interest of researchers for many years. Several studies have linked student questions with engagement, learning, and knowledge construction, but the lack of students' questions in the classroom has also been reported (Almeida, 2012; Chin & Osborne, 2008). This study aims to understand students' spontaneous questioning during online mathematics tutorials, and how this activity is shaped by the context and how it shapes their learning. We observed twelve online tutorials from two first-year undergraduate mathematics units and interviewed three students.

During the interviews, we asked students to share their reasons for asking questions and their perspectives on the factors that affect their questioning during online mathematics tutorials. Using Maslow's hierarchy of needs model (McLeod, 2007), we propose a three-level model to categorise students' questions in terms of their learning potential. We further identify students' motives to ask questions by using Leontev's 'Activity Theory' model (Leont'ev, 1978). The results indicate that the questions asked by students fall into the lower two levels of the three-level hierarchy model. The motives of students' questions have been revealed as sense-making, confused by the content being taught, and linking it to previous knowledge. The factors that affect students' questions identified are shyness, an online setting, a preference to ask questions privately through emails, lack of prior knowledge, and fear of being embarrassed in front of peers.

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USE OF GAME-BASED LEARNING TO ENHANCE CURIOSITY FOR STATISTICS EDUCATION THROUGH UNITED NATIONS SUSTAINABLE DEVELOPMENT GOALS

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KEYWORDS: Statistics Education, Curiosity, Game-based Learning, Feedback, Relevance, United Nations Sustainable Development Goals

The statistics presented in news such as unemployment rate, climate change, interest rates, and recently COVID-19 infection, or vaccination rates are of interest to most people. Logically, we expect more students wanting to study statistics but unfortunately, many students find statistics “boring” or irrelevant to their lives and studies.

Enhancing curiosity through relevance and meaningful feedback (Wang, 2019) can address the issues related to Statistics education. A game designed to educate students in Statistics education based on United Nations Sustainable Development Goals (UNSDG) (Sustainable Development Goals, 2021) could be a solution to examine the effect of feedback, and relevance on enhancing curiosity from a psychological perspective.

The Statistical game in this context consists of four levels including questions developed using Bloom’s taxonomy. The game outlines a story around UNSDG datasets related to poverty, education, and health, where feedback is supplied to learners via various characters. This presentation will report on the design of the game and the feedback strategies to enhance learners’ curiosity. Participants’ data will be collected through system interaction, and self-reporting using validated instruments. This study presents a promising approach for educators in refocusing their efforts to improve Statistics education by fostering a level of curiosity through relevance and meaningful feedback on a digital platform.

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TOWARDS A CONCEPTUALIZATION OF INVERSE PROBLEMS IN MATHEMATICS EDUCATION

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Inverse problems have traditionally been forgotten, despite their essential role in different disciplines (Bunge, 2006). Unfortunately, Mathematics Education is not the exception to this rule as it was observed by different authors (Groestch, 1999, 2001; Martinez-Luaces, 2011).

This situation implies, among other things, the absence of an elaborated theoretical framework. For this purpose, Groestch (1999, 2001) adapted the well-known IPO-model commonly used in Computer Science and other disciplines (see, for example, Martinez-Luaces, Fernandez-Plaza, Rico & Ruiz-Hidalgo, 2021). This first attempt could be a good starting point if only the cultural/conceptual dimension of Didactic Analysis is considered (Rico & Ruiz-Hidalgo, 2018). Nevertheless, it does not take into account the other three dimensions (cognitive, ethical/formative and social), which deserve to be considered.

This work reflects on some examples previously described (Martinez-Luaces, Rico, Ruiz-Hidalgo & Fernandez-Plaza, 2018; Martinez-Luaces, Fernandez-Plaza & Rico, 2020; Martinez-Luaces, Fernandez-Plaza, Rico & Ruiz-Hidalgo, 2021) and also analyzes data obtained in a doctoral thesis fieldwork (Martinez-Luaces, 2021), which concern other dimensions of the Didactic Analysis.

The final purpose of this reflection and analysis is the construction of an appropriate theoretical framework for inverse problems in Mathematics Education and in order to achieve this goal, this communication aims to be a starting point for deeper development in future works.

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KINEMATICS ADVENTURES IN DESMOS

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KEYWORDS: desmos, mathematics, kinematics, foundation education

This presentation demonstrates the use of Desmos teacher activities to teach concepts in foundation mathematics & physics. Activities have been developed[1] using the online education platform Desmos (2021) to introduce students to kinematics. Online exercises are more important than ever in the pandemic era of short-notice lockdowns and remote teaching. Having interactivity available in a template that can be demonstrated in lecture, or, if necessary, can be accessed independently by students at home, is a valuable resource.

One-dimensional kinematics exercises were developed using the teacher-Desmos feature to introduce students to the topic. Students are taken through a range of possibilities with the help of an interactive script to provide input, output, and visual feedback. This is accomplished through the computation layer scripting language built into the platform. Students can type answers into the activity that are stored for feedback or peer viewing. The subsequent 2-D topic builds on the first instance and enhances student understanding of concepts such as vectors and maximum range in physics and derivatives and their applications in mathematics.

Showing students a Desmos graphing calculator version with the equations exposed created confusion and distracted from engaging with the concepts. The activity version puts the code (and equations) in the background leaving students free to discuss and trial outcomes. Displaying and annotating the velocity vectors and range made it easy to highlight characteristics of projectile motion. Student engagement was better using an interactive web-based activity than paper-based structured learning or mixed media and student conversation revealed rich discussion.

1. Activity links available at: https://github.com/millecodex/Delta2021

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MATHEMATICS CURRICULUM ALIGNMENT AND THE TRANSITION FROM SCHOOL TO UNIVERSITY

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KEYWORDS: Mathematics, Curriculum, Transition

Mathematics curriculum reform is a hot topic in many countries. A review of the Australian Curriculum F-10 was announced in June 2020, which while conducted across all eight learning areas, prioritised *Mathematics* and *Technologies.* Considerable consultation has been undertaken in 2021, with implementation from the start of 2022. However, it is notable that the higher years have not been included in this review, nor are stakeholders from higher-education levels featured noticeably in the advisory groups, with respect for example to informing horizon knowledge of where students are headed after Year 10 and through Years 11, 12 and onto first-year university (Years 12 and 13 in New Zealand).

In New Zealand, a Royal Society Te Apārangi Expert Advisory Panel on Mathematics and Statistics was convened between January and June 2021, with a brief to provide advice to the Ministry of Education on the English-medium Mathematics and Statistics curriculum in Aotearoa New Zealand. While this review is again focused at the school level, a key focus of this review was to build and strengthen the pathways for students towards studying higher level mathematics. The report is due for publication in mid-to-late 2021, so we anticipate recommendations from this panel will be available for discussion in this presentation.

The presentation will consider and discuss ways in which we might better facilitate mathematical curriculum alignment and communication between teachers in the higher school years and those responsible for first-year courses in the quantitative sciences at tertiary level.

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ASSESSMENT OF GRADUATE PROFILE ATTRIBUTES IN A STATISTICS CAPSTONE COURSE

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KEYWORDS: Capstone, statistics, graduate profile

Compulsory capstone courses for all undergraduate pathways in the Faculty of Science at the University of Auckland were introduced in 2019. One of the main goals of the Science capstone courses was to provide a vehicle for students to demonstrate the attributes of the B.Sc graduate profile.

In semester one, 2021, the first iteration of the statistics capstone course was offered, and the second iteration was completed in semester 2. Details about the structure of the course will be shared as will student feedback from the first cohort who completed pre- and post-course surveys.

One research question was to establish whether graduate profile attributes could be demonstrated in a statistics capstone course. To do this each assessment task in the course was classified by graduate profile attribute and level of importance. Student marks for each assessment enabled the production of a ‘grade’ by attribute and an overall graduate profile ‘grade’ for each student. To test the validity of this automated method, student work was qualitatively coded and then scored using a framework synthesised from the American Statistical Association (2014) guidelines for statistics graduates, the VALUES rubrics (AACU, n.d.) and the University of Auckland graduate profile. Work to compare the two classification methods is on-going but some preliminary results will be shared.

The statistics capstone course is still a work in progress, with new challenges in the second iteration because of larger numbers of students, overseas online students, and changing from an in-person to an online working environment due to Covid lockdowns.

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STaMPS IN INTRODUCTORY STATISTICS COURSES

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KEYWORDS: Statistical Literacy, COVID-19, Foundation course

Improving levels of statistical literacy has been a focus for many introductory statistics courses in our data rich world. The COVID-19 pandemic has resulted in an elevation in the mathematical and statistical demands of items appearing in the media. Gal and Geiger's (2021) research revealed a dearth of information on exactly what is involved in these new demands. Their research analysed statistical and mathematical products (STaMPs) from four different countries and discovered new or enhanced types of knowledge and skill demands. A question that emerges from this research is how students can be supported to develop their ability to critically evaluate media articles that they read, view, or listen to.

A Foundation Statistics course has been offered on the Tertiary Foundation Certificate programme at the University of Auckland for the last 3 years. This programme offers students a second chance at a tertiary qualification and has a diverse student cohort. Successful graduates of the programme go on to a range of undergraduate study options including Science, Social Science, Engineering and Arts. One of the goals of the Statistics course is to improve students' ability to critically evaluate media articles. Hence, a range of STaMPs were selected, for students to work on throughout the semester, with the support of question prompts and criteria for evaluating statistically based reports.

This presentation will share some examples of STaMPs used in this course, examples of student responses to these at the beginning and end of the course and feedback from the students.

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EXPANDING OUR INSIGHT INTO STUDENTS' ABILITY AND QUESTION DIFFICULTY IN AN ONLINE MATHEMATICS TEST

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KEYWORDS: mathematics as service subject, mathematics for financial students, Rasch measurement model, Rasch analysis, student ability, question difficulty, online mathematics multiple-choice test

The value of mathematics as a service subject for Bachelor of Commerce students might be underestimated, despite the fact that the skills these students need to work with mathematical concepts in a financial context are becoming increasingly important. The aim of the investigation unpacked in this paper was to expand our insight into students' acquisition of these important skills by exploring what value an additional measuring tool based on the Rasch measurement model can add to students' raw scores in an online multiple-choice test. The first focus was to explore new ways to identify the difficulties distance learning students' have with the content of the module by shedding more light on the possible knowledge patterns of struggling students. The second focus was to gain more insight into the difficulty levels of our first attempt at multiple-choice online assessment for this group of students. The Rasch measurement model has been designed to construct a variable measured in units called logits that places student ability and question difficulty on the same continuum or linear scale. A Rasch analysis was done using Winsteps 4.8.0.0 software. With the Rasch measurement tools, such as summary measure for person (student) and item (question), a Wright map, item measures and fit statistics, we could quickly gain more insight into students' ability and into question difficulty. These measurements prove that a Rasch analysis offers a window into the mathematical skills of these participating students. This is helpful for identifying at-risk students and aids test development.

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OPEN-SOURCE ONLINE TOOLS TO VISUALISE AND EXPLORE COMPLEX FUNCTIONS WITH DOMAIN COLOURING

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KEYWORDS: complex, functions, domain, colouring, online, tools, dynamic, visualization, open-source

Complex functions are essential mathematical objects not only in complex analysis but also in algebra, differential geometry and in many other areas such as numerical mathematics and physics. Visualising complex functions is a non-trivial task since they will produce a graph existing in a four-dimensional space. In this presentation, I provide an overview of the method known as domain colouring to visualise and explore complex functions. I also present a set of open-source online tools which main goal is to help students, and anybody interested in this topic, to create significant connections between visual representations, algebraic calculations and abstract mathematical concepts about complex functions.

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SECONDARY SCHOOL TEACHERS' USE OF MULTISENSORY LEARNING

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KEYWORDS: multisensory learning; secondary school teachers; classroom activities

Facilitating the exploration of multiple representations, improving inclusivity for students with learning difficulties and disabilities, and aiding in learners' recall of concepts, multisensory learning has many benefits. Twenty mathematics and statistics secondary school teachers from across New Zealand responded to an anonymous online questionnaire investigating their use of a range of senses in their classroom. While visual media were almost always used, sounds, tactile components, interactive elements, olfactory senses, and active tasks were less commonly implemented. Typically, this was due to a lack of resources or too much time being required for creating tasks. To help introduce more constructivism-based learning opportunities, this research promotes the use of multisensory aspects in task design for high school students and within a small class undergraduate setting.

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FROM INQUIRY-BASED LEARNING TO STORY TELLING FOR ONLINE DELIVERY IN A FOUNDATION STATISTICS COURSE

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KEYWORDS: statistics, simulating inquiry-based learning online, story telling

Inquiry-based learning promotes engagement, curiosity, and experimentation. Rather than being 'instructed to,' students are empowered to explore subjects by asking questions and finding or creating solutions. It's more a philosophy and general approach to education than a strict set of rules and guidelines.

Inquiry-based learning works well when you are in small class situations where students can work together on tabletop activities, but how do you adapt this approach when you are forced to teach online? How can you simulate classroom activities designed to explore statistical concepts?

Going online meant I had to change delivery and I have found myself combining the exploration of concepts through inquiry-based activities with more storytelling. Storytelling is not used much in mathematics and statistics and yet can be very beneficial. It can provide the background needed to create context and has learning beyond a topic or technique. I have found that many assessments that I create for students are about telling that story. Beyond the statistical skills and knowledge, I want my students to be able to tell the story for their data and make decisions that are personal or community changing.

I will share my experiences of teaching online using inquiry-based learning and storytelling.

ASSESSMENT-RELATED SELF-EFFICACY IN MATHEMATICS: A REPEATED MEASURES ANALYSIS

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KEYWORDS: self-efficacy, assessment, university mathematics

The construct of self-efficacy, developed by Bandura (1977; 1997) as part of his social cognitive theory, is a central construct in mathematics education and known as one of the best predictors of mathematical performance (Siegel et al., 1985). Self-efficacy refers to learners' expectations and beliefs about their ability to learn new material, develop new skills, and master tasks. As self-efficacy is interwoven with cognition and achievement (Usher & Pajares, 2009), it is vital to understand how it develops and changes. The dominant instructional design theories recognise the importance of self-efficacy and posit that enhancing self-efficacy leads to improved achievement.

For tertiary educators, it is of particular concern to promote, and not hinder, the development of student self-efficacy within the time constraints of a single semester. We conducted a quasi-experimental, repeated measures study in a second-year university service mathematics course to test the effects of frequent online quizzes (Evans et al., 2021; Riegel & Evans, 2021) on assessment-related self-efficacy in students (N = 277). Modelling demonstrated that self-efficacy around one form of assessment influences self-efficacy in another form of assessment and performance on one form of assessment indirectly influences self-efficacy in another. The results suggest that repeated experiences on low-risk summative assessment can influence the efficacy of students going into an exam. However, high-weight, exam-like assessments during the semester can overwhelm these effects. The findings are discussed together with implications that educators should plan courses so that assessments are designed to support the development of student assessment self-efficacy.

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MATHEMATICIANS' METAPHORS FOR ISOMORPHISM AND HOMOMORPHISM

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KEYWORDS: isomorphism, homomorphism, conceptual metaphor

Isomorphism and homomorphism are topics central to the teaching of abstract algebra (Melhuish, 2015) but research on students' and, especially, mathematicians' understandings of these topics remain limited. Weber and Alcock (2004) examined mathematicians' reasoning around isomorphism while proving theorems and highlighted the use of sameness and relabeling language to describe isomorphism. Hausberger (2017) highlighted the use of structure-preservation language in textbooks to describe homomorphism. More recently, Rupnow (2021) examined two mathematicians' language for both isomorphism and homomorphism in interview and teaching contexts and observed four clusters of metaphors: sameness, mapping, sameness/mapping, and formal definition. Nevertheless, these studies leave questions about whether other types of reasoning remain undiscovered. To address this, we examine nine mathematicians' language for isomorphism and homomorphism based on interviews focused on their understandings and ways of teaching isomorphism and homomorphism in abstract algebra. In order to analyze language use, we use a conceptual metaphor lens (e.g., Lakoff & Núñez, 1997), in which the cross-domain transfer of ideas between well-developed source domains and target domains of interest is centered. Here the target domains of isomorphism and homomorphism are informed by source domains such as sameness and relabeling. Building on Rupnow's (2021) prior work, we use thematic analysis (Braun & Clarke, 2006) and consensus coding to highlight new metaphors within the sameness cluster, including connections to other mathematical branches' analogues of isomorphism, as well as a fifth metaphor cluster centered around isomorphism and homomorphism as tools for changing perspectives.

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CHANGE IN HASTE, REGRET AT YOUR LEISURE? MEASURING THE CHANGES IN SUPPORT WORKSHOPS FORCED BY COVID-19

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KEYWORDS: mathematics support, online vs face-to-face delivery, metrics

The COVID-19 pandemic has provided an imperative for change in the way we teach and support mathematics. The Mathematics Education Support Hub, Western Sydney University, offers a diverse array of support workshops for students. All workshops were delivered face to face before March 2020, and then moved to online delivery, with most remaining online to now (September 2021).

In the context of a series of nursing numeracy workshops and a series of refresher workshops for incoming undergraduates, we will discuss some metrics which have been used in an attempt to measure the effects of this change of delivery mode in terms of effectiveness and reach. These include results of pre- and post-tests used in both modes of delivery, face-to-face attendance, time spent in Zoom sessions, and the plethora of data provided by the learning management system. We will also discuss dimensions of the change from face-to-face to online delivery that are not measured by these metrics, and how the metrics might inform future development of the workshops.

DEFENDING UNINVIGILATED ONLINE EXAMS AGAINST COMPUTER ALGEBRA SYSTEMS AND COMMUNICATION

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KEYWORDS: assessment design, online, uninvigilated

One of the many challenges posed by the COVID-19 pandemic has been the need to alter assessments to run in an online environment. In this talk, I focus on the course for which I found this process most challenging – a general mathematics course taught to a large audience of students from a number of different degree programs.

In the past, most test and exam questions in the course were computation-based, and the teaching materials are designed with this in mind. However, in an online open book examination, students can access computer algebra systems capable of carrying out most computations, and assessment design must take this into account. An additional constraint is that our test and exam must consist of multiple choice problems. This means we are vulnerable to students sharing solutions, and must also defend against misconduct of that kind. This talk will discuss how we worked within these constraints to design fair tests and exams.

TEACHING SUBSPACES IN LINEAR ALGEBRA: BLENDING EMBODIMENT, SYMBOLISM, AND FORMALISM

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KEYWORDS: linear algebra, subspaces, blending, embodied, symbolic, formal

Linear algebra is a core topic for mathematics students. Many science and engineering students are required to take a course in linear algebra at university. Research in linear algebra education shows that many undergraduate students find linear algebra difficult and may only gain a shallow understanding of its powerful concepts and often forget them soon after completing the course. In this study, we investigated introducing subspaces to linear algebra students in one single lecture. To examine the nature of students' thought processes, we employed Tall's (2008) three worlds of mathematical thinking as a theoretical framework. The main challenge for teaching was engaging students to blend the embodied, symbolic, and formal worlds meaningfully.

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DEVELOPMENT OF PROSPECTIVE ELEMENTARY TEACHERS' MATHEMATICAL MODELING COMPETENCIES AND CONCEPTIONS

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KEYWORDS: mathematical modeling, elementary teacher education, prospective teachers

Over the last two decades, the mathematics education community increased research on and attention to the education of prospective teachers in mathematical modeling. Within teacher education, much research is devoted to better preparing future and current middle and high school teachers in teaching modeling, yet standards across the United States include mathematical modeling in elementary grades, and the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) report (2016) emphasizes modeling in elementary grades to help students develop skills in modeling for later grades. To better understand elementary teachers' conceptions of mathematical modeling, research was conducted using mathematical modeling curricular units that were implemented in mathematics content courses for prospective elementary teachers. The curricular units emphasized the mathematical modeling process focusing on the various elements of and approaches to modeling rather than solely on the final models. In addition to student-created models and reports, questionnaires assessing conceptions and reflections on the modeling process were collected during two modeling units across the course of a semester. In this session, we discuss the findings from the implementation of these mathematical modeling curricular units and their impact on prospective elementary teachers' conceptions of mathematical modeling.

IMPORTANCE OF MATHEMATICS AND STATISTICS IN ENGINEERING

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KEYWORDS: importance of mathematics, mathematics and engineering, performance, attitudes

Mathematics is traditionally considered necessary for engineering courses. Over the last three decades the mathematics requirements for entry into engineering programmes has steadily weakened in Australia. Further, the mathematics component of engineering programmes has progressively decreased. This research aims to investigate the following two questions. Firstly, is mathematics a barrier for students to complete an engineering programme? And secondly, is performance in mathematics associated with performance in engineering?

We investigated the significant factors associated with weighted average mark (WAM) and the completion status of engineering studies at both an undergraduate level and a Masters level. Of particular interest was student mathematical background.

Furthermore, a survey of students in enrolled in engineering at the University of Western Australia was conducted to obtain more in depth views of student attitudes and perceptions towards how mathematics and statistics has affected their engineering studies. Binary logistic models were fitted to the survey data. Additionally, focus group interviews was conducted to gain insight on student perspectives of how effective mathematics is taught in their courses. The results are discussed in relation to the importance of mathematics and statistics for the engineering curriculum.

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IMPLEMENTING GEOGEBRA LEARNING TOOLS INTO ASYNCHRONOUS ONLINE MODULES ON EVALUATING PROBABILITIES FROM PROBABILITY TABLES

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KEYWORDS: Geogebra, Asynchronous Online Learning, Probability Tables

In this study, four-step design-based instruction cycles of question, experiment, comparison and analysis, and reflection were developed and gradually operated in Probability and Statistics courses in an Applied Mathematics Department in Taiwan. This study focused on embedding four-step cycles of finding probabilities from tables on asynchronous learning platform. Scripts of instruction videos and Geogebra tools(shorted as GGB hereafter) for experiments were developed according to learning objectives of learning cycles. Each module of the learning platform consisted of 4 parts. Construction video, GGB tools, and probability tables were shared on the same screen so that students could follow instructions and operate GGB tools for experiment at the same time. Modules of Binomial and normal distributions were taught in Probability course in 2020. Results of paper and pencil tests showed that students performed well on Binomial distributions and adequate on Normal distribution. Students showed high satisfaction rate on GGB tools. Out of 44 questionnaires, the average scores were 5.94 for Binomial tools and 5.65 for Normal tools in forty-four 7-point Likert scale. With these modules, class time could focus on problem solving. Modules could also serve as review on Statistics course, before introducing errors of hypothesis testing. Complete operation would be conducted on fall semester 2021 and spring semester of 2022 for various discrete and continuous distributions. Effectiveness and efficacy of this type of learning will be then analyzed.

TEACHING WRITING TO MATHEMATICS STUDENTS

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KEYWORDS: Academic writing, Metaphor, Writing Mathematician

For the past 10 years, I have taught academic writing to students, many of whom identify more as mathematicians than as writers. Over this time, I have developed mathematics-based metaphors to teach productive writing behaviours, such as Writing-as-Modelling, Writing-as-Problem Solving, and Writing-as-Proving (Yoon, 2019). In this presentation, I share practical workshop-style writing activities that draw on similarities between writing and mathematics. These include: structuring an argument; writing as a social activity; using other peoples' texts in writing. I show how these activities can help mathematics students develop some of the behavioural, artisanal, social and emotional features of productive academic writing (Sword, 2018).

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POSTER ABSTRACTS

IMPACT OF SOME RESTRICTIVE MEASURES ON ACTIVE COVID-19 CASES IN THE CITY OF JOINVILLE: POSSIBILITIES FOR TEACHING HYPOTHESIS TESTING

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KEYWORDS: Restrictive measures. COVID-19. Student's *t*-test.

This work presents a teaching sequence to teach hypothesis testing to compare two groups. The idea is based on the COVID-19 pandemics and the locally-applied restrictive measures in the city of Joinville. The proposal seeks, using Student's *t*-test, to identify any differences between the average active COVID-19 cases before and after certain restrictive measures. Three specific periods were analyzed regarding public transportation restrictions and a curfew. All the data are available in Joinville's City Hall's website. The proposal addresses data preparation and a tutorial on comparative boxplots and Student's *t*-test with an Excel spreadsheet. The results showed significant differences between the average of active cases before and after the selected events, evidencing the reduction in the number of cases after restrictive measures were applied. The proposal can be extended for other places and the results allow for extensive discussion, both in the context of the problem or the limitations of the performed statistical analysis.

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USE OF DATA IN THE UNDERGRADUATE STATISTICS EDUCATION

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KEYWORDS: undergraduate statistics, real data, teachers' opinion

The American Statistical Association recommends the use of real data with context and purpose for teaching statistics. The context of the data has an important role in understanding how and why the data were generated or collected, and to give meaning to the statistical results in solving real problems. Based on this recommendation, the goal of this study is to explore, among other topics, how Mexican undergraduate teachers use real data in their course of statistics. The study involved 750 teachers who solved an online questionnaire with Likert scale items. Teachers' answers were organized into items that allowed us to know: 1) if the teacher uses real data in his or her statistics classes, and 2) how they seem to use these data. In the first case, it was found that 88.2% of the teachers use real data in their classes; furthermore, 94.5% of the teachers consider that the data are related to the profession of their students. In the second case, 97.1% of the teachers propose activities for students to apply statistical methods that allow them to find patterns, relationships, trends, or characteristics of interest in the data. Results also showed that disciplinary areas with the lowest use of real data were mathematics, technology, and engineering. These results offer us a snapshot of the status of undergraduate statistics education in Mexico and provide us with reference points around which to conduct future research to deepen our understanding of teachers' use of data in their classrooms.

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BICYCLABILITY: A PROPOSAL FOR TEACHING CORRELATION

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KEYWORDS: Bicycle. Correlation Analysis. Quality of bicyclable streets.

This work presents a teaching sequence to teach Correlation in undergraduate Basic Statistics courses, which aims to identify a relationship between the bicycle infrastructure quality in the city of Joinville, measured through the BEQI score, and demographic and socio-economic factors. The use of bicycles as a means of transportation is a regular practice in many cities such as Joinville, Southern Brazil, nationally known as “the City of Bicycles”. An appropriate infrastructure, however, is fundamental to promote the use of bicycles. In Joinville, the quality of bicyclable streets was evaluated through the Bicycle Environmental Quality Index (BEQI) (Henning et al., 2019). The guiding question for the proposal is: Is there a relationship between streets with better bicycle infrastructure and factors such as income, number of inhabitants and number of commercial activities? Starting from the problem design, the proposed sequence addresses data collection, scatter plot construction, coefficient correlation calculation and the subsequent results discussion. The BEQI scores are presented using spreadsheets and viewed on city maps. The remaining data are extracted from official municipal documentation. To complement the activities, an R (R CORE TEAM, 2021) tutorial, with the RStudio interface, shows the basic R functions for scatter plots and for correlation coefficient calculation, along with specific packages for that purpose.

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