

Multiple scattering attenuation and anisotropy of ultrasonic surface waves

John A. Scales^{a)} and Kasper Van Wijk^{b)}

Physical Acoustics Laboratory and Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines, Golden, Colorado 80401

(Received 29 December 1998; accepted for publication 28 April 1999)

Multiple scattering of waves induces bulk effects such as attenuation and anisotropy that are important in seismology, optics, medical imaging, and other fields involving propagation in disordered media. We report here measurements of ultrasonic surface wave propagation in a strong-scattering, quasiperiodic medium consisting of a grooved surface of aluminum. Using noncontacting optical methods we have tracked the evolution of surface wave pulses within the scattering medium. Waves propagating parallel to the grooves propagate nearly attenuation and dispersion free, whereas waves propagating normal to the grooves are dispersed and exponentially attenuated with distance as energy is transferred from the direct pulse into the multiple-scattering coda. We measure this attenuation length and show that there is, in addition, a scattering induced anisotropy in the phase velocity. © 1999 American Institute of Physics.

[S0003-6951(99)04425-3]

In the past few years measurements aimed at quantifying strong scattering in disordered media have begun to be published. For example, Wiersma *et al.*¹ recently reported the first observations of an Anderson transition for light. Tourin *et al.*² measure coherent backscattering in ultrasonic waves. Scheffold and Maret³ give the first evidence of the classical wave analog of universal conductance fluctuations. Yodh and Chance⁴ describe recent advances in the use of diffuse wave medical imaging. Bizheva *et al.*⁵ use low-contrast interferometry to discriminate between singly and multiply scattered photons and study the transition between the ballistic and diffusive regimes of propagation. An overview of recent theoretical and experimental work in this field can be found in Ref. 6. A comprehensive theoretical treatment of multiple-scattering of classical waves can be found in the textbook of Ishimaru.⁷

Here we describe measurements of strong multiple scattering in a nearly one-dimensional (1D) quasiperiodic medium consisting of a Fibonacci pattern of grooves cut in the surface of an aluminum block. These sequences are frequently used in studies of disorder since they are quasiperiodic yet exhibit considerable complexity as the order of the sequences increases (e.g., Refs. 8 and 9). Further, renormalization group analysis has been used to derive scaling properties of such systems.¹⁰ Fibonacci layered dielectrics have been studied experimentally by Gellermann *et al.*⁹ who observed localizing behavior as the number of layers increased. A clear discussion of disorder-induced localization of waves in layered media is given by Berry and Klein,¹¹ who describe analogous optical experiments. The current work was inspired by the results of Burke *et al.*¹² who measured ultrasonic crack-induced anisotropy, but in the process also showed surprisingly strong scattering attenuation of the surface waves. Using noncontacting optical methods we have

followed the evolution of a surface wave pulse as it propagates through a strong scattering medium.

The experimental setup is shown in Fig. 1. A 200 V repetitive pulse is used to excite an angle-beam transducer mounted on the surface of an aluminum block of dimensions 28 cm×23 cm×21.5 cm. The transducer (Panametrics 0.5 MHz Videoscan ABT) consists of a piezoelectric compressional wave transducer attached to a wedge cut at an angle such that the horizontal component of the velocity of *P* waves generated in the wedge matches the surface wave velocity in the aluminum, resulting in efficient production of mode-converted surface waves.¹³ The wedge used had a footprint of 7 cm (in the forward direction) by 4.2 cm (Fig. 1).

To record the wave forms we used a laser-Doppler vibrometer (Polytec sensor head and vibrometer controller with a high-frequency decoder). The signal was amplified with a low-noise preamplifier (SR 560 with 12 db/octave 10 kHz high-pass filter) and digitized at 8-bit resolution using a

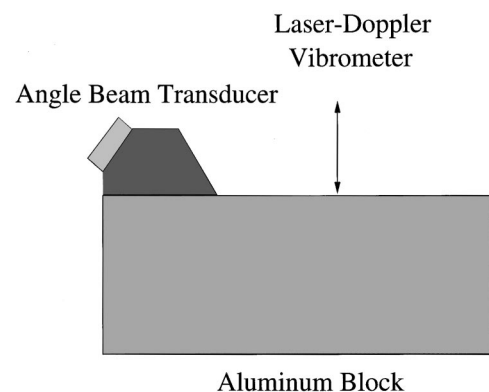


FIG. 1. The source consists of a piezoelectric transducer coupled to a wedge cut at an angle such that the horizontal component of the velocity of *P* waves generated in the wedge matches the surface wave velocity in the aluminum. The aluminum block is rectangular with dimensions 28 cm×23 cm×21.5 cm. The transducer wedge is 7 cm in the direction of wave propagation and 4.2 cm wide.

^{a)}Electronic mail: jscales@mines.edu

^{b)}Electronic mail: kasper@dix.mines.edu

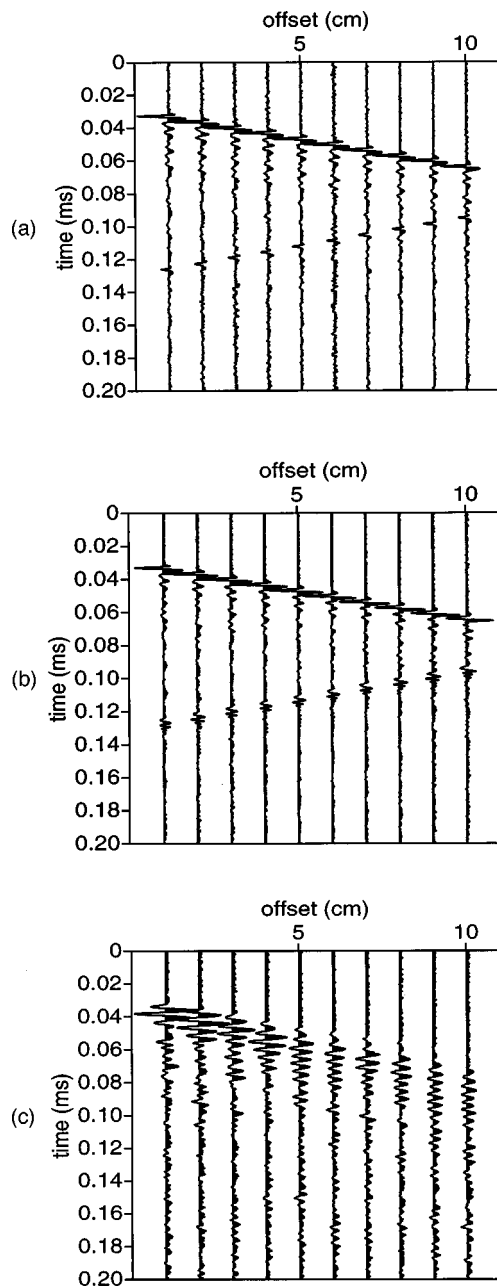


FIG. 2. Traces recorded at 10 locations on the aluminum surface: (a) traces recorded on the smooth face, (b) for propagation parallel to the grooves, and (c) for propagation perpendicular to the grooves. The source-receiver offsets are measured from the leading edge of the transducer wedge. For (a) and (b) the only events visible are the direct surface wave and a surface wave reflected off opposite end of the block. For propagation perpendicular to the grooves the offsets are not exactly at 1 cm spacing since the beam could not be focused in a groove. In this case measured receiver locations are listed in Table I. Each trace is the average of 100 shots.

Gage digital oscilloscope card attached to a PC.

Measurements were made on both an ungrooved side of the aluminum and on one with a pattern of grooves cut in a Fibonacci sequence. A Fibonacci sequence can be made by concatenating the previous two sequences in the series. Let $S(0)=A$ and $S(1)=B$ be the base elements of the sequence. Then the n th order sequence is obtained via

$$S(j)=\{S(j-1),S(j-2)\}, \quad j=2,3,\dots,n,$$

where the curly braces denote concatenation (as opposed to addition for the usual Fibonacci numbers). For example, A

TABLE I. Source-receiver distances (offset) measured from the leading edge of the transducer for the waves propagating perpendicular to the grooves. In this case, due to the presence of the grooves, it was not possible to use precisely 1 cm spacing as was the case for the other two geometries.

Receiver Number	Offset (mm)
1	9
2	18
3	28.5
4	40
5	51
6	59
7	68
8	76.5
9	87
10	99

and B might denote the value of some physical property characterizing each of the base units of the sequence. In the present problem A and B are used to denote the presence or absence of a groove; for example, the sequence ABB would denote a groove followed by two nongrooves. If the basic unit of spacing (the width of a groove or a nongroove) is h , then ABB would denote a groove of width h followed by ungrooved surface of width $2h$. In the present case $h=1$ mm. The dominant frequency is about 200 kHz. This corresponds to a wavelength of about 15 mm. Thus, we are in a regime in which there are many scatterers per wavelength and the wave penetrates some distance below the base of the grooves, giving effectively a frequency-dependent transmission coefficient. For 1D (layered) systems, any degree of randomness is sufficient to cause localization since there are only two Lyapunov exponents, they sum to zero, and the randomness ensures that the exponents are nonzero.¹⁴ In higher dimensions the situation is more complicated and experimental evidence of classical localization remains elusive. In addition, the grooved aluminum is not perfectly 1D since there will be diffractions from the base of the grooves.

Figure 2 shows *common source gathers* (i.e., wave forms or traces recorded at different receiver locations for a fixed source location) for three different configurations (Table I). Each trace is the average of 100 repeated pulses. Figure 2(a) shows data for propagation on a smooth (un-grooved) side of the same aluminum block. Figure 2(b) shows propagation parallel to the grooves. Finally, Fig. 2(c) shows propagation normal to the grooves. In the first two figures the only visible events are the direct surface wave and a surface wave reflected off the far end of the aluminum block. Propagation parallel to the grooves is very similar to propagation on a smooth face, and in both cases there is little attenuation or dispersion. In contrast, the scattering attenuation and dispersion is clearly evident in the third figure. This is a purely elastic (and frequency dependent) effect as energy is simply being transferred from the first arriving pulse into the multiple-scattering coda.

Figure 3 shows a plot of the peak amplitude of the first pulse as a function of distance from the source for the three propagation geometries. For propagation normal to the grooves the decay is well fit by an exponential. The decay length will vary according to the properties and distribution of the scatterers; in this particular case the $1/e$ decay length is about 30 mm.

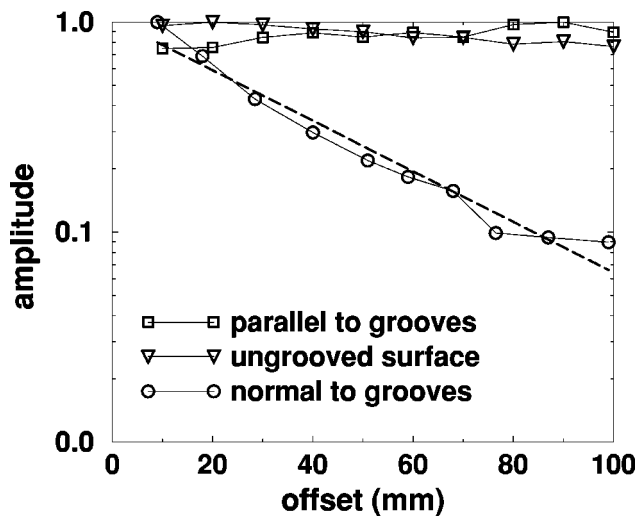


FIG. 3. Attenuation of the first pulse and the best fitting exponential. The $1/e$ attenuation is achieved at about the fourth trace, corresponding to an average decay length of about 30 mm. Also shown are measurements for waves propagating parallel to the grooves and on the ungrooved face.

Figure 4 shows the travel time of the first-arriving surface wave peak as a function of the source-receiver offset for the three geometries. We estimated the phase velocity from the slopes of the best fitting straight lines. For propagation on the smooth face, or for propagation parallel to the grooves,

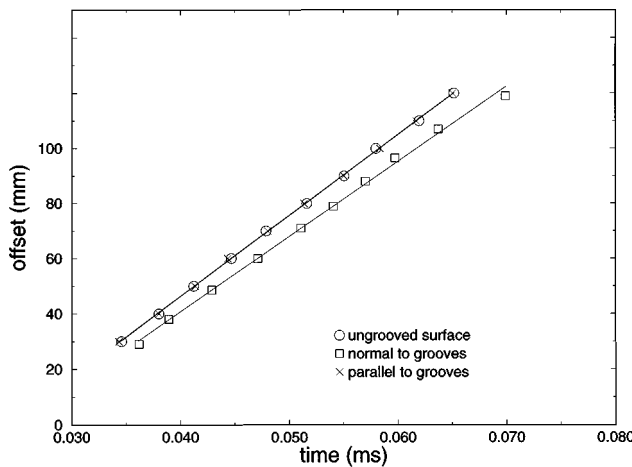


FIG. 4. Measured phase velocity for the grooved (perpendicular and parallel) and ungrooved data. For propagation parallel to the grooves linear regression yields a phase velocity of 2.93 ± 0.02 mm/ μ s. For propagation normal to the grooves the phase velocity is only 2.72 ± 0.05 mm/ μ s.

regression yields a phase velocity of 2.93 ± 0.02 mm/ μ s. Whereas for propagation normal to the grooves the phase velocity is only 2.72 ± 0.05 mm/ μ s. Since the depth of the medium is many wavelengths and the medium itself is homogeneous, except for the grooves (which are perturbations of the free surface), this reduction in the phase velocity is due to the interference of the multiply scattered waves. In other words, for propagation normal to the grooves we see a long wavelength effective anisotropy caused by the multiple scattering. This effect is well-known in seismology where geologic layering gives rise to long wavelength transverse isotropy.¹⁵

Using noncontacting optical methods we have tracked the evolution of a surface wave pulse as it passes through a strong-scattering, quasiperiodic medium. Multiple scattering gives rise to substantial dispersion and velocity anisotropy as well as an exponential decay of the pulse for propagation normal to the aligned scatterers. This exponential decay is a purely elastic effect as energy in the pulse is being shifted backward into the multiple scattering coda.

We thank Bob Kranz and Mike Batzle of the Colorado School of Mines for their considerable help in performing these experiments. This work was partially supported by the US Army Research Office under Grant No. DAAG55-98-1-0070.

- ¹D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, *Nature (London)* **390**, 671 (1997).
- ²A. Tourin, A. Derode, P. Roux, B. van Tiggelen, and M. Fink, *Phys. Rev. Lett.* **79**, 3637 (1997).
- ³F. Scheffold and G. Maret, *Phys. Rev. Lett.* **81**, 5800 (1998).
- ⁴A. Yodh and B. Chance, *Phys. Today* **48**, 34 (1995).
- ⁵K. K. Bizheva, A. M. Siegel, and D. A. Boas, *Phys. Rev. E* **58**, 7664 (1998).
- ⁶*New Aspects of Electromagnetic and Acoustic Wave Diffusion*, Springer Tracts in Modern Physics, edited by POAN Research Group (Springer, New York, 1998), p. 144.
- ⁷A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Oxford University Press, Oxford, 1997).
- ⁸D. Toet, M. Potemski, Y. Y. Wang, and J. C. Maan, *Phys. Rev. Lett.* **66**, 2128 (1991).
- ⁹W. Gellermann, M. Kohmoto, B. Sutherland, and P. C. Taylor, *Phys. Rev. Lett.* **72**, 633 (1994).
- ¹⁰M. Kohmoto, L. Kadanoff, and C. Tang, *Phys. Rev. Lett.* **50**, 1870 (1983).
- ¹¹M. V. Berry and S. Klein, *Eur. J. Phys.* **18**, 222 (1997).
- ¹²S. K. Burke, L. R. F. Rose, and C. M. Scala, *Appl. Phys. Lett.* **65**, 1349 (1994).
- ¹³R. W. Damon, W. T. Maloney, and D. H. McMahon, *Excitation, Detection, and Attenuation of High-Frequency Elastic Surface Waves*, in *Physical Acoustics: Principles and Methods*, edited by W. P. Mason and R. N. Thurston (Academic, New York, 1970), Vol. VII.
- ¹⁴J. A. Scales and E. S. Van Vleck, *J. Comput. Phys.* **133**, 27 (1997).
- ¹⁵G. Backus, *J. Geophys. Res.* **67**, 4427 (1962).