Surface wave dispersion from small vertical scatterers

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[1] Heterogeneity in the subsurface creates conflicting types of dispersion of seismic waves. A laboratory and numerical experiment show that multiple scattering of elastic waves from isolated heterogeneities near the surface not only attenuates, but also delays coherent events. Because scattering off these impedance contrasts is frequency dependent, multiple scattering is a source of dispersion. If ignored, multiple scattering dispersion could be erroneously attributed to a model with horizontal homogeneous layers of different wave speeds. *INDEX TERMS:* 0935 Exploration Geophysics: Seismic methods (3025); 7212 Seismology: Earthquake ground motions and engineering; 7255 Seismology: Surface waves and free oscillations; 7260 Seismology: Theory and modeling. **Citation:** van Wijk, K., and A. L. Levshin (2004), Surface wave dispersion from small vertical scatterers, *Geophys. Res. Lett.*, *31*, L20602, doi:10.1029/2004GL021007.

1. Introduction

[2] In a homogeneous medium, surface waves are non-dispersive. In the earth, however, due to the general increase in velocity with depth, the longer wavelength components of a surface wave train penetrate deeper and hence travel faster. This effect is the basis for the widely used surface wave dispersion analysis used to infer horizontally layered earth structure in global earth seismology [*Dorman and Ewing*, 1962; *Knopoff*, 1961; *Snieder*, 1987] and shallower geotechnical applications [*Park et al.*, 1999; *Xia et al.*, 1999; *Ritzwoller and Levshin*, 2002]. *Knopoff* [1961] already pointed out that more than one horizontally layered model can explain dispersion of surface waves equally well. Therefore, in global seismology where the 1D assumption is more than likely violated anyway, more sophisticated methods incorporate higher-order Rayleigh wave modes [*Gabriels et al.*, 1987; A. L. Levshin et al., The use of crustal higher modes to constrain crustal structure across central Asia, to appear in *Geophysical Journal International*, 2004] or 3D tomographic models [e.g., *Bijwaard et al.*, 1998].

[3] In geotechnical applications surface wave dispersion is ever popular, because of its relatively straightforward experimentation and processing. On the other hand, it has been acknowledged that small isolated impedance discontinuities do exist in the near surface [*Gucunski et al.*, 1996; *Herman and Perkins*, 2004] and thus contaminate otherwise coherent seismic events. Solutions have been presented that image the subsurface when scattering can be described by the Born approximation [e.g., *Snieder*, 1987], but multiple scattering from near-surface heterogeneity is a mechanism for dispersion: using a laboratory model, we show that multiple-scattering dispersion can be confused with layer-based dispersion in a frequency/time analysis using the wrong a priori assumption of a layered model.

2. Physical and Numerical Experiment

[4] The setup of the experiment is shown in Figure 1: A 200-V repetitive pulse is used to excite an angle-beam transducer mounted on the surface of an aluminum block of dimensions $x = 232$ mm $\times$ $y = 215$ mm $\times$ $z = 280$ mm. The transducer wedge has a footprint of 70 mm by 42 mm. The angle of the transducer is such that its output in the aluminum block is mainly a broad-band Rayleigh wave, effectively planar in the transverse (y) direction, with a dominant wavelength around 6 mm.

[5] The wave field is detected along the x-direction by a scanning laser vibrometer that measures absolute particle velocity on the surface of the sample via the Doppler shift [e.g., *Nishizawa et al.*, 1997; *Scales and van Wijk*, 1999]. The signal is digitized at 14-bit resolution using a digital oscilloscope card, while the entire setup is positioned on a vibration isolation table to reduce background noise.

[6] The aluminum block has a Fibonacci pattern of aligned linear grooves machined into one face. The grooves are nominally 1-mm wide (x-direction), 2.75-mm deep (z-direction) and 1 or 2 mm apart, but to represent the actual groove pattern more accurately in numerical simulations, we scanned the surface at 2400 dots per inch (90 dots per mm), allowing us to include variations – coming from mechanical machining – in the average width of the grooves and the surface between grooves. The Fibonacci sequence is quasi-periodic, but increases in complexity as it gets longer [*Carpena et al.*, 1995]. Theoretical and experimental results for transmission through Fibonacci multi-layers show that minima in the transmission coefficient (as a function of wavenumber) become deeper as the number of layers in the Fibonacci multilayer increases, asymptotically leading to true band gaps [*Gellermann et al.*, 1994]. Analytic solutions for the 1D Fibonacci scattering problem exist [*Dal Negro et al.*, 2003], although they were not used in this article.

[7] Figure 2 shows the laboratory data (top) and numerical simulations (bottom) using the spectral-element method [see *van Wijk et al.*, 2004b, and references therein]. The wave fields look qualitatively similar in the observations and numerical simulations. The earlier events show coherence in the sense that a single phase can be tracked from one

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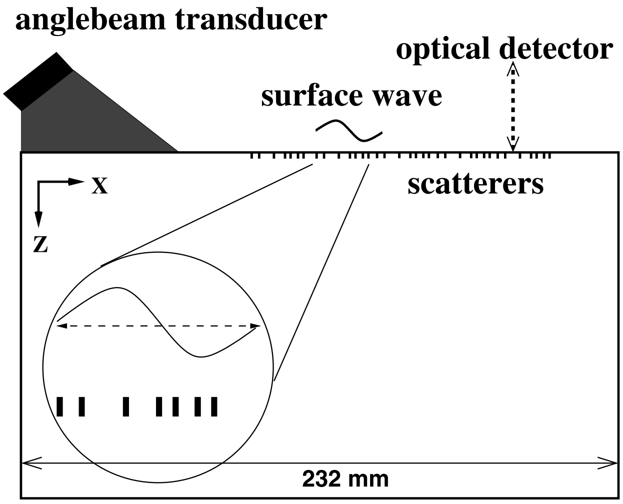

Figure 1. Schematic setup of the experiment. The anglebeam transducer generates a Rayleigh wave that is multiply scattered by the grooves cut across one face of the aluminum block in the *y*-direction. Vertical particle velocity is recorded by the laser Doppler vibrometer.

detector location to the next, whereas at later times, scattering causes arrivals to be incoherent from trace to trace. A detailed comparison between the measurements and numerical simulations [*van Wijk et al.*, 2004b] show that random fluctuations in the laboratory data are negligible for our purposes.

[8] At each groove, energy is partially reflected, causing the direct arrival to be attenuated. In fact, the strongest event in Figure 2 is not the direct wave, because scattering has reduced the amplitude of the transmitted wave to below the noise level. Instead, the strongest energy in the wave form is the result of constructively interfering multiply scattered surface waves, much as the wavelet built from peg-leg multiples of body waves in a finely layered Earth model discussed by *O'Doherty and Anstey* [1971].

[9] The faster surface waves are those not disturbed by the grooves at all. These are mainly longer wavelengths that sample a deeper part of the model, beneath the depth of the grooves. Higher frequencies sample a shallower part of the medium and are slowed down by multiple scattering between the grooves. Whereas single scattering would only extract energy from the transmitted wave, MS causes the forward energy to be delayed. Since the scattering strength is frequency dependent, dispersion is the result. While we know that this is the mechanism for dispersion in this otherwise homogeneous medium, in the next section we present models that fit the observed dispersion, under the wrong a priori assumption of a horizontally layered model.

3. Interpretation: A Layered Model

[10] To analyze the dispersion in the records of Figure 2 we use the Frequency-Time Analysis technique (FTAN) developed at CIEI/CU [*Levshin et al.*, 1989; *Ritzwoller and Levshin*, 1998]. As an output of FTAN for each time-windowed record (in this case between 0.02 ms and 0.08 ms), we obtain the phase velocity $C(\nu)$ as a function of frequency ν (top panel of Figure 3). At each frequency, the phase velocity is an average over six detector pairs. The error bars are one standard deviation of the phase velocity extracted from different detector pairs: local differences of the groove pattern are a likely source of variance, but the FTAN is robust with respect to the time- and frequency windows used.

[11] Differential phase velocity curves described above are used for inversion in the framework of the surface wave theory for vertically-heterogeneous layered models [*Herrmann*, 1978]. The search for the best solution is carried out in the space of several unknown parameters, namely shear velocities and thicknesses of layers. Other parameters (i.e., densities and longitudinal velocities) are either fixed or functionally tied to shear velocities. The search procedure includes an iterative two-step approach. The initial layered model chosen by intuitive reasoning is perturbed by the conjugate gradient technique [e.g., *Nolet*, 1987] to get the model with the best fit of the predicted dispersion curve to the observed curve. To make results independent of the selection of the initial model, this model is perturbed by the Monte Carlo technique before the next

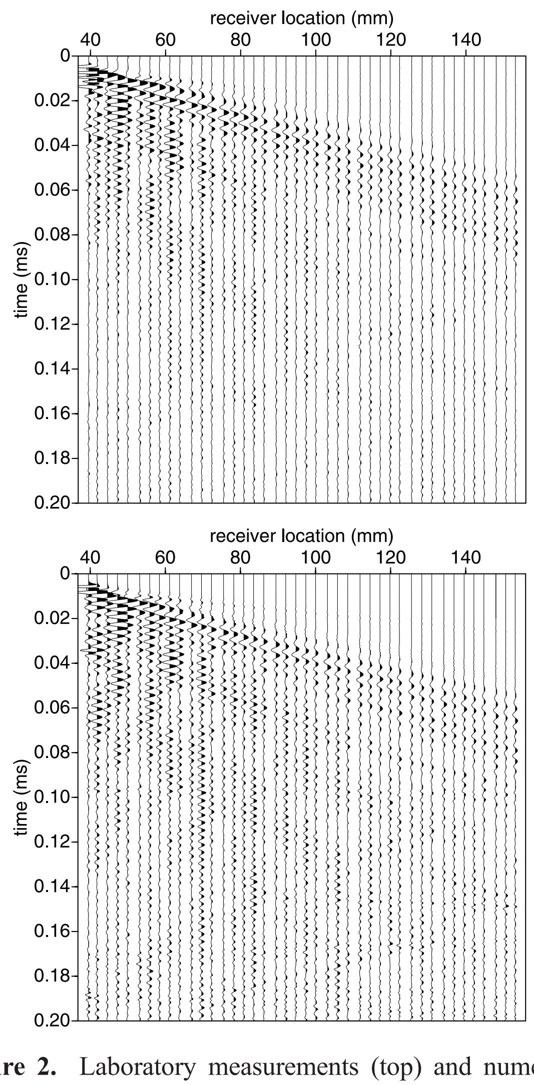

Figure 2. Laboratory measurements (top) and numerical simulations (bottom) of the vertical component of the particle velocity between the first 42 grooves in an aluminum block.

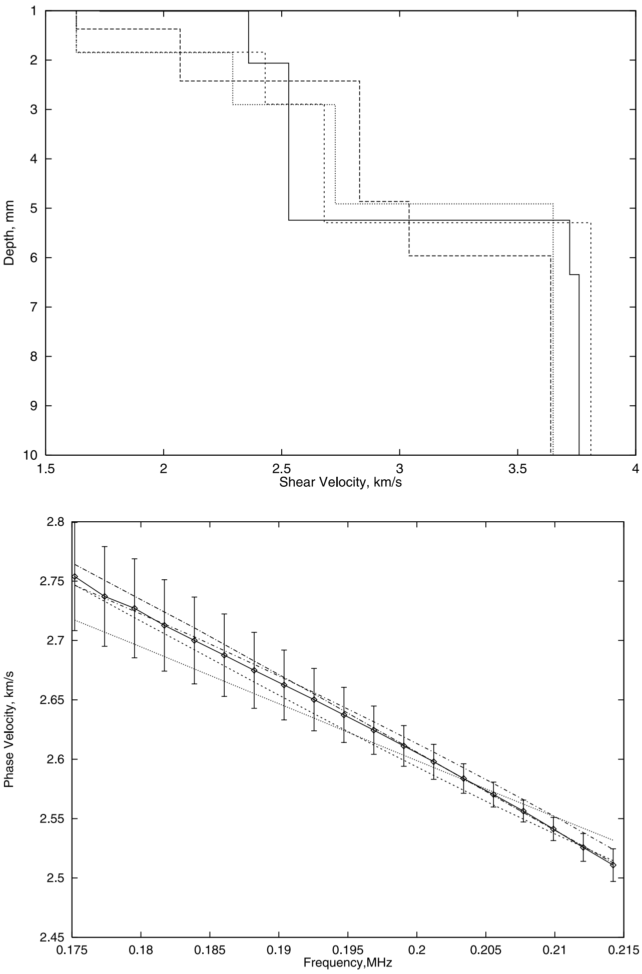

Figure 3. Layered shear-wave models (top) that fit the observed dispersion of surface waves (bottom) within one standard deviation. The true shear-wave velocity in aluminum is around 3 km/s, and the grooves are 2.75 mm deep.

search by the conjugate gradient technique. Several types of initial models are tried, some of them with small number of layers and sharp boundaries, others with a smooth transition provided by a multi-layered structure. Results of inversion for the normal incidence in a grooved model are shown in the top panel of Figure 3. A significant non-uniqueness in the inversion is evident, as different types of models fit the observations within the estimated error in the dispersion curves. All found solutions, however, exhibit a common feature: shear velocity in the bottom layer half space is equal to $\approx$3.5 km/s, which is significantly higher than the known value for aluminum ($\approx$3 km/s). In addition, at least three layers are necessary to fit the observed dispersion.

4. Discussion and Conclusions

[12] The inversion of surface wave dispersion in layered media is not unique, even in the highly reproducible milieu of the laboratory. Different layered models can explain the same observed dispersion, but we have shown a model with isolated scatterers near the surface resulting in similar dispersion that is not caused by layering at all. In reality, dispersion of surface waves is likely to be due to a combination of isolated scatterers and layering in the near surface. Strong topography, near-surface fractures, voids, and other near-surface heterogeneity show wave fields with scattering strength that is quantitatively comparable to these laboratory conditions [*Larose et al.*, 2004; *Herman and Perkins*, 2004]. Thus, an interpretation based on layering alone or including just single scattering leads to an erroneous interpretation of the subsurface.

[13] In reality, strong scattering from topography variations or subsurface heterogeneity might be easily identifiable. For example, the densely sampled wave fields of Figure 2 show many incoherent arrivals, especially at late times. While this is no formal proof of multiple scattering (such as coherent backscattering [*Larose et al.*, 2004], equipartitioning of wave modes [*Hennino et al.*, 2001], and randomization of surface waves [*Campillo and Paul*, 2003]), it is clear that the a priori layered model is not a valid assumption here, and a more extensive analysis is required. However, deterministic methods such as full wave-form inversion in the presence of strong scattering might be too ambitious, as small errors in the model result in large deviations between the data and predicted data [*van Wijk et al.*, 2004b]. Alternatively, statistical methods such as radiative transfer theory can provide us with ensemble-averaged model parameters in the presence of strong scattering [*Goedecke*, 1977; *Wu*, 1985; *Margerin et al.*, 1999; *van Wijk et al.*, 2004a].

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