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Eliminating the Weakest Link: Making Manipulation Intractable?

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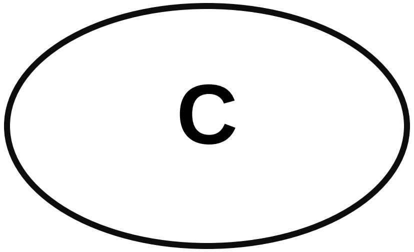
Motivation

Motivation



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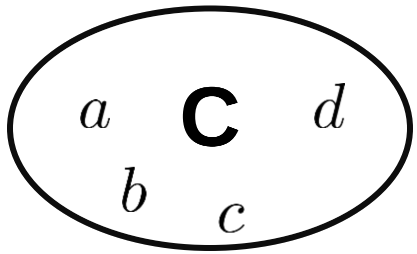

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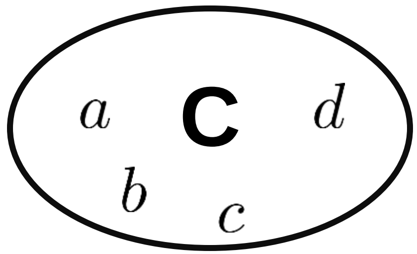
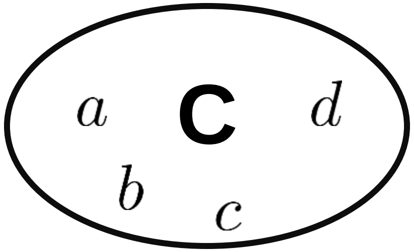


E

$$d \succ b \succ a \succ c$$
$$a \succ c \succ b \succ d$$
$$c \succ a \succ b \succ d$$
$$b \succ c \succ a \succ d$$

Motivation

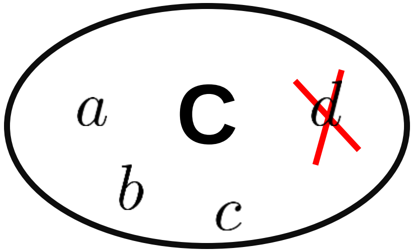


E

$$d \succ b \succ a \succ c$$
$$a \succ c \succ b \succ d$$
$$c \succ a \succ b \succ d$$
$$b \succ c \succ a \succ d$$

$$\{a \succ b \succ c \succ d\}$$

Motivation



E

$\not{d} \succ b \succ a \succ c$

$a \succ c \succ b \succ \not{d}$

$c \succ a \succ b \succ \not{d}$

$b \succ c \succ a \succ \not{d}$

Example

$$\{a, b, c, d\}$$

Example

$\{a, b, c, d\}$

Example




$$\{a, b, c, d\}$$

$$d \succ b \succ a \succ c$$
$$a \succ d \succ b \succ c$$
$$c \succ a \succ d \succ b$$
$$b \succ c \succ a \succ d$$

Example




$d \succ b \succ a \succ c$

$a \succ d \succ b \succ c$

$c \succ a \succ d \succ b$

$b \succ c \succ a \succ d$

$\{a, b, c, d\}$

Base rule - Borda

$(n-1, n-2, \ldots, 0)$

$a \succ b \succ \ldots \succ d$

Example





$\{a, b, c, d\}$

Base rule - Borda

$d \succ b \succ a \succ c$	$\{1, 2, 0, 3\}$
$a \succ d \succ b \succ c$	$\{3, 1, 0, 2\}$
$c \succ a \succ d \succ b$	$\{2, 0, 3, 1\}$
$b \succ c \succ a \succ d$	$\{1, 3, 2, 0\}$

Example




Base rule - Borda

$$\{a, b, c, d\}$$

$$
\begin{array}{cc}
d \succ b \succ a \succ c & \{1,2,0,3\} \\
a \succ d \succ b \succ c & \{3,1,0,2\} \\
c \succ a \succ d \succ b & \{2,0,3,1\} \\
b \succ c \succ a \succ d & \{1,3,2,0\} \\
\hline
 & \{7,6,5,6\}
\end{array}
$$

Example




$$d \succ b \succ a$$
$$a \succ d \succ b$$
$$a \succ d \succ b$$
$$b \succ a \succ d$$

$$\{a, b, \quad, d\}$$

$$\{0, 1, \quad, 2\}$$
$$\{2, 0, \quad, 1\}$$
$$\{2, 0, \quad, 1\}$$
$$\{1, 2, \quad, 0\}$$

$$\{5, 3, \quad, 4\}$$

Base rule - Borda

Example






Base rule - Borda

$$\{a, \ , \ , d\}$$

$d \succ a$ $\quad \{0, \ , \ , 1\}$

$a \succ d$ $\quad \{1, \ , \ , 0\}$

$a \succ d$ $\quad \{1, \ , \ , 0\}$

$a \succ d$ $\quad \{1, \ , \ , 0\}$

$$\{3, \ , \ , 1\}$$

Example

More Examples

Motivation


WEAKEST
LINK


СЛАБОЕ
ЗВЕНО


50000
40000
30000
20000
10000
5000
2000
1000
8000
БАНК
1:04
5

Motivation

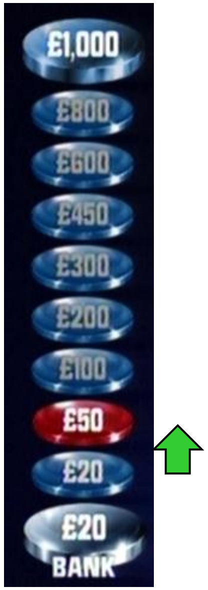
Motivation

Motivation

Motivation

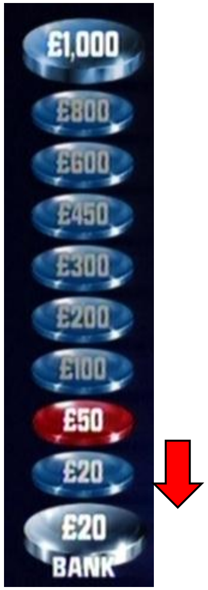
Motivation

Right answer



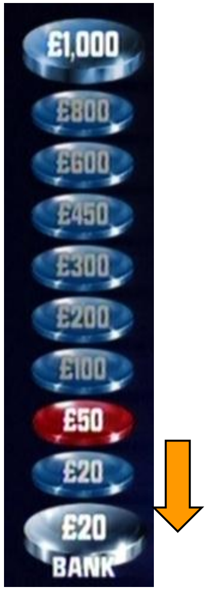
Motivation

Wrong answer



Motivation

Bank (before the next question)



Motivation

Statistics: the strongest link, the weakest link

Motivation

Voting
(Veto)

Motivation

The player with the max veto-score is eliminated

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Motivation

The winner takes all the money

Motivation

There is no point to play truthfully

Motivation

Is it computationally difficult
to play strategically?

Motivation

Is it computationally difficult
to manipulate such voting rules?

Motivation

Eliminate(X): successively eliminates the candidate placed in last place by X

Motivation

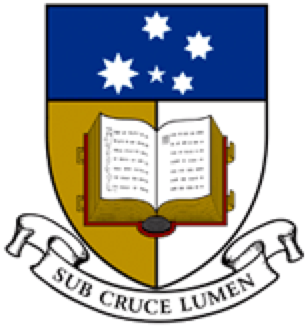
Eliminate(Veto)

Motivation

Divide(X): successively eliminates those candidates with the mean or smaller score

Motivation

Divide(Borda)



Motivation

Sequential(X): runs a sequence of elections using X to eliminate the last placed candidate from each successive election. In each round, voters can change their votes.

Motivation

Sequential(X):

Motivation

#manipulators	One	
Copeland	P	[Bartholdi & Orlin 91]
STV	**NPC**	[Bartholdi et al. 89]
Veto	P	[Zuckerman et al. 08]
Plurality with runoff	P	[Zuckerman et al. 08]
Cup	P	[Conitzer et al. 07]
Maximin	P	[Bartholdi & Orlin 91]
Ranked pairs	**NPC**	[Xia et al. 09]
Bucklin	P	[Xia et al. 09]
Borda	P	[Bartholdi & Orlin 91]
Nanson's rule	**NPC**	**[AAAI'11]**
Baldwin's rule	**NPC**	**[AAAI'11]**

Motivation

Eliminate(Plur-ty) Eliminate(Borda)	NP-complete[1991] NP-complete[AAAI'11]
Divide(Borda)	NP-complete[AAAI'11]

Motivation

Eliminate(Plur-ty)	NP-complete[1991]
Eliminate(Borda)	NP-complete[AAAI'11]
Eliminate(Veto)	?
Coombs	?
Eliminate(scoring rule*)	?
Divide(Borda)	NP-complete[AAAI'11]
Divide(scoring rule*)	?
Sequential (Plurality)	?

Unweighted Coalitional Manipulation (UCM)

UCM under X is NP-complete
UCM under Eliminate(X) is P?

X is an artificial rule

UCM

Elections	1-in-3 SAT
0 > 1 > 2 > 5 > 4> ….	(1,2,5)
1 > 2 > 5 > 0 > 4> ….	1,2,5,-4,….

UCM

Elections	1-in-3 SAT
0 > 1 > 2 > 5 > 4> ….	(1,2,5)
1 > 2 > 5 > 0 > 4> ….	1,2,5,-4,….
Rule X: if `assignment' vote is a solution in 1-in-3 SAT then 0 is a winner otherwise 1	Rule Eliminate(X) 0 or 1 always reach the last round.

X is an artificial rule

Eliminate(veto)

Eliminate(veto)

Theorem: UCM under eliminate(Veto) is **NP-complete** with a single manipulator

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Eliminate(veto)

…the difficulty of WCM on Coombs for unlimited candidates as an open question.
Coleman and Teague [2007]

Inspired by STV proof [Bartholdi and Orlin 1991]

3-Cover

$S = \{d_1, \ldots, d_n\}$, $|S| = n$ and $S_1, S_2, \ldots, S_m \subset S$ with $|S_i| = 3$ for $i \in [1, m]$.

Does there exist an index set I with $|I| = n/3$ and $\bigcup_{i \in I} S_i = S$

Eliminate(veto)

1. Make choice of sets S_i: (a_i,-a_i, b_i,-b_i, p_i), i=1..m
   - (choice) b_i or -b_i, i=1..m
   - (memory) elimination of p_i increases veto scores of all candidates except {b_j ,-b_j} j < i+1
2. Check that cover is valid:
   - size of the cover is n/3
   - a dangerous candidate is eliminated iff we selected 3-COVER

Coombs rule

Coombs' rule is eliminate(veto) which stops when one candidates has a majority.

Coombs rule

Theorem: UCM under Coombs is **NP-complete** with a single manipulator

Coombs rule

UCM under Coombs	UCM under Elim(Veto)
At least n manipulators	0 manipulators

Coombs rule

UCM under Coombs

UCM under Elim(Veto)

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$	$b \succ$	a
	2	c	d_2	d_3		d_{n-1}	b	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$		$\vdots$	$\vdots$
	n	c	d_n	d_{n-1}		d_1	b	a
Group 2	$n+1$	b	c	d_1	d_2		d_n	a
	$n+2$	b	c	d_2	d_3		d_{n-1}	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$2n$	b	c	d_n	d_{n-1}		d_1	a
Group 3	$2n+1$	a	b	c	d_1		d_{n-1}	d_n
	$2n+2$	a	b	c	d_2		d_{n-2}	d_{n-1}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$	a	b	c	d_n		d_2	d_1

Coombs rule

UCM under Coombs

UCM under Elim(Veto)

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$	$b \succ$	a
	2	c	d_2	d_3		d_{n-1}	b	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$		$\vdots$	$\vdots$
	n	c	d_n	d_{n-1}		d_1	b	a
Group 2	$n+1$	b	c	d_1	d_2		d_n	a
	$n+2$	b	c	d_2	d_3		d_{n-1}	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$2n$	b	c	d_n	d_{n-1}		d_1	a
Group 3	$2n+1$	a	b	c	d_1		d_{n-1}	d_n
	$2n+2$	a	b	c	d_2		d_{n-2}	d_{n-1}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$	a	b	c	d_n		d_2	d_1

Coombs rule

UCM under Coombs

UCM under Elim(Veto)

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$	$b \succ$	
	2	c	d_2	d_3		d_{n-1}	b	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$		$\vdots$	
	n	c	d_n	d_{n-1}		d_1	b	
Group 2	$n+1$	b	c	d_1	d_2		d_n	
	$n+2$	b	c	d_2	d_3		d_{n-1}	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	
	$2n$	b	c	d_n	d_{n-1}		d_1	
Group 3	$2n+1$		b	c	d_1		d_{n-1}	d_n
	$2n+2$		b	c	d_2		d_{n-2}	d_{n-1}
	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$		b	c	d_n		d_2	d_1

Coombs rule

UCM under Coombs

UCM under Elim(Veto)

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$		
	2	c	d_2	d_3		d_{n-1}		
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$			
	n	c	d_n	d_{n-1}		d_1		
Group 2	$n+1$		c	d_1	d_2		d_n	
	$n+2$		c	d_2	d_3		d_{n-1}	
	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	
	$2n$		c	d_n	d_{n-1}		d_1	
Group 3	$2n+1$			c	d_1		d_{n-1}	d_n
	$2n+2$			c	d_2		d_{n-2}	d_{n-1}
	$\vdots$			$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$			c	d_n		d_2	d_1

Coombs rule

UCM under Coombs

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$	$b \succ$	a
	2	c	d_2	d_3		d_{n-1}	b	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$		$\vdots$	$\vdots$
	n	c	d_n	d_{n-1}		d_1	b	a
Group 2	$n+1$	b	c	d_1	d_2		d_n	a
	$n+2$	b	c	d_2	d_3		d_{n-1}	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$2n$	b	c	d_n	d_{n-1}		d_1	a
Group 3	$2n+1$	a	b	c	d_1		d_{n-1}	d_n
	$2n+2$	a	b	c	d_2		d_{n-2}	d_{n-1}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$	a	b	c	d_n		d_2	d_1

UCM under Elim(Veto)

Group 1	1	$c \succ$	$d_1 \succ$	$d_2 \succ$		$d_n \succ$	$b \succ$	a
	2	c	d_2	d_3		d_{n-1}	b	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$		$\vdots$	$\vdots$
	n	c	d_n	d_{n-1}		d_1	b	a
Group 2	$n+1$	b	c	d_1	d_2		d_n	a
	$n+2$	b	c	d_2	d_3		d_{n-1}	a
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$2n$	b	c	d_n	d_{n-1}		d_1	a
Group 3	$2n+1$	a	b	c	d_1		d_{n-1}	d_n
	$2n+2$	a	b	c	d_2		d_{n-2}	d_{n-1}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
	$3n$	a	b	c	d_n		d_2	d_1

Coombs rule

UCM under Coombs	UCM under Elim(Veto)
1 manipulator	At least n manipulators

Eliminate (truncated scoring rule)

Truncated scoring rule

Given a fixed k, a *truncated scoring rule*
has a scoring vector $(s_1, \ldots, s_m)$ with $s_i = 0$ for all $i > k$.

Truncated scoring rule

k-approval	(1..1,0,…)
the Heisman Trophy	(3,2,1,0,…)
the presidential election in Kiribat	(4,3,2,1,0,…)
Formula One points	(10,…,1,0…)

Eliminate(Truncated scoring rule)

Theorem: UCM under eliminate (truncated s.r.) is **NP-complete** with a single manipulator

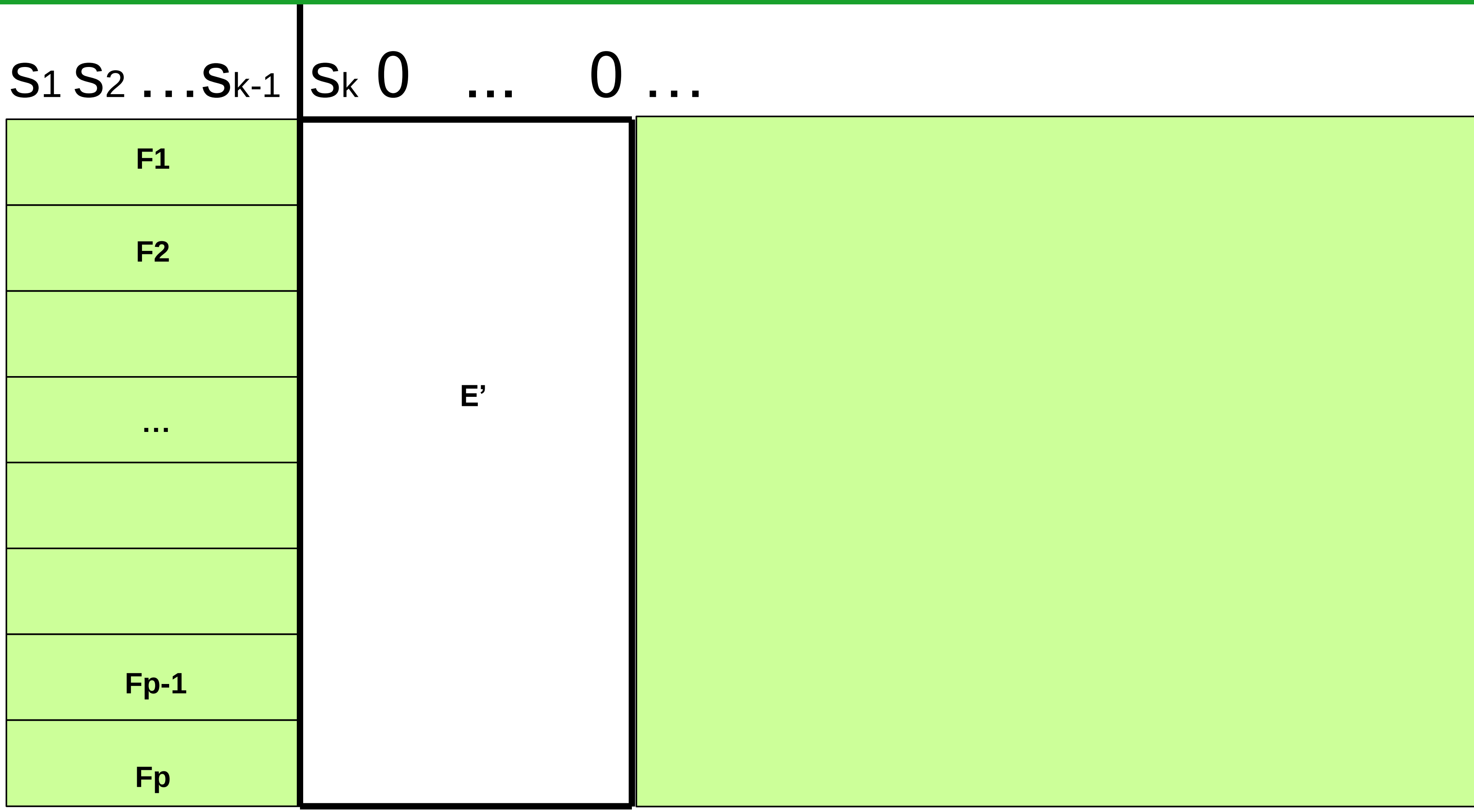
Eliminate(Truncated scoring rule)

Inspired by STV proof [Bartholdi and Orlin 1991]

Eliminate(Truncated scoring rule)

A manipulator **only** makes a choice at the i-th,i=1..m, rounds between two candidates that are tied

Eliminate(Truncated scoring rule)



Divide(Truncated scoring rule)

Theorem: UCM under divide(truncated s.r.) is **NP-complete** with a single manipulator

Sequential rules

2012 Summer Olympics bidding results [hide]					
City	**NOC**	**Round 1**	**Round 2**	**Round 3**	**Round 4**
London	Great Britain	**22**	27	**39**	**54**
Paris	France	21	25	33	50
Madrid	Spain	20	**32**	31	—
New York City	United States	19	16	—	—
Moscow	Russia	15	—	—	—

2012 Summer Olympics bidding results [hide]					
City	**NOC**	**Round 1**	**Round 2**	**Round 3**	**Round 4**
London	Great Britain	**22**	27	**39**	**54**
Paris	France	21	25	33	50
Madrid	Spain	20	**32**	31	—
New York City	United States	19	16	—	—
Moscow	Russia	15	—	—	—

An election in which a manipulator can only change the result
if
the manipulator votes differently in some rounds

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0			
1	a	w	c	
1	d_1	a	w	c
1	d_2	a	w	c
3	g	a	w	c
2	b	w	c	
2	f_1	b	w	c
2	f_2	b	w	c
6	w	c		
5	c	w		

$$\sigma(w) = 6$$
$$\sigma(c) = 5$$
$$\sigma(g) = 3$$
$$\sigma(b) = 2$$
$$\sigma(f_1) = 2$$
$$\sigma(f_2) = 2$$
$$\sigma(a) = 1$$
$$\sigma(d_1) = 1$$
$$\sigma(d_2) = 1$$

Manipulator

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0			
1	a	w	c	
1	d_1	a	w	c
1	d_2	a	w	c
3	g	a	w	c
2	b	w	c	
2	f_1	b	w	c
2	f_2	b	w	c
6	w	c		
5	c	w		

$$\sigma(w) = 6$$
$$\sigma(c) = 5$$
$$\sigma(g) = 3$$
$$\sigma(b) = 2$$
$$\sigma(f_1) = 2$$
$$\sigma(f_2) = 2$$
$$\sigma(a) = 1$$
$$\sigma(d_1) = 1$$
$$\sigma(d_2) = 1$$

Manipulator

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1			
1	a	w	c		w	c		
1	d_1	a	w	c	d_1	w	c	
1	d_2	a	w	c	d_2	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		b	w	c	
2	f_1	b	w	c	f_1	b	w	c
2	f_2	b	w	c	f_2	b	w	c
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 7$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 2$

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

~~$\sigma(a) = 1$~~

$\sigma(d_1) = 1$

$\sigma(d_2) = 1$

Manipulator

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Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0			
1	a	w	c	
1	d_1	a	w	c
1	d_2	a	w	c
3	g	a	w	c
2	b	w	c	
2	f_1	b	w	c
2	f_2	b	w	c
6	w	c		
5	c	w		

$$\sigma(w) = 6$$
$$\sigma(c) = 5$$
$$\sigma(g) = 3$$
$$\sigma(b) = 2$$
$$\sigma(f_1) = 2$$
$$\sigma(f_2) = 2$$
$$\sigma(a) = 1$$
$$\sigma(d_1) = 1$$
$$\sigma(d_2) = 1$$

Manipulator

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0			
1	a	w	c	
1	d_1	a	w	c
1	d_2	a	w	c
3	g	a	w	c
2	b	w	c	
2	f_1	b	w	c
2	f_2	b	w	c
6	w	c		
5	c	w		

$\sigma(w) = 6$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 2$

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

$\sigma(a) = 1$

$\sigma(d_1) = 1$

$\sigma(d_2) = 1$

Manipulator $a > \ldots .$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-2			
1	a	w	c		a	w	c	
1	d_1	a	w	c	a	w	c	
1	d_2	a	w	c	a	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		b	w	c	
2	f_1	b	w	c	f_1	b	w	c
2	f_2	b	w	c	f_2	b	w	c
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 6$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 2$

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

$\sigma(a) = 3$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $a > \ldots$.

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-2			
1	a	w	c		a	w	c	
1	d_1	a	w	c	a	w	c	
1	d_2	a	w	c	a	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		b	w	c	
2	f_1	b	w	c	f_1	b	w	c
2	f_2	b	w	c	f_2	b	w	c
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 6$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 2$

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

$\sigma(a) = 3$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $a > \ldots.$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-2			
1	a	w	c		a	w	c	
1	d_1	a	w	c	a	w	c	
1	d_2	a	w	c	a	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		w	c		
2	f_1	b	w	c	f_1	w	c	
2	f_2	b	w	c	f_2	w	c	
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 8$

$\sigma(c) = 5$

$\sigma(g) = 3$

~~$\sigma(b) = 2$~~

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

$\sigma(a) = 3$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $a > \ldots.$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-2			
1	a	w	c		a	w	c	
1	d_1	a	w	c	a	w	c	
1	d_2	a	w	c	a	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		b	w	c	
2	f_1	b	w	c	f_1	b	w	c
2	f_2	b	w	c	f_2	b	w	c
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 6$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 2$

$\sigma(f_1) = 2$

$\sigma(f_2) = 2$

$\sigma(a) = 3$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $b > \ldots.$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-4			
1	a	w	c		a	w	c	
1	d_1	a	w	c	a	w	c	
1	d_2	a	w	c	a	w	c	
3	g	a	w	c	g	a	w	c
2	b	w	c		b	w	c	
2	f_1	b	w	c	b	w	c	
2	f_2	b	w	c	b	w	c	
6	w	c			w	c		
5	c	w			c	w		

$\sigma(w) = 6$

$\sigma(c) = 5$

$\sigma(g) = 3$

$\sigma(b) = 6$

~~$\sigma(f_1) = 2$~~

~~$\sigma(f_2) = 2$~~

$\sigma(a) = 3$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $b > \ldots.$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-5		
1	a	w	c		a	w	c
1	d_1	a	w	c	a	w	c
1	d_2	a	w	c	a	w	c
3	g	a	w	c	a	w	c
2	b	w	c		b	w	c
2	f_1	b	w	c	b	w	c
2	f_2	b	w	c	b	w	c
6	w	c			w	c	
5	c	w			c	w	

$\sigma(w) = 6$

$\sigma(c) = 5$

~~$\sigma(g) = 3$~~

$\sigma(b) = 6$

~~$\sigma(f_1) = 2$~~

~~$\sigma(f_2) = 2$~~

$\sigma(a) = 6$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $a > \ldots.$

Sequential (Plurality)

Tie-breaking $c > g > d_1 > d_2 > a > f_1 > f_2 > b > w.$

# Votes	Round 0				Round 1-5		
1	a	w	c		a	w	c
1	d_1	a	w	c	a	w	c
1	d_2	a	w	c	a	w	c
3	g	a	w	c	a	w	c
2	b	w	c		b	w	c
2	f_1	b	w	c	b	w	c
2	f_2	b	w	c	b	w	c
6	w	c			w	c	
5	c	w			c	w	

$\sigma(w) = 6$

$\sigma(c) = 5$

~~$\sigma(g) = 3$~~

$\sigma(b) = 6$

~~$\sigma(f_1) = 2$~~

~~$\sigma(f_2) = 2$~~

$\sigma(a) = 6$

~~$\sigma(d_1) = 1$~~

~~$\sigma(d_2) = 1$~~

Manipulator $c > \ldots.$

Conclusions

Motivation

Eliminate(Borda)	NP-complete[AAAI'11]
Eliminate(Veto)	NP-complete
Coombs	NP-complete
Eliminate(scoring rule*)	NP-complete
Divide(Borda)	NP-complete[AAAI'11]
Divide(scoring rule*)	NP-complete
Sequential (Plurality)	NP-complete

Hard in theory!

Hard in theory!
Hard in practice?

Thank you!