Finding your feet: modelling the batting abilities of cricketers using Gaussian processes

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The basics

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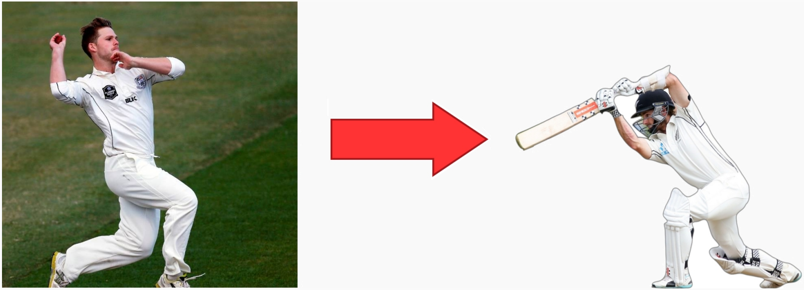

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- **Batting average** $= \frac{\text{Total \# runs scored}}{\text{Total \# dismissals}}$

‘Getting your eye in’

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- The local pitch and weather conditions

Pitch conditions

Day 1 pitch.

Day 5 pitch.

‘Getting your eye in’

Batting is initially difficult due to external factors such as:

- The local pitch and weather conditions
- The specific match scenario

The process of batsmen familiarising themselves with the match conditions is nicknamed **‘getting your eye in’**.

Predicting the hazard

- **Hazard** = probability of a batsmen being dismissed on their current score

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- **Hazard** = probability of a batsmen being dismissed on their current score
- Due to the 'eye in' process, a constant hazard model is no good for predicting when a batsman will get out
  - Will under predict dismissal probability for low scores
  - Will over predict dismissal probability for high scores (i.e. when a player has their 'eye in')

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1. How well a player bats when they first begin an innings
2. How much better a player bats when they have their ‘eye in’
3. How long it takes them to get their ‘eye in’

Kane Williamson's career record

Kane Williamson

New Zealand

Full name Kane Stuart Williamson

Born August 8, 1990, Tauranga

Current age 27 years 322 days

Major teams New Zealand, Barbados Tridents, Gloucestershire, Gloucestershire 2nd XI, New Zealand Under-19s, Northern Districts, Sunrisers Hyderabad, Yorkshire

Playing role Top-order batsman

Batting style Right-hand bat

Bowling style Right-arm offbreak

Relation Cousin - D Cleaver

insights Explore Kane Williamson's performance

Batting and fielding averages

	Mat	Inns	NO	Runs	HS	Ave	BF	SR	100	50	4s	6s	Ct	St
Tests	65	116	10	5338	242*	50.35	10590	50.40	18	26	585	13	58	0
ODIs	127	121	11	5156	145*	46.87	6195	83.22	11	33	474	43	53	0
T20Is	51	49	7	1316	73*	31.33	1088	120.95	0	8	142	21	24	0
First-class	128	220	17	9821	284*	48.37	19144	51.30	28	49	1149	30	119	0
List A	188	178	19	7277	145*	45.76	8838	82.33	15	45	640	64	80	0
T20s	153	145	16	3808	101*	29.51	3120	122.05	1	25	365	85	64	0

Credit: *www.cricinfo.com*

Initial aim

1. Develop models which quantify a player's batting ability at any stage of an **innings**

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- Models should provide a better measure of player ability than the batting average
- Fitted within a Bayesian framework:
  - Nested sampling (Skilling, 2006)
  - C++, Julia & R

Deriving the model likelihood

If $X \in \{0, 1, 2, 3, ...\}$ is the number of runs scored by a batsman:

$$\begin{aligned}\text{Hazard function} &= H(x) \\ &= P(X = x | X \geq x)\end{aligned}$$

$H(x) =$ The probability of getting out on score x, given you made it to score x

Data

Fit the model to player career data:

Runs	**Out/not out**
13	0
42	0
53	0
104	1
2	0
130	0
2	0
1	0
176	0

- $0 =$ out, $1 =$ not out

Deriving the model likelihood

Assuming a functional form for $H(x)$, conditional on some parameters θ, the model likelihood is:

$$L(\theta) = L_{Out}(\theta) \times L_{NotOut}(\theta)$$

$$L_{Out}(\theta) = \prod_{i=1}^{I-N} \left(H(x_i) \prod_{a=0}^{x_i - 1} [1 - H(a)] \right)$$

$$L_{NotOut}(\theta) = \prod_{i=1}^{N} \left(\prod_{a=0}^{y_i - 1} [1 - H(a)] \right)$$

$\{x_i\}$ = set of out scores

$\{y_i\}$ = set of not out scores

I = Total number of innings

N = Total number of not out innings

Parameterising the hazard function

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- To reflect our cricketing knowledge of the ‘getting your eye in’ process, $H(x)$ should be higher for low scores, and lower for high scores
- From a cricketing perspective we often refer to a player’s ability in terms of a batting average

The effective average function, $\mu(x)$

- Instead, we can model the hazard function in terms of an 'effective batting average' or 'effective average function', $\mu(x)$.

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$\mu(x) =$ **batsman's ability on score x, in terms of a batting average**

- Relationship between the hazard function and effective average function:

$$H(x) = \frac{1}{\mu(x) + 1}$$

- This allows us to think in terms of batting averages, rather than dismissal probabilities

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- Reasonable to believe that batsmen begin an innings playing with some initial batting ability, $\boldsymbol{\mu_1}$
- Batting ability increases with number of runs scored, until some peak batting ability, $\boldsymbol{\mu_2}$, is reached
- The speed of the transition between μ_1 and μ_2 can be represented by a parameter, **L**

The effective average function, $\mu(x)$

$$\mu(x; \mu_1, \mu_2, L) = \mu_2 + (\mu_1 - \mu_2) \exp\left(-\frac{x}{L}\right)$$

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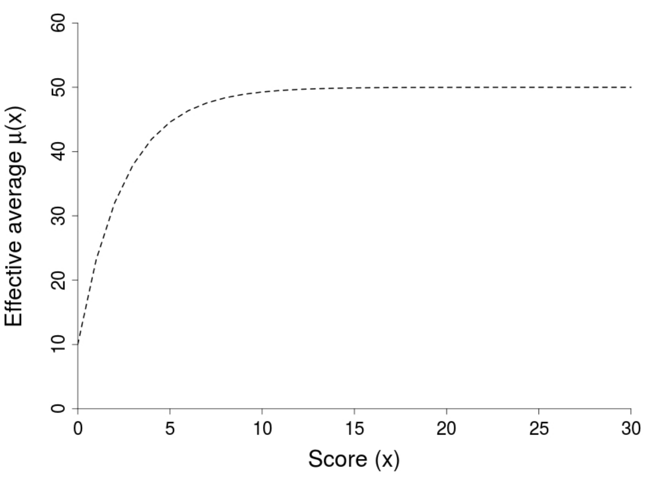

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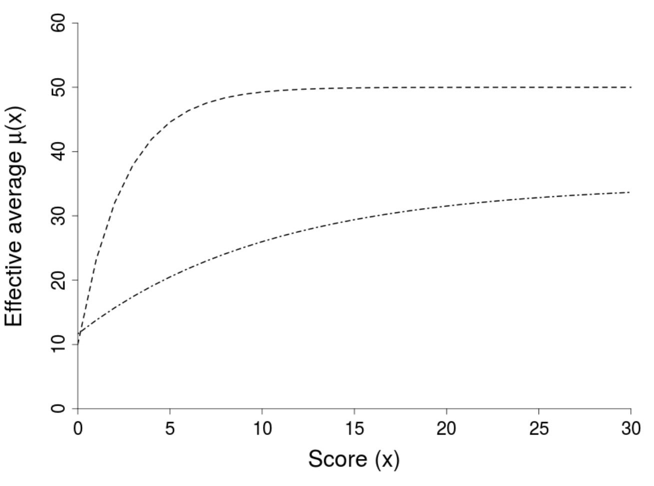

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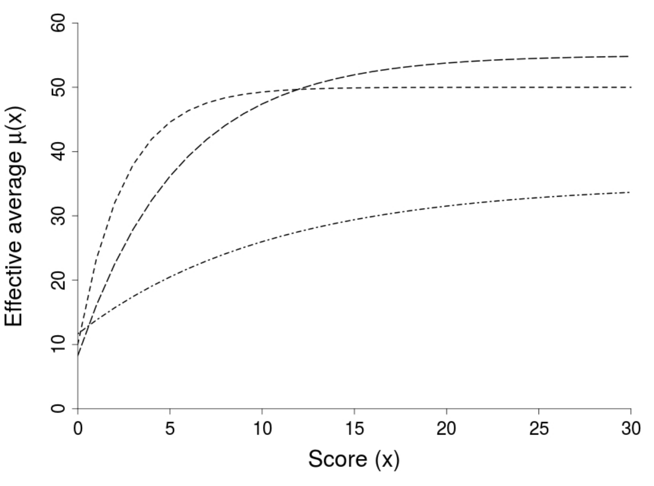

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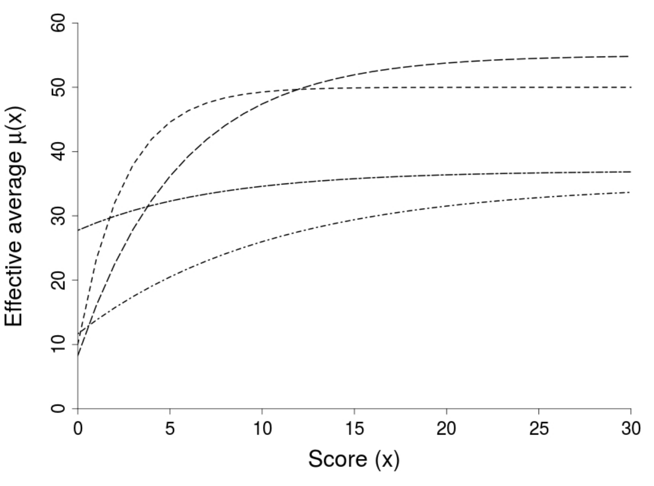

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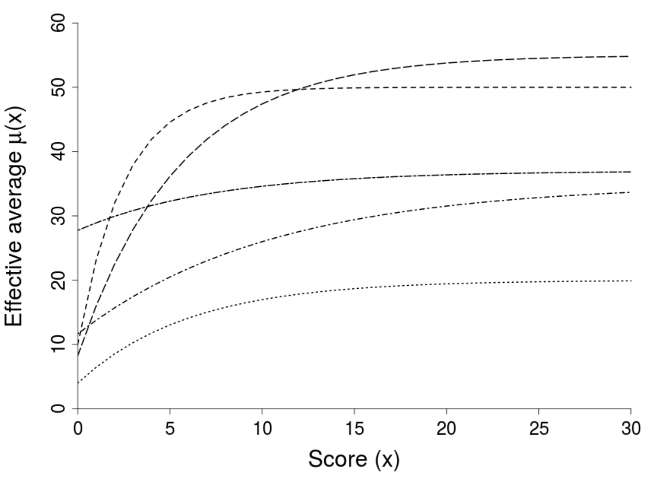

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Model specification

Set of parameters, $\theta = \{\mu_1, \mu_2, L\}$

- Assign conservative, non-informative priors
- Model implemented in C++ using a nested sampling algorithm that uses Metropolis-Hastings updates

Posterior summaries

Table 1: Posterior parameter estimates and uncertainties (68% C.Is) for current top four Test batsmen (December 2018). Current top Test all-rounder* included for comparison. 'Prior' indicates the prior point estimates and uncertainties.

Player	μ_1	μ_2	L	Average
V. Kohli (IND)	$22.7^{+9.7}_{-6.9}$	$61.0^{+8.8}_{-6.4}$	$6.5^{+10.0}_{-4.5}$	54.6
S. Smith (AUS)	$33.2^{+10.6}_{-9.7}$	$68.9^{+11.2}_{-8.2}$	$11.6^{+13.2}_{-7.8}$	61.4
K. Williamson (NZL)	$18.2^{+6.8}_{-5.1}$	$58.3^{+7.7}_{-6.7}$	$6.8^{+5.9}_{-3.5}$	50.4
J. Root (ENG)	$24.4^{+7.9}_{-6.3}$	$56.6^{+6.6}_{-5.7}$	$7.7^{+5.9}_{-3.9}$	50.4
S. Al-Hasan* (BAN)	$24.4^{+7.1}_{-6.8}$	$43.4^{+6.2}_{-4.7}$	$5.8^{+9.1}_{-4.2}$	39.7
Prior	$6.6^{+12.8}_{-5.0}$	$25.0^{+27.7}_{-13.1}$	$3.0^{+6.7}_{-2.3}$	N/A

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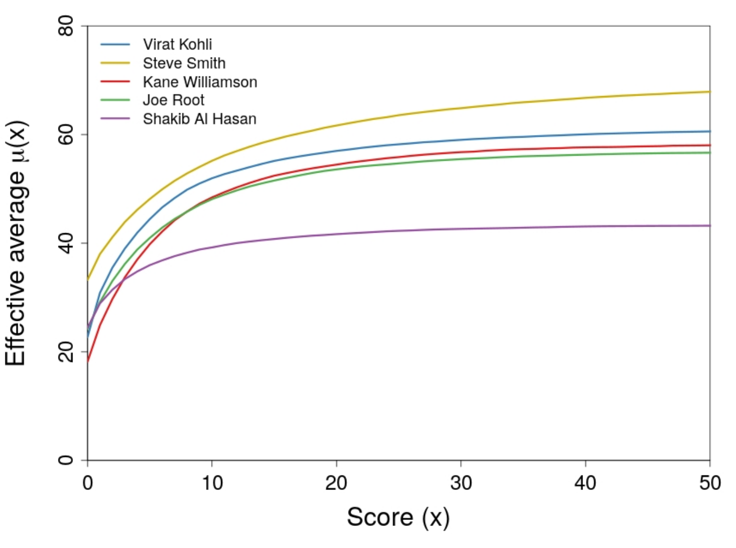

Figure 6: Posterior predictive effective average functions, $\mu(x)$.

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- Root appears to begin an innings batting with greater ability
  - $\mu_1 = 18.2$ vs. 24.4
- However, Williamson gets his 'eye in' quicker and appears to be the superior player once familiar with match conditions
  - $L = 6.8$ vs. 7.7
  - $\mu_2 = 58.3$ vs. 56.6

Predictive effective average functions

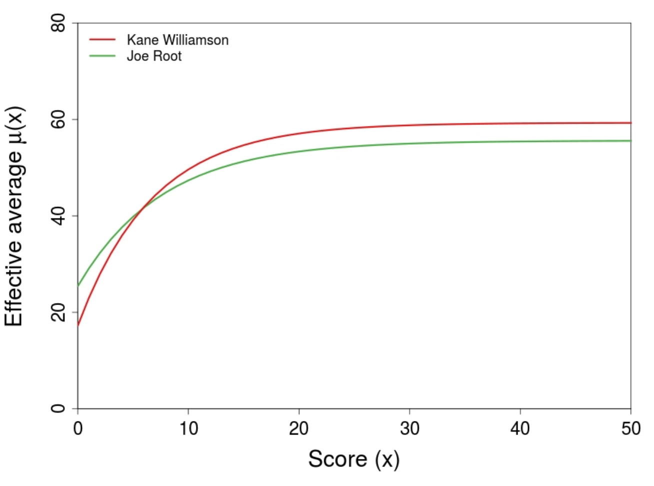

Figure 7: Posterior predictive effective average functions, $\mu(x)$, for Williamson and Root.

Looking at the bigger picture

So far the effective average allows us to quantify how the batting abilities of players change *within* an innings, in terms of a batting average.

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What about how batting ability changes across a player's career?

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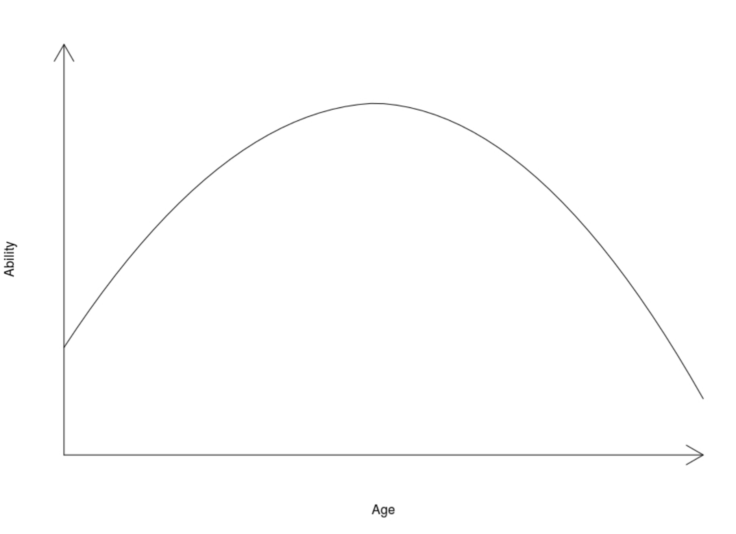

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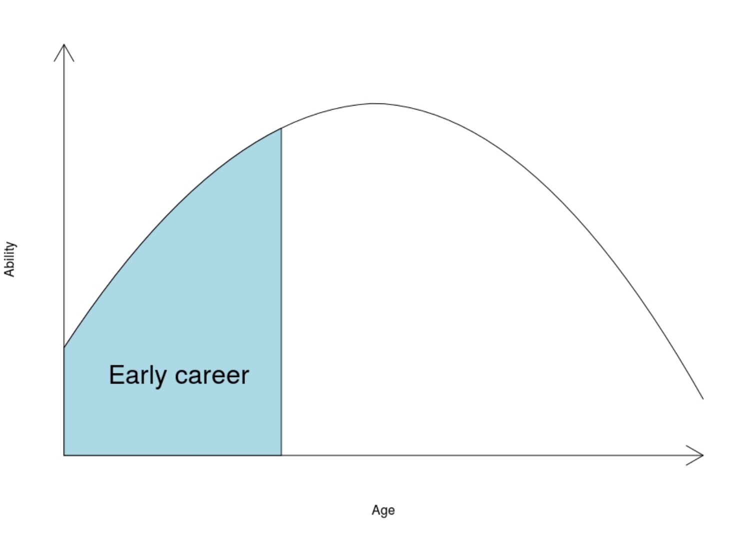

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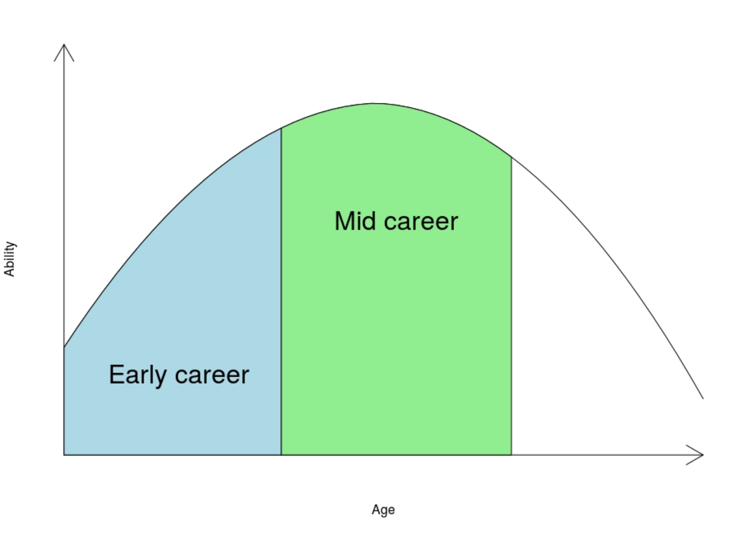

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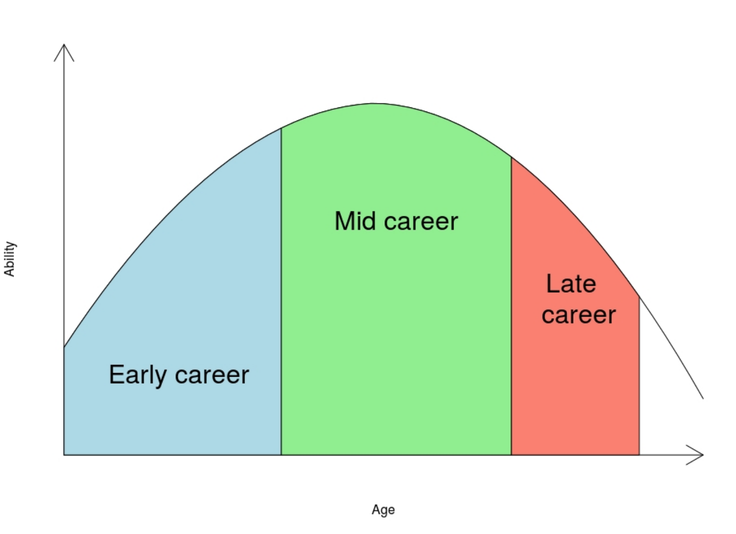

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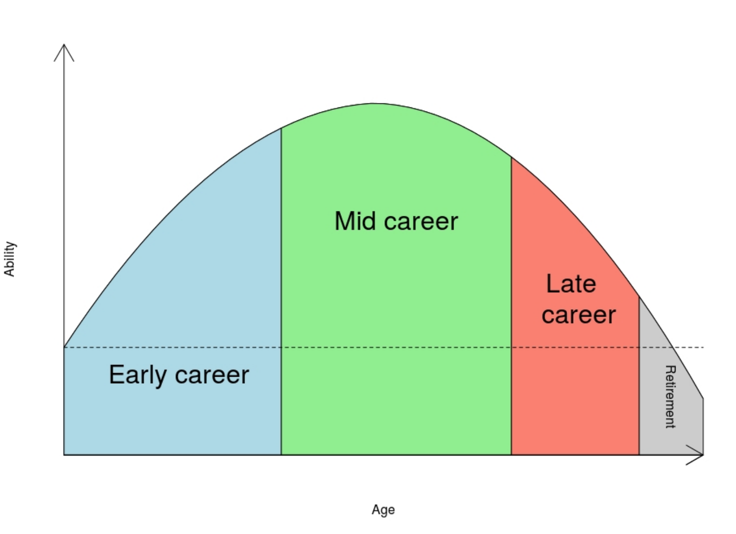

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- Interestingly, players frequently string numerous strong performances together
- Suggests external factors such as a player's current form is an important variable to consider

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Now, our aim is to derive a secondary model which can measure and predict player batting ability at any given stage of a *career*.

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Needs to be able to handle random fluctuations in performance due factors such as:

- Player form
- Player fitness (both mental and physical)
- Random chance!

Gaussian processes

Gaussian processes are a class of schotastic process, made up of a collection of random variables, such that every finite collection of those random variables has a multivariate normal distribution (Rasmussen & Williams, 2006).

A Gaussian process is completely specified by its:

- Mean value, m
- Covariance function, $K(x, x)$

Matérn$_{\frac{3}{2}}$ covariance function

The Matérn$_{\frac{3}{2}}$ covariance function:

$$K_{\frac{3}{2}}(X_i, X_j) = \sigma^2 \left(1 + \frac{\sqrt{3}\ |X_i - X_j|}{\ell}\right) \exp\left(-\frac{\sqrt{3}\ |X_i - X_j|}{\ell}\right)$$

$\sigma =$ 'signal variance', determines how much a function value can deviate from the mean
$\ell =$ 'length-scale', roughly the distance required to move in the input space before the function value can change significantly

Example: Gaussian processes

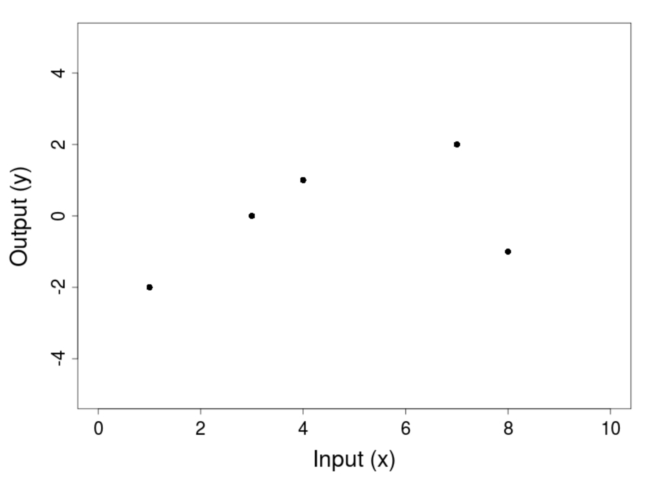

Figure 8: Some 'noiseless' observed data in the input/output space.

Example: Gaussian processes

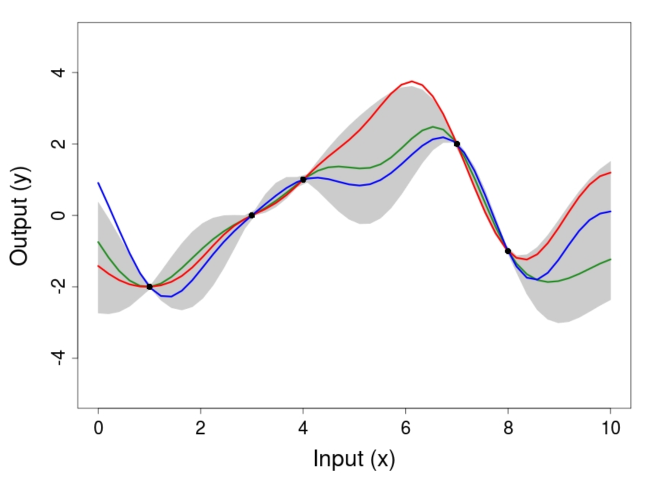

Figure 9: Example Gaussian processes fitted to some noiseless data. Shaded area represents a 95% credible interval.

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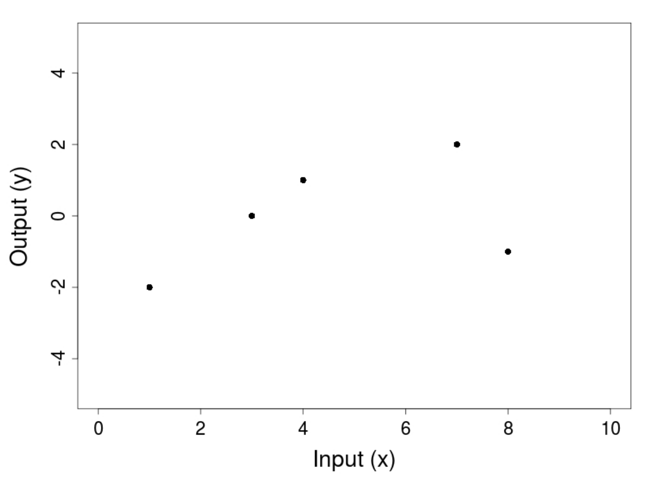

Figure 10: Some ‘noisy’ observed data in the input/output space.

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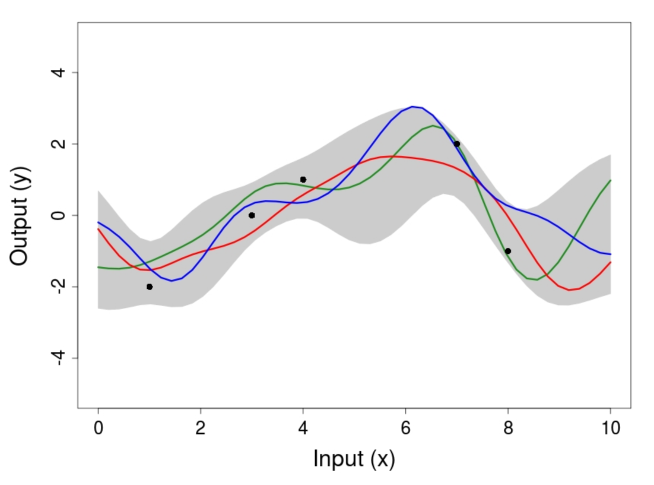

Figure 11: Example Gaussian processes fitted to some noisy data. Shaded area represents a 95% credible interval.

Modelling batting career trajectories

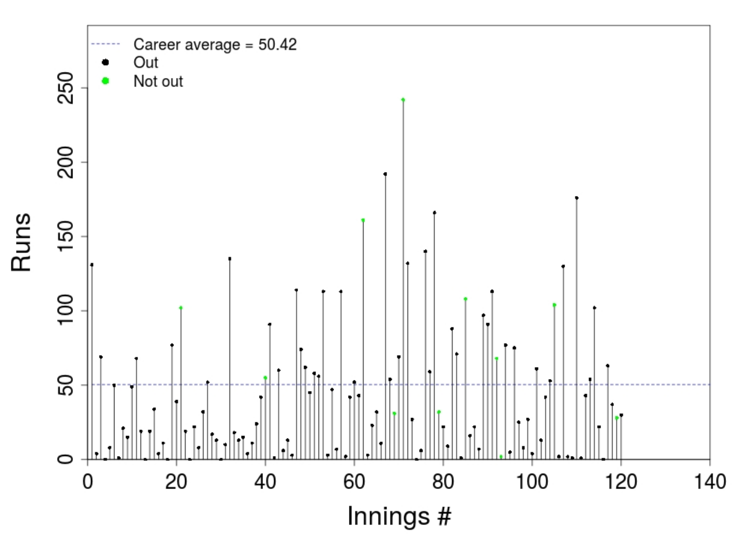

Figure 12: Plot of Test career scores for Kane Williamson.

Modelling batting career trajectories

Recall the 'within-innings' effective average function, $\mu(x)$:

$$\mu(x; \mu_1, \mu_2, L) = \text{player batting ability on score } x$$

- $\mu_2 =$ 'peak' batting ability *within* an innings

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- $\mu_2 =$ 'peak' batting ability *within* an innings

Define a 'between-innings' effective average function, $\nu(x, t)$:

$\nu(x, t) =$ **player batting ability on score x, in t^{th} career innings, in terms of a batting average**

- $\mu_{2_t} =$ 'peak' batting ability within batsman's t^{th} career innings

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$$\begin{aligned} \nu(t) &= \text{expected number of runs scored in } t^{th} \text{ innings} \\ &= \text{expected } \underline{\text{batting average}} \text{ in } t^{th} \text{ innings} \end{aligned}$$

Model specification

Set of parameters, $\theta = \{\mu_1, \{\mu_{2_t}\}, L, m, \sigma, \ell\}$

- Assign conservative, non-informative priors to μ_1, L, m, σ and ℓ

  $\{\mu_{2_t}\} \sim \mathrm{GP}(m,\ K(X_i, X_j; \sigma, \ell))$

- Model implemented in C++ using a nested sampling algorithm that uses Metropolis-Hastings updates

Predictive effective average function

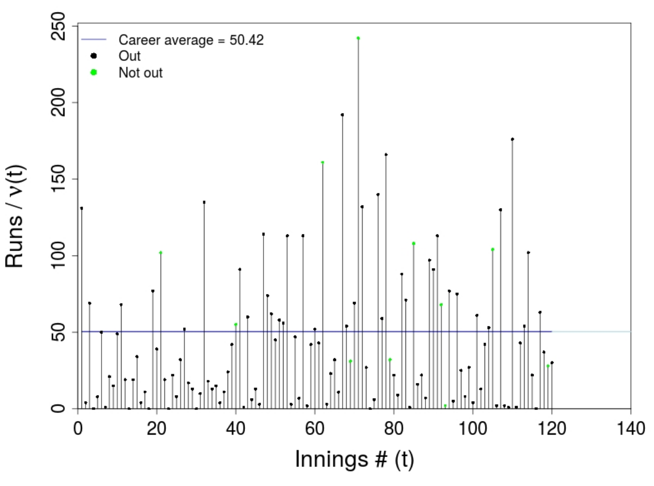

Figure 13: Test career batting data for Kane Williamson, including career average (blue).

Predictive effective average function

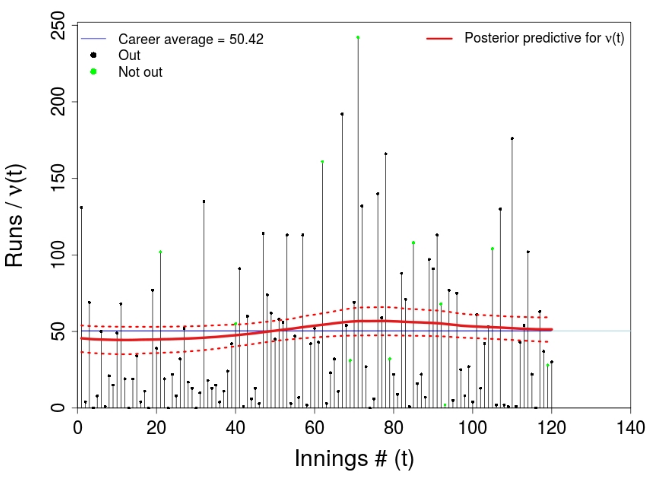

Figure 14: Posterior predictive effective average function, $\nu(t)$, for Kane Williamson (red), with 68% credible intervals (dotted).

Predictive effective average function

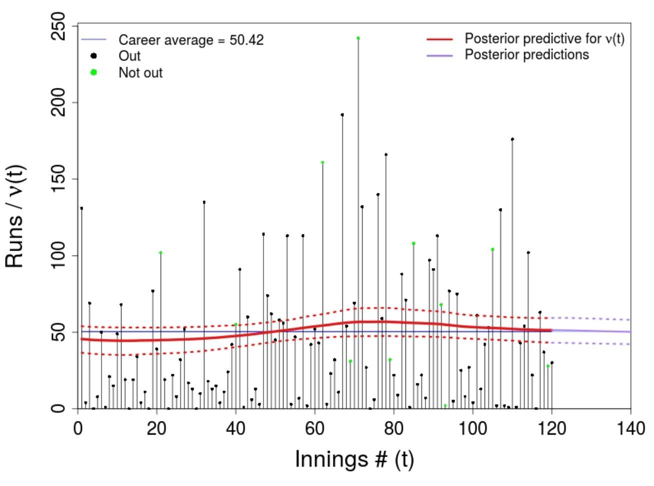

Figure 15: Posterior predictive effective average function, $\nu(t)$, for Kane Williamson (red), including predictions for the next 20 innings (purple), with 68% credible intervals (dotted).

Predictive effective average function

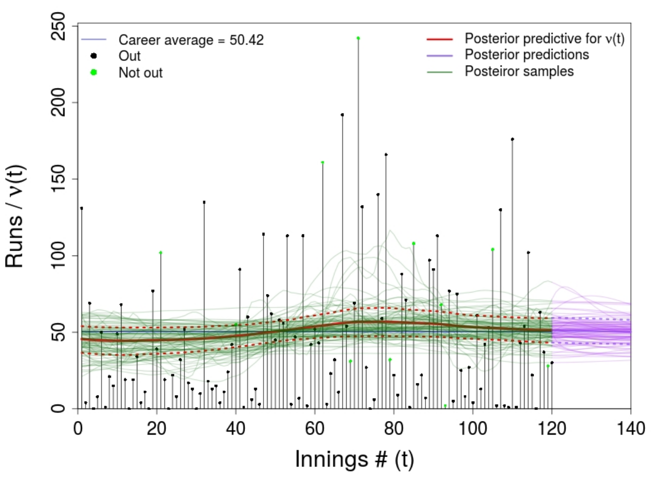

Figure 16: Posterior predictive effective average function, $\nu(t)$, for Kane Williamson (red), including a subset of posterior samples (green) and predictions for the next 20 innings (purple).

Predictive effective average functions

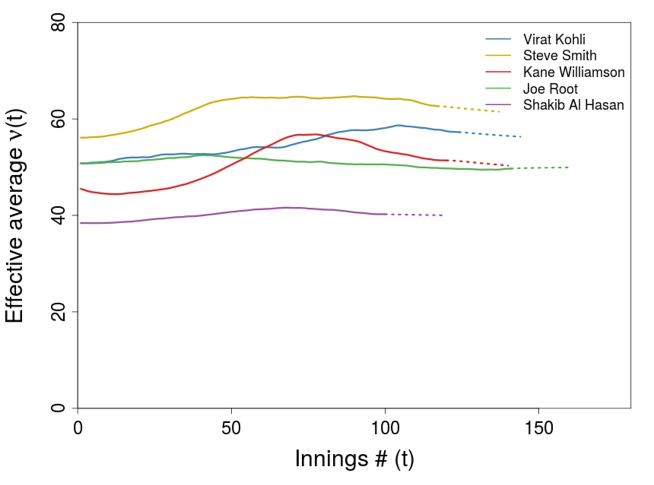

Figure 17: Posterior predictive effective average functions, $\nu(t)$. Dotted lines are predictions for the next 20 innings.

Predicting future abilities

Table 3: Posterior predictive point estimates for the effective average $\nu(t)$, for the next career innings. The official ICC Test batting ratings (and rankings) are shown for comparison.

Player	**Career Average**	**Predicted ν(next innings)**	**ICC Rating (#)**
V. Kohli (IND)	54.6	57.2	935 (1)
S. Smith (AUS)	61.4	62.6	910 (2)
K. Williamson (NZ)	50.4	51.3	847 (3)
J. Root (ENG)	50.5	49.7	808 (4)
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- Virat Kohli has a 18.3% chance of scoring 100 or more in his next innings, while Steve Smith has a 20.6% chance
- There is a 32.2% chance that Virat Kohli outscores Steve Smith in their next respective innings

Concluding statements, limitations and further work

Limitations and conclusions

- Models ignore variables such as balls faced and minutes batted

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  - Limits usage to longer form Test/First Class matches

Concluding statements

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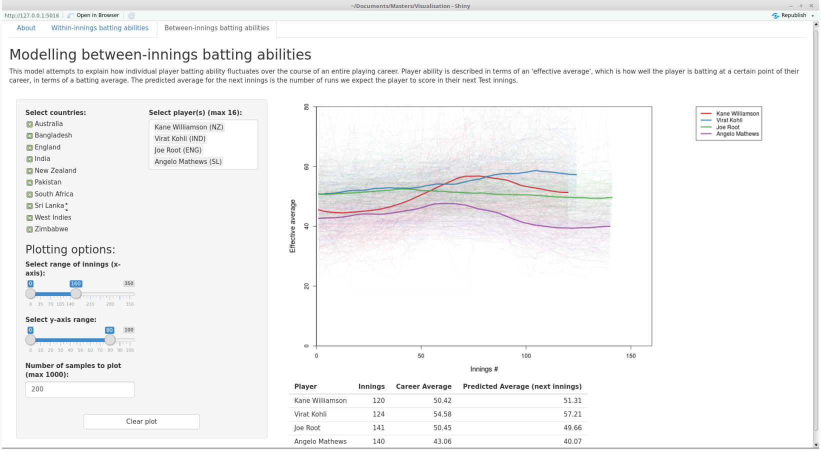
- There has been a recent boom in statistical analysis in cricket, particularly around T20 cricket
- However, many analyses stray away from maintaining an easy to understand, cricketing interpretation
- We have developed tools which allow us to quantify player batting ability both within *and* between innings, supporting several common cricketing beliefs
  - 'Getting your eye in'
  - 'Finding your feet'

Effective average visualisations

Stevenson & Brewer (2017)

www.oliverstevenson.co.nz



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