Dynamic aspects of apparent attenuation and wave localization in layered media

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Summary

We present a theory for multiply-scattered waves in layered media which takes into account wave interference. The inclusion of interference in the theory leads to a new description of the phenomenon of wave localization and its impact on the apparent attenuation of seismic waves. We use the theory to estimate the localization length at a CO_2 sequestration site in New Mexico at sonic frequencies (2 kHz) by performing numerical simulations with a model taken from well logs. Near this frequency, we find a localization length of roughly 180 m, leading to a localization-induced quality factor Q of 360.

Introduction

Intrinsic seismic attenuation bears the direct imprint of rheological properties and fluid conditions in the subsurface and is thus a valuable parameter to measure in the field. Such a measurement is complicated by the fact that subsurface conditions not necessarily related to rheology or fluids, for example heterogeneity, also attenuate seismic waves. As a result, heterogeneity causes field measurements of attenuation to reflect an apparent instead of an intrinsic attenuation (Gorich and Muller, 1987). In layered media, the apparent attenuation is a weighted combination of intrinsic attenuation and scattering attenuation due to reflection and transmission at interfaces. As famously shown by O'Doherty and Anstey (1971), the multiple scattering of waves must be taken into account to properly gauge the attenuation due to scattering. White *et al*. (1990) and Shapiro and Zien (1993) have further shown that a particularly strong type of multiple scattering, known as wave localization, is key to the understanding of scattering attenuation in layered media.

We adapt a recently published theory (Haney and van Wijk, 2007) for multiply-scattered waves to describe scattering attenuation in a general layered subsurface model. An example of such a subsurface model is one constructed from well logs. The modifications are needed since the original theory shown in Haney and van Wijk (2007) used a model of identical thin layers randomly located within a homogeneous background medium. Here, this restriction is relaxed and a model consisting of layers of random density, P-wave velocity, and thickness is assumed.

The theory takes into account wave interference and is therefore able to represent wave localization. We find that scattering attenuation is in fact a combination of two distinct scattering mechanisms: one due to scattering out of the main direction of wave propagation which would exist in the absence of interference and the other due to wave localization. The length scale over which the latter mechanism acts is called the localization length and is critical to assessing the amount of scattering attenuation in a particular model. We show an application of the theory to the estimation of the localization length in a 1D model taken from well logs at the West Pearl Queen Field, a CO_2 sequestration site in New Mexico.

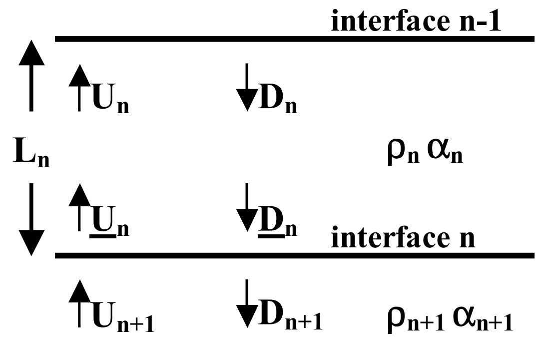

Figure 1: Up- and down-going waves near two interfaces in a layered model. The depth coordinate z is taken as positive downward. Waves are assumed to be normally-incident and therefore only density ρ and P-wave velocity α are needed in each layer of thickness L. Note that the underlined wavefields in layer *n* are just above interface *n* and the other wavefields are just below interface *n*-1.

Theory

We consider a simple model of plane waves propagating at normal incidence to planar interfaces. The model is specified in Figure 1 by the density ρ and P-wave velocity α in each layer, as well as the layer thickness L. Our theory for multiply-scattered waves begins with the scattering matrix describing the mixing of up- and down-going wavefields at the *n*-th interface

$$\begin{bmatrix} \underline{U}_n \\ D_{n+1} \end{bmatrix} = \begin{bmatrix} R_A & T_B \\ T_A & R_B \end{bmatrix} \begin{bmatrix} \underline{D}_n \\ U_{n+1} \end{bmatrix}, \qquad (1)$$

where R_A and R_B are the reflection coefficients and T_A and T_B are the transmission coefficients from above (subscript

A) and below (subscript B) the n-th interface. The locations of the up- and down-going wavefields in equation 1 are shown in Figure 1. We assume that particle velocity is being measured and take the reflection and transmission coefficients to be

$$R_A = \frac{\rho_n \alpha_n - \rho_{n+1}\alpha_{n+1}}{\rho_n \alpha_n + \rho_{n+1}\alpha_{n+1}} = -R_B , \qquad (2)$$

$$T_A = \frac{2\rho_n \alpha_n}{\rho_n \alpha_n + \rho_{n+1}\alpha_{n+1}} = \frac{\rho_{n+1}\alpha_{n+1}}{\rho_n \alpha_n} T_B . \qquad (3)$$

For simplicity, we do not include intrinsic attenuation in the formulation here since its effects are well known (Haney *et al*., 2005) and can be taken into account later.

The scattering matrix in equation 1 is not well-suited for developing a scattering theory because it is not unitary; that is, the matrix (denoted by M) does not satisfy $MM^T = M^TM = I$, where the superscript T represents the matrix transpose and I is the identity matrix. Physically, the unitary property is an expression of energy conservation and reciprocity. This problem can be solved by using wavefields scaled by the square-root of the local impedance, as shown by Frasier (1970). We therefore consider scaled wavefields (no italics) in terms of the original particle velocity wavefields (in italics) as

$$\underline{\mathrm{U}}_n = \sqrt{\rho_n \alpha_n}\, \underline{U}_n , \qquad (4)$$

with similar scaling for the other up- and down-going wavefields U_{n+1}, $\underline{\mathrm{D}}_n$, and D_{n+1}. Note that wavefields defined in the (n+1)-th layer are scaled by the impedance in that layer.

With the use of scaled wavefields, equation 1 can be rewritten as

$$\begin{bmatrix} \underline{\mathrm{U}}_n \\ \mathrm{D}_{n+1} \end{bmatrix} = \begin{bmatrix} r & t \\ t & -r \end{bmatrix} \begin{bmatrix} \underline{\mathrm{D}}_n \\ \mathrm{U}_{n+1} \end{bmatrix}, \qquad (5)$$

where $r = R_A$ and t is given by

$$t = \frac{2\sqrt{\rho_n \alpha_n \rho_{n+1}\alpha_{n+1}}}{\rho_n \alpha_n + \rho_{n+1}\alpha_{n+1}} . \qquad (6)$$

For simplicity of notation, we suppress the subscript n on r and t for the time being. In contrast to the matrix in equation 1, the matrix in equation 5 is unitary, since the sum of r^2 and t^2 can be shown to be equal to one. The matrix equation is therefore well-suited for developing a scattering theory. We show some of the main points of the derivation here; however, the logic is basically the same as that presented in Haney and van Wijk (2007) and so we defer to that paper for more detail.

Multiple scattering effects can be examined by studying the second statistical moment of the wavefield – that is, the energy (Haney *et al*., 2005). In order to study the energy of the wavefield, we take the squared magnitude of the two equations making up the matrix equation in equation 6. This gives the following two equations:

$$\left|\underline{\mathrm{U}}_n\right|^2 = r^2\left|\underline{\mathrm{D}}_n\right|^2 + t^2\left|\mathrm{U}_{n+1}\right|^2 + 2rt\mathrm{Re}\left(\underline{\mathrm{D}}_n \mathrm{U}^*_{n+1}\right), \qquad (7)$$

and

$$\left|\mathrm{D}_{n+1}\right|^2 = t^2\left|\underline{\mathrm{D}}_n\right|^2 + r^2\left|\mathrm{U}_{n+1}\right|^2 - 2rt\mathrm{Re}\left(\underline{\mathrm{D}}_n \mathrm{U}^*_{n+1}\right) \qquad (8)$$

where the asterisk represents complex conjugation and Re refers to the real part. Note that the squared-magnitude of the scaled wavefield quantities in equation 4 represent seismic energy. Adding equations 7 and 8 and using the fact that the sum of r^2 and t^2 is equal to one gives

$$\left|\mathrm{D}_{n+1}\right|^2 - \left|\mathrm{U}_{n+1}\right|^2 = \left|\underline{\mathrm{D}}_n\right|^2 - \left|\underline{\mathrm{U}}_n\right|^2 , \qquad (9)$$

where the interference terms in equations 7 and 8 (the last terms on the righthand side) have cancelled each other. Since the wavefields at the base of layer n, $\underline{\mathrm{D}}_n$ and $\underline{\mathrm{U}}_n$, are related to the wavefields at the top layer n, D_n and U_n, by simple phase advance or delay (as shown in Figure 1), the squared magnitude of either $\underline{\mathrm{D}}_n$ or $\underline{\mathrm{U}}_n$ is equal to the squared magnitude of either D_n or U_n (Haney and van Wijk, 2007). Equation 9 can therefore be rewritten as

$$\left|\mathrm{D}_{n+1}\right|^2 - \left|\mathrm{U}_{n+1}\right|^2 = \left|\mathrm{D}_n\right|^2 - \left|\mathrm{U}_n\right|^2 . \qquad (10)$$

This equation, as pointed out by Haney and van Wijk (2007), is a statement of energy flux conservation and has been used previously by Claerbout (1985) to derive the method of acoustic daylight imaging.

We obtain a second energy equation, in addition to equation 10, by subtracting instead of adding equations 7 and 8. After some algebraic manipulation, this gives

$$\left(\left|\mathrm{D}_{n+1}\right|^2+\left|\mathrm{U}_{n+1}\right|^2\right)-\left(\left|\mathrm{D}_n\right|^2+\left|\mathrm{U}_n\right|^2\right)= -\frac{2r^2}{t^2}\left(\left|\mathrm{D}_n\right|^2-\left|\mathrm{U}_n\right|^2\right)-\frac{4r}{t}\mathrm{Re}\left(\underline{\mathrm{D}}_n\mathrm{U}^*_{n+1}\right). \quad (11)$$

Note that, in contrast to equation 10, this equation contains an interference term (the last term on the righthand side). Haney and van Wijk (2007) describe the proper treatment of this term; it requires a relation between U_{n+1} and $\underline{\mathrm{D}}_n$ and depends on whether the observation point is above or below the source. The correct treatment of this interference term allows the phenomenon of wave localization to be included in the theory.

Once wave interference is taken into account in equation 11, we average equations 10 and 11 over all realizations of the random medium and a limiting procedure is invoked to move from discrete variables (e.g., a wavefield in the *n*-th layer) to continuous ones (e.g., a wavefield as a function of depth coordinate z). This procedure is shown in detail in Haney and van Wijk (2007). The result is two coupled differential equations in terms of the up- and down-going energies (the average squared magnitudes of the scaled directional wavefields). However, instead of up- and down-going energies, we choose to use total energy, the sum of the up- and down-going energies, and the energy flux, the down-going energy minus the up-going energy. These two wavefield quantities are denoted by T and F respectively. Note that they are functions of the depth coordinate z. For time harmonic wavefields, the two differential equations are

$$\frac{\mathrm{d}F}{\mathrm{d}z}=0, \quad (12)$$

$$\frac{\mathrm{d}T}{\mathrm{d}z}=-\left[\frac{2B}{l_s}+\frac{1}{l_{loc}}\right]F-\frac{\mathrm{sgn}(z-z_s)}{l_{loc}}T, \quad (13)$$

where z_s is the source depth and the scattering directivity B, the scattering mean free path l_s, and the localization length l_{loc} are given by

$$B=\frac{\left\langle r_n^2\right\rangle}{\left\langle r_n^2\right\rangle+\left\langle (t_n-1)^2\right\rangle}, \quad (14)$$

$$\frac{1}{l_s}=\frac{1}{\left\langle L_n\right\rangle}\left\langle\frac{r_n^2}{t_n^2}\right\rangle, \quad (15)$$

$$\frac{1}{l_{loc}}=\frac{2}{\left\langle L_n\right\rangle}\mathrm{Re}\left\langle\frac{R_1 r_n}{1-R_2 r_n}\right\rangle, \quad (16)$$

and where brackets denote ensemble averaging. The parameters R_1 and R_2 represent reflection coefficients at a point for the entire layered medium toward the direction of the source and away from the direction of the source, respectively. The connection between equations 10 and 12 and equations 11 and 13 should be evident by comparing their respective expressions. In the case of a time-dependent source, equations 12 and 13 become two partial differential equations:

$$\frac{\partial F}{\partial z}+\frac{1}{v}\frac{\partial T}{\partial t}=\frac{\delta(z-z_s)w(t)}{v}, \quad (17)$$

$$\frac{\partial T}{\partial z}+\frac{1}{v}\frac{\partial F}{\partial t}=-\left[\frac{2B}{l_s}+\frac{1}{l_{loc}}\right]F-\frac{\mathrm{sgn}(z-z_s)}{l_{loc}}T. \quad (18)$$

where $w(t)$ is the (energy) source waveform (the squared magnitude of the analytic signal of the amplitude source

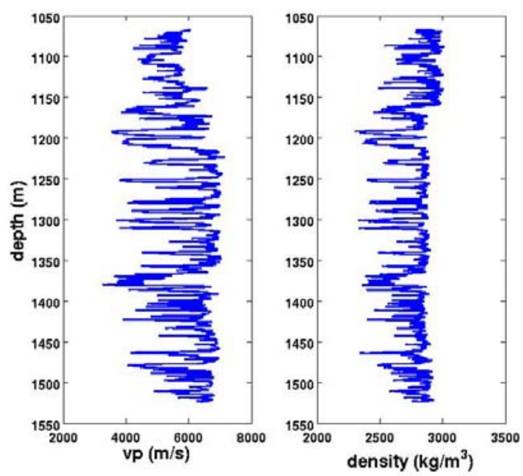

Figure 2: P-wave velocity and density logs from the West Pearl Queen Field. The depth interval shown here includes the Yates, Seven Rivers, and Queen formations.

waveform) and v is the group velocity. By solving equations 17 and 18, we find that a direct wave propagates outward from the source with a squared-amplitude that decays with distance exponentially over a characteristic distance we call the extinction mean free path l_{ext} given by

$$\frac{1}{l_{ext}} = \frac{B}{l_s} + \frac{1}{l_{loc}} \quad (19)$$

This equation shows that a pulse in a layered medium experiences two types of scattering attenuation. The first type (first term on the righthand side) is due to scattering out of the forward propagation direction and it exists even without wave interference. The second type is due to the phenomenon of wave localization, which is entirely the result of wave interference. Note that, had our derivation included intrinsic attenuation, there would be an extra factor of $1/l_i$ in equation 19, where l_i would be the characteristic distance over which intrinsic attenuation acts to exponentially dampen a propagating pulse. Thus, the apparent attenuation of seismic waves is the result of three distinct mechanisms: intrinsic attenuation and two types of scattering attenuation.

Further analysis of the solution of equations 17 and 18 shows that the energy of the multiply-scattered coda waves in a layered medium becomes "trapped" near the source location with a characteristic exponential distance equal to the localization length. This is a hallmark of wave localization in the time-dependent case. We exploit this property in the following numerical example to estimate the localization length at the West Pearl Queen Field in New Mexico in the sonic frequency range (2 kHz).

Numerical example

We use a velocity-pressure, staggered-grid finite difference code suited for 1D normal-incidence wave propagation to simulate full-waveforms in a model taken from the P-wave velocity and density well logs shown in Figure 2. These well logs are from the West Pearl Queen Field, a CO_2 sequestration site in New Mexico. The well logs show the dolostones and interbedded sandstones of the Yates (base at 1160 m), Seven Rivers (1160-1360 m), and Queen formations (top at 1360 m). The well logs are sampled with a depth interval of 0.1524 m (0.5 ft). At the edges of the model, we place absorbing boundaries so as to avoid artificial reflections from the edge of the numerical model. In 1D, such absorbing boundaries are trivial to implement.

Shown in Figure 3 is the spatial distribution of the seismic energy at a late time due to a 2 kHz (Ricker wavelet) planar explosion source at a depth of 1300 m. The time shown in Figure 3 is long after the large amplitude direct waves have been absorbed at the upper and lower edges of the numerical grid. Note that the y-axis, plotting energy, is logarithmic. Thus, exponential decay of the coda wave energy would show up as a triangle in this plot peaked at the source position. Such a gross exponential trapping of the multiply scattered coda waves is evident in Figure 3. Since the theory we have presented here is statistical (relying on averages over random realizations) there is some fluctuation about the spatial exponential decay away from the source position. However, estimates of the decay length, the localization length, can be obtained by fitting straight lines to the energy on either side of the source. By averaging these two estimates, we come away with a rough estimate of 180 m for the localization length at the dominant frequency of the source (2 kHz).

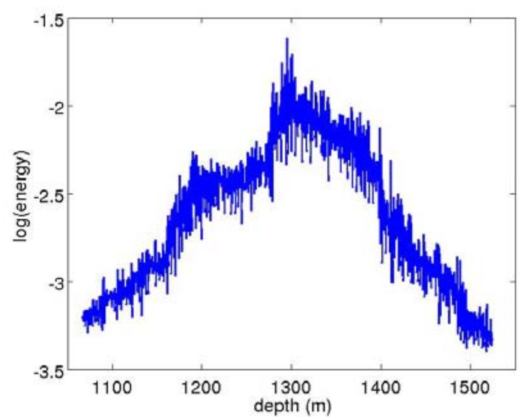

Figure 3: The total energy of the wavefield at late times in the model of Figure 2 taken from the West Pearl Queen field.

Using the relation between localization length and localization Q ($l_{loc} = vQ_{loc}/2\pi f$) we find a localization Q of roughly 360, a realistic value for seismic Q. Thus, wave localization should play a significant role in the attenuation of seismic waves in the sonic frequency range and higher (> 2 kHz) at the West Pearl Queen Field.

In conclusion, we have described a multiple scattering theory for layered media which includes wave interference and, as a result, the phenomenon of wave localization. In the future we plan to explore its applicability in the coda wave interferometry method (Pacheco and Snieder, 2005) in layered media for mapping subtle spatial variations in time-lapse subsurface material properties.

Acknowledgements

We thank Professor Bob Benson of the Colorado School of Mines for access to the well logs from the West Pearl Queen Field.

EDITED REFERENCES

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