Plenary talks of the Joint Meeting of the NZMS, AustMS and AMS

all plenaries will take place in the Fisher & Pakel Auditorium 260-115

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Effective algorithms for linear groups: success and challenges

Eamonn O'Brien[1]

[1]University of Auckland

Monday 9 December: 10:30-11:20

Matrix groups defined over finite fields arise in many contexts, perhaps most naturally in representation theory, a fundamental area of research. Despite their ubiquity, we have long faced a paucity of effective algorithms for their study.

In this lecture I will survey a long-running and highly successful program designed to address this need. It exploits randomization, deep results from the classification of finite simple groups, and remarkable estimates from the relatively new discipline of statistical group theory to provide algorithms which answer many natural geometric and structural questions about such groups. These algorithms have well-understood complexity and demonstrate practical efficiency.

A striking outcome is that, subject to the existence of certain number-theoretic oracles, we can now construct in polynomial time the composition factors of a linear group. The work also has important implications for groups defined over infinite domains. I identify how we can use this machinery to exploit theoretical solutions to long-standing questions, and mention some challenging open questions.

Around the generalised Lax conjecture

James Saunderson[1]

[1]Monash University

Monday 9 December: 11:30-12:20

Hyperbolic polynomials are multivariate polynomials with certain real-rootedness properties. Originally studied in the context of PDEs, these polynomials also arise naturally in areas such as optimisation and combinatorics. Associated with a hyperbolic polynomial is a convex geometric object called a hyperbolicity cone. Important special cases include polyhedral cones, the humble ice-cream cone, and the cone of symmetric positive semidefinite matrices.

The generalised Lax conjecture asserts that every hyperbolicity cone is the intersection of a cone of positive semidefinite matrices with a linear space. This is known to be true in three dimensions, a result of Helton and Vinnikov, but is open in general. This talk aims to introduce these objects and ideas and survey a selection of recent results that are related, in various ways, to the generalised Lax conjecture.

Exploring pattern formation

Priya Subramanian[1]

[1]University of Auckland

Tuesday 10 December: 9:10-10:00

A wide array of physical systems spanning atomic to atmospheric scales, including many we encounter in daily life as physical, chemical or biological systems, display a variety of the same regular repeating patterns, e.g., stripes, squares, hexagons, etc. How are such diverse systems able to organise into the same ordered structures? Does the display of a particular pattern at the large scale inform us of structural/physical restrictions at a smaller scale? Questions like these motivate the investigation of patterns as an entity independent of the specific system in with they arise.

The study of pattern formation concerns itself with developing a range of tools to model and analyse minimal mechanisms and then to characterise the obtained patterns, irrespective of any particular application. It borrows methods and tools heavily across multiple areas of mathematics, such as partial differential equations, dynamical systems and numerical computing.

My talk will start with the idea of how to model a prototypical pattern-forming system and then to analyse the possible patterns that can arise within it in two and three dimensions. I will then explore some of the limitations of this model and what ingredients can be added to allow more complicated patterns to arise, e.g., superlattice patterns and quasipatterns. The rest of my talk will focus on my use of two new connections within mathematics that help explore and classify patterns. The first is the use of computational algebraic geometry to obtain all equilibria with a chosen symmetry. The second is the use of topological data analysis to characterise obtained patterns quantitatively.

A new computational paradigm for computing the persistent homology of Rips filtrations

Katharine Turner[1]

[1]Australian National University

Tuesday 10 December: 10:30-11:20

Given a point cloud in Euclidean space and a fixed length scale, we can create simplicial complexes (called Rips complexes) to represent that point cloud using the pairwise distances between the points. By tracking how the homology classes evolve as we increase that length scale, we summarise the topology and the geometry of the "shape" of the point cloud in what is called the persistent homology of its Rips filtration.

A major obstacle to more widespread take up of persistent homology as a data analysis tool is the long computation time and, more importantly, the large memory requirements needed to store the filtrations of Rips complexes and compute its persistent homology. We bypass these issues by finding a "Reduced Rips Filtration" which has the same degree-1 persistent homology but with dramatically fewer simplices.

The talk is based off joint work is with Musashi Koyama, Facundo Memoli and Vanessa Robins.

The mathematics of solitaire

Persi Diaconis[1]

[1]Stanford University

Tuesday 10 December: 18:00-19:00

People play 'ordinary solitaire' (Klondike) millions of times a day. Yet, we mathematicians can't figure out 'what is the chance of winning?' or 'how can I play well?'

Indeed, even the chess and go playing programs 'Alpha zero' can't solve these problems (thank goodness that there are some things the computer can't do). I'll introduce an easier solitaire where we can figure things out. The math involved is some of the deepest of the past 50 years. It has applications to the way fire burns and the rings left by coffee cups. AND of course, it's beautiful in its own right. I'll try to explain all to a general audience 'in English'.

This is a story of how representation theory interacts with statistical mechanics

Richard Kenyon[1]

[1]Yale University

Wednesday 11 December: 9:10-10:00

The dimer model is a classical model of statistical mechanics, involving the study of random perfect matchings (dimer covers) of a graph. When the graph is planar, determinantal and other linear algebra techniques can be used to compute many quantities of probabilistic interest.

We extend this model to planar graphs with a GL(n) connection, showing how the associated determinant enumerates traces of "multiwebs", which are representation-theoretic objects.

All terms will be defined, and some applications will be discussed.

Searching for interesting mathematical objects with neural networks

Geordie Williamson[1]

[1]University of Sydney

Thursday 12 December: 9:10-10:00

Finding proofs and finding examples are critical parts of the mathematical process. Over the last few years there has been enormous effort in academia and industry to try to use AI to discover proofs. The focus of my talk will be different: can we use AI to find interesting examples?

I will focus on three different areas where AI has proven useful: extremal combinatorics, combinatorial geometry and knot theory. I will explain the role played by the neural network in each search task. A common theme is that although the neural network improves search in every example (and in several cases allows one to discover the best known examples), the effect is often subtle and interesting. By looking at several examples in parallel, I hope to give a sense of where neural networks might help the mathematician interested in finding new phenomena using computers and AI.

This is joint work with lots of people: the extremal combinatorics work is joint with Charton, Ellenberg and Wagner. The knot theory work is joint with Charton, Narayanan and Yacobi, and the combinatorial geometry work is joint with Davies, Gupta, Racanière, Swirszcz, Weber and Wagner.

Te ara o te reo Māori: Modelling the trajectory of the Māori language

Rachael Ka'ai-Mahuta[1] *and Michael Miller*[2]

[1] Auckland University of Technology

[2] Victoria University of Wellington

Thursday 12 December 10:30-11:20

Te reo Māori (the Māori language) is the language of the Indigenous people of Aotearoa New Zealand, however, the number of speakers of this language has undergone a decline since colonisation. As part of the movement to revitalise the Māori language, various interventions have been suggested and put into practice over the past few decades, such as introducing Māori language week, and the establishment of Kohanga Reo pre-schools (immersion-based "language nests"). In recent years, the New Zealand Government has released a series of goals for the language, including the audacious goal of achieving one million Māori language speakers by 2040. However, the likelihood of achieving the Government's goals remains uncertain. In this presentation, we introduce the context (historical and contemporary) for the need to model the language, before presenting a compartmental ODE model to analyse the trajectory of the Māori language, in terms of the number of speakers at different levels of proficiency. By incorporating existing data on language proficiency and usage, the model will evaluate the future trajectory of the prevalence of the language. We will show how this model can be used to inform strategies for achieving the goal of one million speakers by 2040, and whether achieving this goal gives a reliable indicator that the language is on the trajectory to recovery.

The hidden structure of disorder as seen by waves and minimizers

Svitlana Mayboroda[1]

[1]University of Minnesota and ETH Zurich

Friday 13 December: 9:10-10:00

The talk will concentrate on the effect of interior disorder and boundary irregularity of the behaviour of waves, minimizers, and Brownian travelers. The waves see geometric obstacles differently than the particles. The Brownian travelers might land only on a small subset of the boundary. What is the picture that they secretly recognize and obey? We shall discuss a range of phenomena, from wave localization in disorder to air transport in a lung, and underlying mathematical challenges and techniques.

Reasoning about Mathematical 'If-Then' Statements

Lara Alcock[1]

[1]Loughborough University

Friday 13 December: 10:30-11:20

Mathematics routinely uses 'if-then' statements to express theorems and conjectures, stipulating that a proposition 'if A then B' is false only if A is true and B is false. Everyday language routinely uses 'if-then' statements too, but there interpretations depend upon content and context. What does this mean for mathematics students? How do they interpret and reason with the conditionals they meet in undergraduate mathematics?

This talk will shed light on this question, presenting empirical data from studies involving mathematical versions of conditional reasoning tasks used in cognitive psychology. It will also provide attendees with an opportunity to reflect upon whether professional mathematicians' reasoning is always as 'logical' as we might think, and on how logical reasoning develops with mathematical expertise.