Elastic scattering by planar fractures

Thomas E. Blum[1], Roel Snieder[2], Kasper van Wijk[1], and Mark E. Willis[3]

[1]Physical Acoustics Lab
Boise State University
Boise, ID

[2]Center for Wave Phenomena
Colorado School of Mines
Golden, CO

[3]Formerly at ConocoPhillips, Houston, TX,
now at Halliburton, Houston, TX

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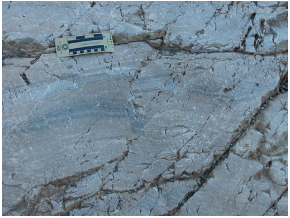
Outline

Faults and fractures

- Controls fluid flow: hydrocarbons, water, magma. . .
- Characterization of fracture properties with elastic waves
- Active or passive monitoring

http://makel.org/fractures/



http://phobos.ramapo.edu/

http://http://geotripper.blogspot.com/

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Theoretical expression and laboratory modeling

- Single fracture

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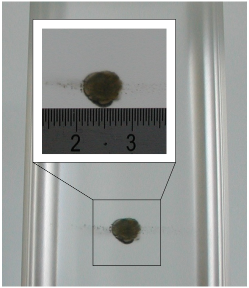
- Frequency domain: $f(t) = \int F(\omega) e^{-i\omega t} d\omega$
- Previous work: small fractures $\Rightarrow$ effective medium (Crampin, 1981; Hudson, 1981),
  or large fractures $\Rightarrow$ reflection coefficients (Pyrak-Nolte et al., 1990; 1992)

Theoretical expression and laboratory modeling

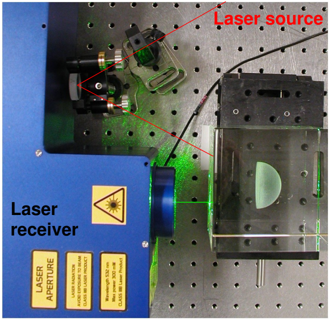
- Fractured plastic samples
- Ultrasonic frequencies (100 kHz - 10 MHz) $\Rightarrow \lambda \sim 10^{-4}$ - 10^{-2} m
- Laser generation and detection of body waves



Units are cm

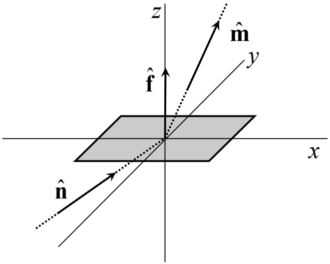
Ultrasonic laser receiver

- Wide bandwidth
- Absolute displacement
- Non-contact and small footprint compared to the wavelength
- No moving parts
- Scanning system



General expression

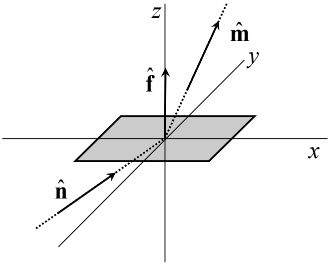
- Decomposition of the compliance η:
  $\eta_{ij} = \eta_N f_i f_j + \eta_T (\delta_{ij} - f_i f_j)$



σ	stress
ω	angular frequency
α	P-wave velocity
ρ	density of the material
k_α	wavenumber
R	distance to the fracture

General expression

- Decomposition of the compliance η:
  $\eta_{ij} = \eta_N f_i f_j + \eta_T (\delta_{ij} - f_i f_j)$
- Displacement as a function of the scattering amplitude:
  $u_n^{(P)}(\mathbf{x}) = f_{PP}(\eta) \frac{e^{ik_\alpha R}}{R} m_n$



σ	stress
ω	angular frequency
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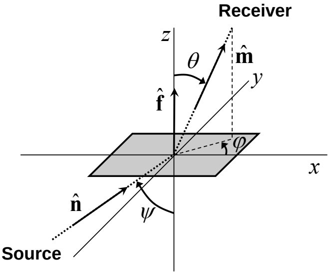
R	distance to the fracture

Scattering amplitude of a circular plane crack

For the experimental geometry:

$$f_{P,P}(\psi,\theta) = \frac{\omega a}{2\rho\alpha^3(\sin\psi - \sin\theta)} J_1\left(\frac{\omega a}{\alpha}(\sin\psi - \sin\theta)\right)$$
$$\times \left[\eta_N \left\{(\lambda+\mu)^2 + (\cos 2\psi + \cos 2\theta)(\lambda+\mu)\mu + \mu^2(\cos 2\psi \cos 2\theta)\right\} + \eta_T \mu^2 \left(\sin 2\psi \sin 2\theta\right)\right] .$$

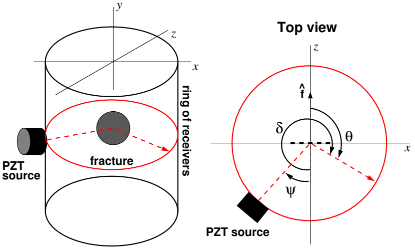
$\Rightarrow$ term in η_N, and term in η_T non-zero for $\psi \neq 0$



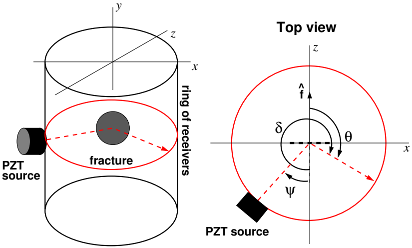
ω	angular frequency
α	P-wave velocity
ρ	density of the material
λ, μ	Lamé parameters
a	fracture radius

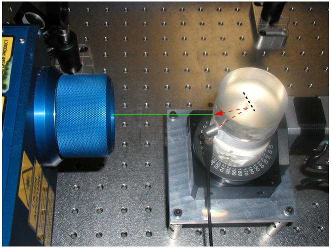
Experimental setup

- Sample: PMMA cylinder (transparent plastic material), 150 mm high x 50.8 mm diameter
- Piezoelectric transducer source, 5 MHz, 400 V pulse
- Laser ultrasonic receiver: wide bandwidth (20 kHz – 20 MHz), absolute vertical displacement, small footprint, sensitivity in Å
- Fixed source-fracture angle ψ and moving receiver (θ changes)



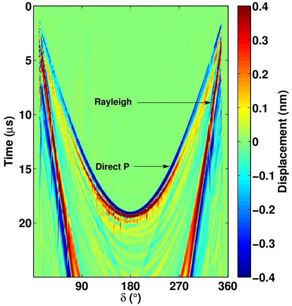
Experimental setup: geometry





Non-fractured sample

- velocities $\alpha = 2600$ m/s and $\beta = 1400$ m/s
- $\rho = 1190$ kg/m^3 $\Rightarrow$ Lamé parameters $\lambda = 3.4$ GPa and $\mu = 2.3$ GPa



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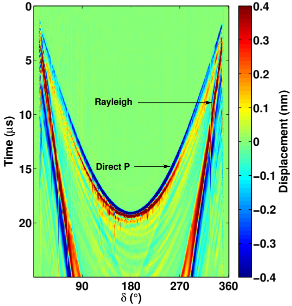
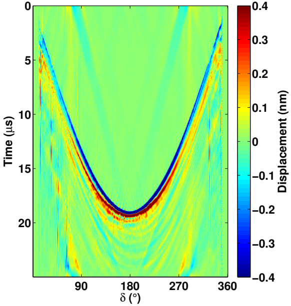
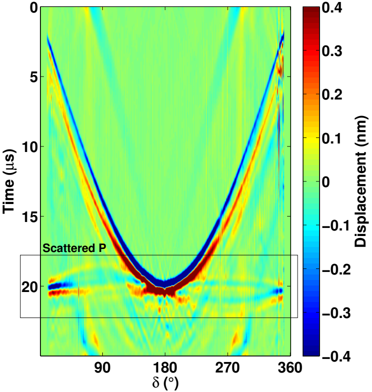
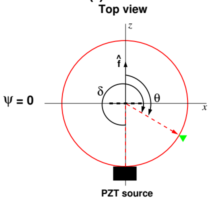
- $\rho = 1190$ kg/m^3 $\Rightarrow$ Lamé parameters $\lambda = 3.4$ GPa and $\mu = 2.3$ GPa
- *f-k* filter to remove surface waves





Fractured sample: data





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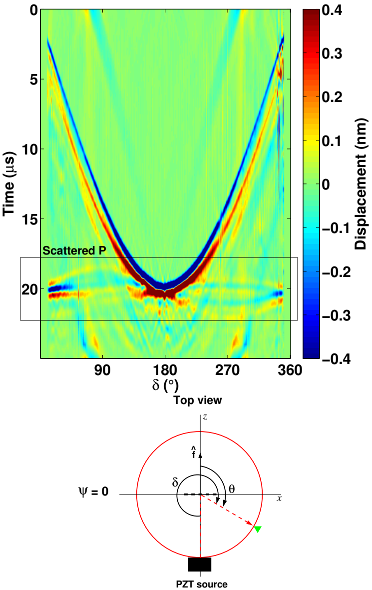
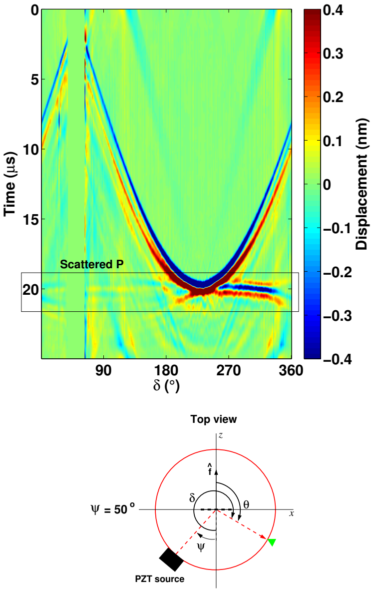
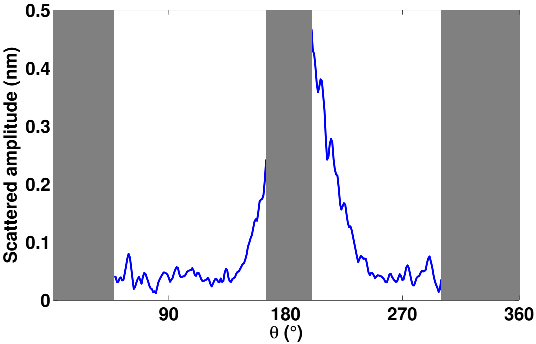




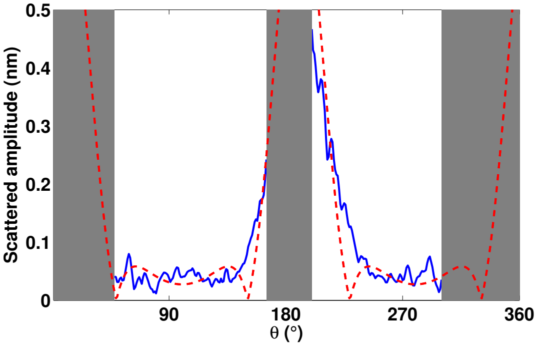
Fractured sample: scattering amplitudes, $\psi = 0^\circ$



Influence of η_N:

experimental amplitude

Fractured sample: scattering amplitudes, $\psi = 0°$

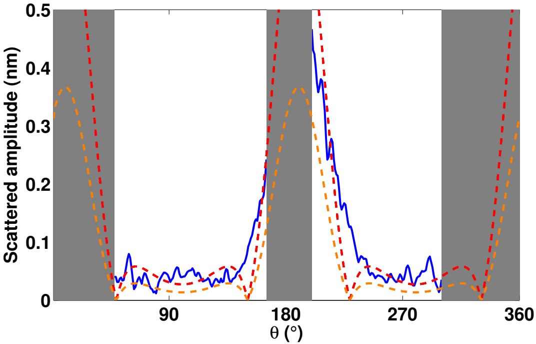


Influence of η_N:

experimental amplitude

$\eta_N = 10^{-11}$ m/Pa

Fractured sample: scattering amplitudes, $\psi = 0°$



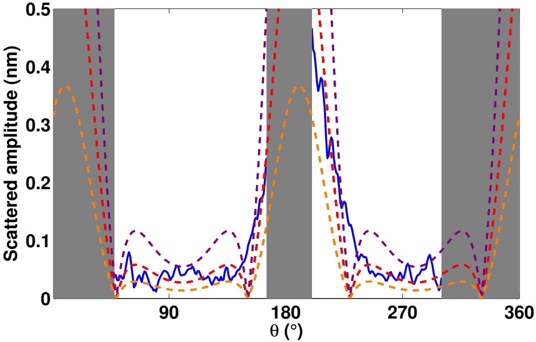
Influence of η_N:

experimental amplitude

$\eta_N = 10^{-11}$ m/Pa

$\eta_N = 0.5\ 10^{-11}$ m/Pa

Fractured sample: scattering amplitudes, $\psi = 0°$



Influence of η_N:

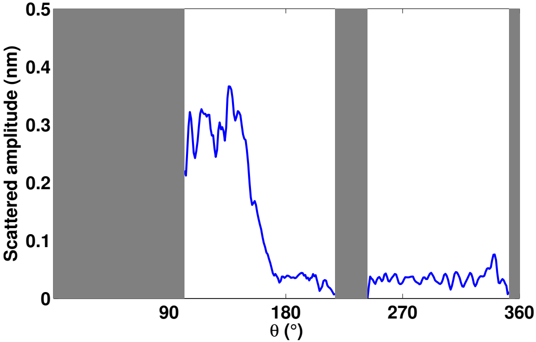
experimental amplitude

$\eta_N = 10^{-11}$ m/Pa

$\eta_N = 0.5\ 10^{-11}$ m/Pa

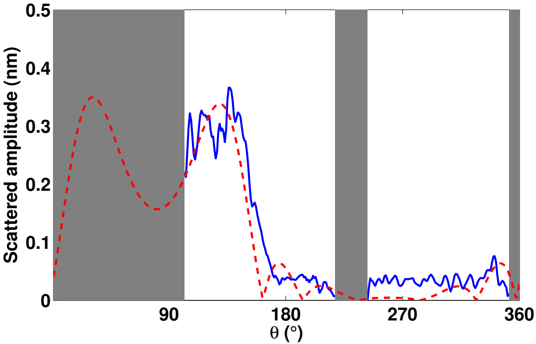
$\eta_N = 2\ 10^{-11}$ m/Pa

Fractured sample: scattering amplitudes, $\psi = 50°$



Influence of η_T:
experimental amplitude

Fractured sample: scattering amplitudes, $\psi = 50^\circ$

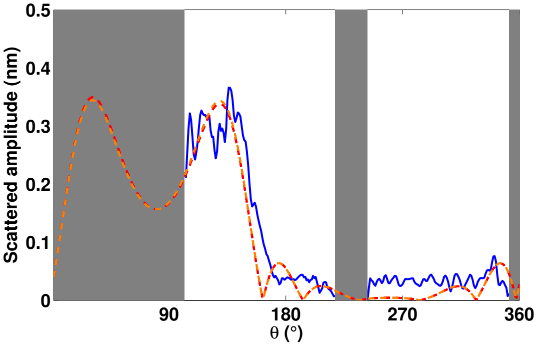


Influence of η_T:

experimental amplitude

$\eta_T = 10^{-12}$ m/Pa

Fractured sample: scattering amplitudes, $\psi = 50°$



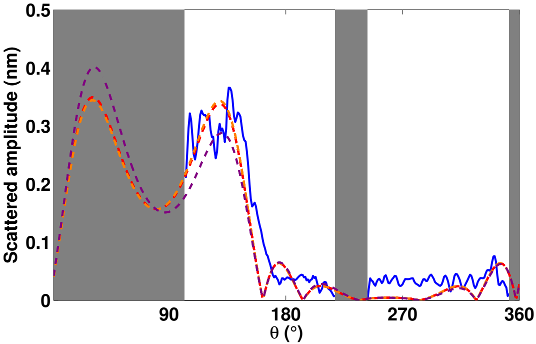
Influence of η_T:

experimental amplitude

$\eta_T = 10^{-12}$ m/Pa

$\eta_T = 10^{-13}$ m/Pa

Fractured sample: scattering amplitudes, $\psi = 50°$



Influence of η_T:

experimental amplitude

$\eta_T = 10^{-12}$ m/Pa

$\eta_T = 10^{-13}$ m/Pa

$\eta_T = 10^{-11}$ m/Pa

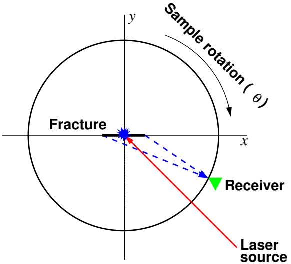
Fracture scattering: summary

- Analytic expression for the scattering amplitude
- Good agreement between theory and laboratory data
- Estimation of the compliance $\eta_N \approx 10^{-11}$ m/Pa
- Same range of compliance as found the literature (Pyrak-Nolte et al., 1990, Worthington, 2007)
- Low sensitivity to η_T

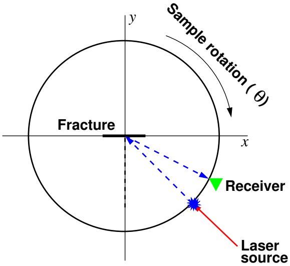
Direct excitation of a fracture

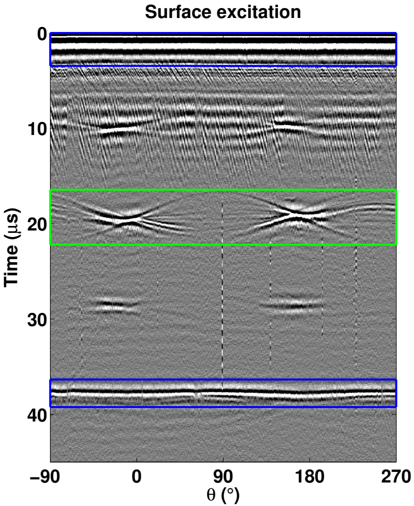
Experimental setup

- Same fractured sample
- Direct excitation by laser-induced thermal expansion
- Pulsed infrared laser source

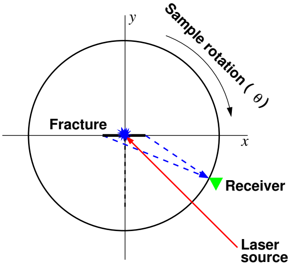


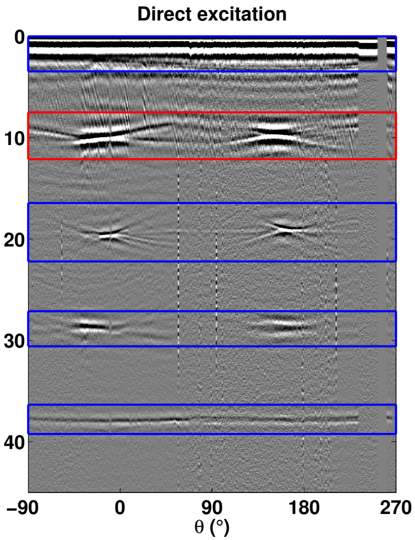
Results


y
Sample rotation (θ)
Fracture
x
Receiver
Laser
source

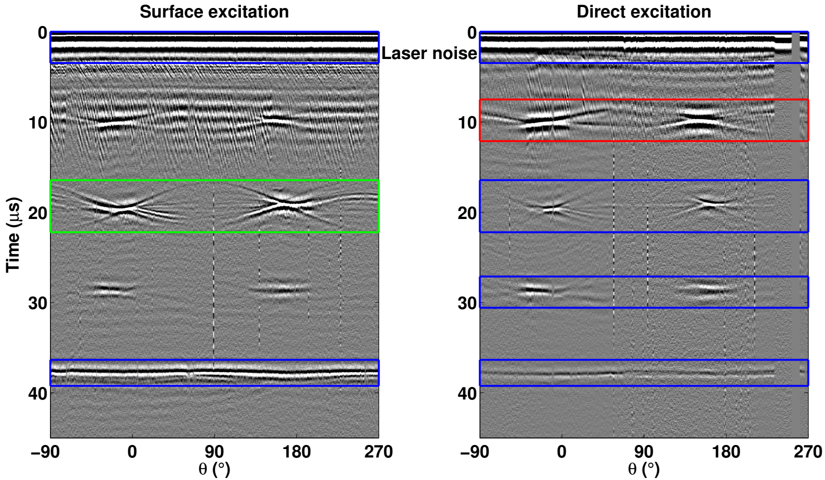

Surface excitation
Time (μs)
0
10
20
30
40
–90
0
90
180
270
θ (°)

Results

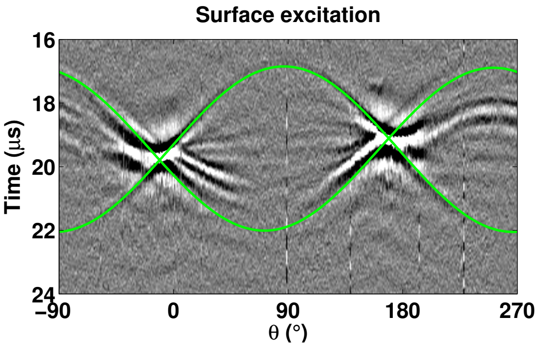

y
x
Sample rotation (θ)
Fracture
Receiver
Laser
source

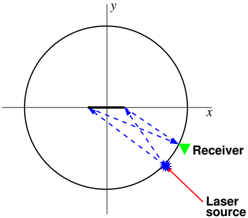

Direct excitation
0
10
20
30
40
–90
0
90
180
270
θ (°)

Results


Surface excitation
Direct excitation
Laser noise
Time (μs)
0
10
20
30
40
–90
0
90
180
270
θ (°)

Tip diffractions $\Rightarrow$ radius estimation $a = 3.5$ mm





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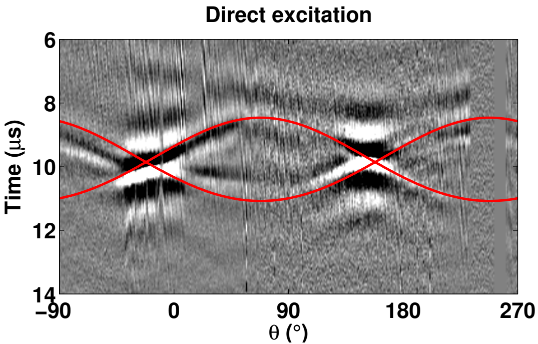

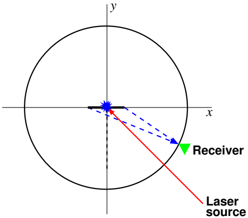

Aknowledgements

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